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Coefficient inequalities for certain subclasses of multivalent functions associated with conic domain

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Abstract

A number of families of q-extensions of analytic functions in the open unit disk $\mathbb{U}$ have been defined by means of basic (or q-)calculus and considered from many distinctive prospectives and viewpoints. In this paper, we generalize and study certain subclasses of analytic functions involving higher-order q-derivative operators. We settle characteristic equations for these presumably new classes and also study numerous coefficient inequalities. For the results obtained in this presentation, we also carry out appropriate connections with those in multiple other concerning works on this subject.

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1 Introduction and definitions

By $\mathcal{A}(p)$ we denote the class of functions with series representation

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{n+p} z^{n+p} \quad \bigl(p \in \mathbb{N} = \{1, 2, \ldots\}\bigr), \tag{1.1}$$

which is analytic and p-valent in the open unit disk

$$\mathbb{U} = \bigl\{z : z \in \mathbb{C} \text{ and } |z| < 1\bigr\}.$$

In particular, we denote

$$\mathcal{A} := \mathcal{A}(1).$$

Moreover, we denote by $\mathcal{S} \subset \mathcal{A}$ the class of all univalent functions in the unit disk $\mathbb{U}$.

For two analytic functions f_j, $j = 1, 2$, in $\mathbb{U}$, a function f_1 is said to be subordinate to the function f_2 and write as

$$f_1 \prec f_2 \quad \text{or} \quad f_1(z) \prec f_2(z)$$

if in $\mathbb{U}$, we can find an analytic Schwarz function w with

$$w(0) = 0 \quad \text{and} \quad \left|w(z)\right| < 1$$

such that

$$f_1(z) = f_2\bigl(w(z)\bigr).$$

Further, if the function f_2 is univalent in $\mathbb{U}$ then following equivalence relation holds true

$$f_1(z) \prec f_2(z) \quad (z \in \mathbb{U}) \quad \Rightarrow \quad f_1(0) = f_2(0) \quad \text{and} \quad f_1(\mathbb{U}) \subset f_2(\mathbb{U}).$$

The noteworthy class of Carathéodory functions $\mathcal{P}$ consists of all analytic functions ψ in $\mathbb{U}$ normalized by

$$\psi(z) = 1 + \sum_{n=1}^{\infty} \psi_n z^n \tag{1.2}$$

and satisfying

$$\Re\bigl(\psi(z)\bigr) > 0 \quad (\forall z \in \mathbb{U}).$$

For a convex function f, it is always true that the image of f under $\mathbb{U}$ and all circles lying within $\mathbb{U}$ centered at the origin are convex arcs. But justification is required whether the characteristic still holds for circles with center at any other point, say ξ. Goodman [4, 5] answered negatively and defined uniformly convex and starlike functions that have this nice characteristic. Analytically, he defined uniformly convex and starlike functions, respectively, as

$$\Re\left\{1 + \frac{(z-\xi)f''(z)}{f'(z)}\right\} > 0 \quad (z \in \mathbb{U})$$

and

$$\Re\left\{1 + \frac{(z-\xi)f'(z)}{f(z) - f(\xi)}\right\} > 0 \quad (z \in \mathbb{U}).$$

We denote the former by $\mathcal{UCV}$ and the later by $\mathcal{US}^*\mathcal{T}$. It is natural to ask whether classical Alexander's result holds for these two classes, but there are counterexamples [4], which show that the relation is not true for these classes. Rønning [24], using $\mathcal{UCV}$, introduced the class

$$\mathcal{S}^*\mathcal{T} = \bigl\{f \in \mathcal{A} : f(z) = zg'(z), g(z) \in \mathcal{UCV}\bigr\}$$

and succeeded in proving that neither $\mathcal{S}^*\mathcal{T} \not\subset \mathcal{US}^*\mathcal{T}$ nor $\mathcal{US}^*\mathcal{T} \not\subset \mathcal{S}^*\mathcal{T}$. Ultimately, Rønning [23] and Ma and Minda [14] introduced the following one-variable characterization of these classes.

Definition 1 Let $f \in \mathcal{A}$. Then $f \in \mathcal{UCV}$ if

$$\Re\left\{\frac{(zf'(z))'}{f'(z)}\right\} > \left|\frac{zf''(z)}{f'(z)}\right| \quad (z \in \mathbb{U}).$$

Definition 2 Let $f \in \mathcal{A}$. Then $f \in \mathcal{S}^*\mathcal{T}$ if

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \left|\frac{zf'(z)}{f(z)} - 1\right| \quad (z \in \mathbb{U}).$$

This led to the basis for conic domains introduced by Kanas and Wiśniowska [10, 11] as

$$\Omega_k = \left\{u + iv : u > k\sqrt{(u-1)^2 + v^2}\right\}, \quad k \geqq 0. \tag{1.3}$$

These domains represent the right half-plane, a parabola, a hyperbola, and an ellipse for $k = 0$, $k = 1$, $0 < k < 1$, and $k > 1$, respectively.

The role of an extremal function is done by the function $p_k(z)$ given by

$$p_k(z) = \begin{cases} \frac{1+z}{1-z} & (k = 0), \\ 1 + \frac{2}{\pi^2}(\log\frac{1+\sqrt{z}}{1-\sqrt{z}})^2 & (k = 1), \\ 1 + \frac{2}{1-k^2}\sinh^2\{(\frac{2}{\pi}\arccos k)\arctan h\sqrt{z}\} & (0 \leq k < 1), \\ 1 + \frac{1}{k^2-1}[1 + \sin(\frac{\pi}{2K(\kappa)}\int_0^{\frac{u(z)}{\sqrt{\kappa}}} \frac{dt}{\sqrt{1-t^2}\sqrt{1-\kappa^2t^2}})] & (k > 1), \end{cases} \tag{1.4}$$

where

$$u(z) = \frac{\sqrt{\kappa} - z}{\sqrt{\kappa}z - 1} \quad (\forall z \in \mathbb{U}),$$

and $\kappa \in (0, 1)$ is chosen so that

$$k = \cosh\bigl(\pi K'(\kappa)/\bigl(4K(\kappa)\bigr)\bigr),$$

where $K(\kappa)$ denotes the first-kind Legendre complete elliptic integral, and its derivative is given by

$$K'(\kappa) = K\bigl(\sqrt{1-\kappa^2}\bigr)$$

and called the complementary integral of $K(t)$. If we set the function

$$p_k(z) = 1 + \delta_k z + \cdots,$$

subsequently, by [9] it is obvious that from (1.4) we have

$$\delta_k = \begin{cases} \frac{8(\arccos k)^2}{\pi(1-k^2)} & (0 \leq k < 1), \\ \frac{8}{\pi^2} & (k = 1), \\ \frac{\pi^2}{4k^2(1+t)\sqrt{t}R^2(t)} & (k > 1). \end{cases} \tag{1.5}$$

Subjected to the above-mentioned conic domain, we define some elementary classes.

Definition 3 Let f be a function from the functional class $\mathcal{A}$. Then $f \in k\text{-}\mathcal{UCV}$ if

$$\Re\left\{\frac{(zf'(z))'}{f'(z)}\right\} > k\left|\frac{zf''(z)}{f'(z)}\right| \quad (\forall z \in \mathbb{U} \text{ and } k \geq 0).$$

Definition 4 A normalized analytic function f belongs to the class $k\text{-}\mathcal{S}^*\mathcal{T}$ if

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > k\left|\frac{zf'(z)}{f(z)} - 1\right| \quad (\forall z \in \mathbb{U} \text{ and } k \geq 0).$$

Definition 5 Let f be a function from the functional class $\mathcal{A}$. Then $f \in k\text{-}\mathcal{UQ}$ if there exists a function $g \in k\text{-}\mathcal{UCV}$ such that

$$\Re\left\{\frac{(zf'(z))'}{g'(z)}\right\} > k\left|\frac{zf''(z)}{g'(z)}\right| \quad (\forall z \in \mathbb{U} \text{ and } k \geq 0).$$

Definition 6 A normalize analytic function f belongs to the class $k\text{-}\mathcal{UK}$ if there exists a function $g \in k\text{-}\mathcal{S}^*\mathcal{T}$ such that

$$\Re\left\{\frac{zf'(z)}{g(z)}\right\} > k\left|\frac{zf'(z)}{g(z)} - 1\right| \quad (\forall z \in \mathbb{U} \text{ and } k \geq 0).$$

Now we recall some firm footing concept details and definitions of the q-difference calculus, which play a vital role in our presentation. Unless otherwise notified, we presume that $0 < q < 1$ and $p \in \mathbb{N} = \{1, 2, 3, \ldots\}$. For a nonnegative number λ, the q-number $[\lambda]_q$ is defined by

$$[\lambda]_q = \sum_{j=0}^{j-1} q^j = 1 + q + q^2 + \cdots + q^{j-1}, \qquad [0]_q = 0.$$

In general, for $\lambda \in \mathbb{C}$, we have $[\lambda]_q = \frac{1-q^\lambda}{1-q}$. The q-factorials $[j]_q!$ are defined by $[0]_q! = 0$ and $[j]_q! = \prod_{k=1}^{j}[k]_q$. It is straightforward to observe that $\lim_{q\to 1-}[\lambda]_q = \lambda$ and $\lim_{q\to 1-}[j]_q! = j!$.

Definition 7 (See [7] and [8]) For a function f from class $\mathcal{A}$, the q-derivative (or q-difference) operator D_q in a subset of complex numbers $\mathbb{C}$ is defined by

$$(D_q f)(z) = \begin{cases} \frac{f(z)-f(qz)}{(1-q)z} & (z \neq 0), \\ f'(0) & (z = 0), \end{cases} \tag{1.6}$$

provided that $f'(0)$ exists.

We observe from Definition 7 that

$$\lim_{q\to 1-}(D_q f)(z) = \lim_{q\to 1-}\frac{f(z)-f(qz)}{(1-q)z} = f'(z)$$

for a differentiable function f in a subset of $\mathbb{C}$. Further, by (1.1) and (1.6) we obtain

$$\bigl(D_q^{(1)}f\bigr)(z) = [p]_q z^{p-1} + \sum_{n=1}^{\infty}[n+p]_q a_{n+p} z^{n+p-1}, \tag{1.7}$$

$$\bigl(D_q^{(2)}f\bigr)(z) = [p]_q[p-1]_q z^{p-2} + \sum_{n=1}^{\infty}[n+p]_q[n+p-1]_q a_{n+p} z^{n+p-2}, \tag{1.8}$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$\bigl(D_q^{(p)}f\bigr)(z) = [p]_q! + \sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n]_q!} a_{n+p} z^{n}, \tag{1.9}$$

where $(D_q^{(p)}f)(z)$ is the pth q-derivative of $f(z)$.

Recently, the studies of q-calculus have inspired an intense interest of researchers because of its advantages in many areas of mathematics and physics. The significance of the operator D_q is quite obvious by its applications in the study of several subclasses of analytic functions. Initially, in 1990, Ismail et al. [6] gave the idea of q-extension of the class of starlike functions; nevertheless, a foothold usage of the q-calculus in the context of geometric function theory was effectively invoked by Srivastava [26]. After that, wonderful studies have been done by numerous mathematicians offering a momentous part in the advancement of geometric function theory. In particular, the study the q-Mittag-Leffler functions for close-to-convex functions was done by Srivastava and Bansal [30] (see also [21]). In [32], they also considered the functional class of q-starlike functions related to conic region σ_k, where the estimate of the third Hankel determinant has been settled in [17] (see also [28]). Recently, Srivastava et al. (see, e.g., [15, 31, 34, 35] published a set of papers, in which they concentrated on the class of q-starlike functions related to the Janowski functions from different aspects. For some more recent investigations on q-calculus, we refer to [13, 16, 27, 29, 33]. In this paper, we mainly generalize the work presented in Srivastava et al. [32].

Definition 8 (See [6]) A function $f \in \mathcal{A}$ belongs to the functional class $\mathcal{S}_q^*$ if

$$f(0) = f'(0) - 1 = 0 \tag{1.10}$$

and

$$\left|\frac{z}{f(z)}(D_q f)z - \frac{1}{1-q}\right| \le \frac{1}{1-q}. \tag{1.11}$$

In view of the last inequality, it is obvious that, in the limiting case $q \to 1-$,

$$\left|w - \frac{1}{1-q}\right| \le \frac{1}{1-q},$$

the above closed disk is merely the right-half plane, and the class $\mathcal{S}_q^*$ of q-starlike functions turns into the prominent class $\mathcal{S}^*$. Analogously, by the principle of subordination we may express relations (1.10) and (1.11) as follows (see [36]):

$$\frac{z}{f(z)}(D_qf)(z) \prec \widehat{p}(z) \quad \left(\widehat{p}(z) = \frac{1+z}{1-qz}\right).$$

The notation $\mathcal{S}_q^*$was first used by Sahoo et al. [25].

Remark 1 In defining the class $\mathcal{C}_q$ of q-convex functions, the most important Alexander theorem [3] for functions $f \in \mathcal{A}$ was used by Baricz and Swaminathan [2] as

$$f(z) \in \mathcal{C}_q \quad \Longleftrightarrow \quad z(D_qf)(z) \in \mathcal{S}_q^*.$$

The generalization of $\mathcal{P}(p_k)$ to k-$\mathcal{P}_q$ presented in Definition 9 is due to Srivastava et al. [32]. He used the conic domain and the earlier discussed q-calculus as follows.

Definition 9 A function $\psi \in \mathcal{P}$ belongs to the class k-$\mathcal{P}_q$ if the following relation holds:

$$\psi(z) \prec \frac{2p_k(z)}{(1+q)+(1-q)p_k(z)}$$

with the function $p_k(z)$ given in equation (1.4).

It is interesting that in geometric characteristics the function $\psi \in k$-$\mathcal{P}_q$ takes all values from the domain $\Omega_{k,q}$, $k \geq 0$, analytically given by

$$\Omega_{k,q} = \left\{w : \Re\left(\frac{(1+q)w}{(q-1)w+2}\right) > k\left|\frac{(1+q)w}{(q-1)w+2} - 1\right|\right\},$$

which represent q-analogues of generalized conic regions.

Note the following easily observable facts about the class k-$\mathcal{P}_q$.

Remark 2 Firstly, we see that

$$k\text{-}\mathcal{P}_q \subseteq \mathcal{P}\left[\frac{2k}{2k+1+q}\right],$$

where $\mathcal{P}[\frac{2k}{2k+1+q}]$ is the famous class of functions with real parts greater than $\frac{2k}{2k+1+q}$. Secondly, we have

$$\lim_{q\longrightarrow 1-} k\text{-}\mathcal{P}_q = \mathcal{P}(p_k),$$

where $\mathcal{P}(p_k)$ is the familiar class given by Wisniowska and Kanas [10]. Thirdly, by taking the limit we obtain

$$\lim_{q\longrightarrow 1-} 0\text{-}\mathcal{P}_q = \mathcal{P},$$

where $\mathcal{P}$ is the class of functions given by (1.2).

Using the q-differential operator, various new classes have been defined. Hence it is natural to give the following definition.

Definition 10 A function $f \in \mathcal{A}$ is said to belong to the class k-$\mathcal{S}_q^*(p)$ if

$$\Re\left(\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}+2}\right) > k\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}+2}-1\right|$$

or, equivalently,

$$\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)} \in k\text{-}\mathcal{P}_q.$$

Remark 3 First of all, it is straightforward to see that

$$0\text{-}\mathcal{S}_q^*(1) = \mathcal{S}_q^*,$$

where the functional class $\mathcal{S}_q^*$ was considered and analyzed by Ismail et al. [6]. Secondly, we easily observe that

$$\lim_{q\longrightarrow 1-} k\text{-}\mathcal{S}_q^*(1) = k\text{-}\mathcal{S}^* = k\text{-}\mathcal{S}^*\mathcal{T},$$

where the class k-$\mathcal{S}^*\mathcal{T}$ was presented and studied by Kanas and Wiśniowska [11]. Thirdly,

$$k\text{-}\mathcal{S}_q^*(1) = k\text{-}\mathcal{S}_q^*,$$

where the function class k-$\mathcal{S}_q^*$ was initially considered and studied by Srivastava et al. [32]. Finally,

$$\lim_{q\longrightarrow 1-} 0\text{-}\mathcal{S}_q^*(1) = \mathcal{S}^*,$$

where $\mathcal{S}^*$ is the essential class of starlike functions.

Definition 11 Just as in Remark 1, by the Alexander theorem [3] the class k-$\mathcal{C}_q$ can be defined by the following relation:

$$f(z) \in k\text{-}\mathcal{C}_q(p) \quad\Longleftrightarrow\quad \frac{z}{p!}\big(D_q^{(p)}f\big)(z) \in k\text{-}\mathcal{S}_q^*(p).$$

Definition 12 Any function $f \in \mathcal{A}(p)$ is said to belong to the class k-$\mathcal{K}_q(p)$ if

$$\Re\left(\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}\right) > k\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}-1\right|$$

or, equivalently,

$$\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)} \in k\text{-}\mathcal{P}_q$$

for some $g \in k\text{-}\mathcal{S}_q^*(p)$.

Definition 13 In similar manner as in Remark 1, by using the idea of Alexander's relation [3] we define the class $k\text{-}\mathcal{C}_q^*(p)$ by the following relation:

$$f(z) \in k\text{-}\mathcal{C}_q^*(p) \quad \Longleftrightarrow \quad \frac{z}{p!}\big(D_q^{(p)}f\big)(z) \in k\text{-}\mathcal{K}_q(p).$$

Remark 4 First of all, we can see that

$$0\text{-}\mathcal{K}_q(1) = \mathcal{K}_q,$$

where $\mathcal{K}_q(p)$ is the function class defined and examined by Raghavendar et al. [20]. Secondly, in the limit case, we have

$$\lim_{q\longrightarrow 1-} k\text{-}\mathcal{K}_q(1) = k\text{-}\mathcal{K} = k\text{-}\mathcal{UK} \quad \text{and} \quad \lim_{q\longrightarrow 1-} k\text{-}\mathcal{C}_q^*(1) = k\text{-}\mathcal{C}^* = k\text{-}\mathcal{UQ},$$

where $k\text{-}\mathcal{UK}(p)$ and $k\text{-}\mathcal{UQ}(p)$ are the function classes introduced and studied by Acu [1]. Thirdly, we have

$$\lim_{q\longrightarrow 1-} 0\text{-}\mathcal{K}_q(1) = \mathcal{K} \quad \text{and} \quad \lim_{q\longrightarrow 1-} 0\text{-}\mathcal{C}_q^*(1) = \mathcal{C}^*,$$

where $\mathcal{C}^*$ and $\mathcal{K}$ are the function classes of quasi-convex and close-to-convex functions; for details, see [12, 19].

2 A set of lemmas

Each of the lemmas given further will be helpful in demonstrating our main results.

Lemma 1 ([22]) *Let ψ be a function of the form*

$$\psi(z) = 1 + \sum_{n=1}^{\infty} \psi_n z^n$$

subordinate to a function $\mathfrak{H}$ of the form

$$\mathfrak{H}(z) = 1 + \sum_{n=1}^{\infty} C_n z^n.$$

In particular, when $\mathfrak{H}$ is univalent in the unit disk $\mathbb{U}$ and $\mathcal{H}(\mathbb{U})$ is convex, then

$$|\psi_n| \leq |C_1| \quad (n \in \mathbb{N}).$$

Lemma 2 *Suppose that the sequence* $\{a_k\}_{k=0}^{\infty}$ *is defined by*

$$a_p = 1$$

and

$$a_{n+p} = \frac{\delta_k(q+1)[n+1]_q!}{2[n+p]_q!([n+1]_q - 1)} \sum_{l=1}^{n-1} \frac{[p+l]_q!}{[l+1]_q!} a_{p+l}. \tag{2.1}$$

Then

$$a_{n+p-1} = \prod_{j=2}^{n} \frac{[j]_q\{2([j]_q - 1) + \delta_k(q+1)\}}{2\{[j]_q - 1\}[j+p-1]_q}.$$

Proof By (2.1) we easily get

$$\frac{[n+p]_q!}{[n+1]_q!}\big([n+1]_q - 1\big)a_{n+p} = \sum_{l=1}^{n} \frac{[n+p-l]_q!}{[n+1-l]_q!} a_{n+p-l}c_l \tag{2.2}$$

and

$$\frac{[n+p-1]_q!}{[n+1]_q!}\big([n+1]_q - 1\big)a_{n+p-1} = \sum_{l=1}^{n} \frac{[n+p-1-l]_q!}{[n+1-l]_q!} a_{n+p-1-l}c_l. \tag{2.3}$$

Combining (2.2) and (2.3), we obtain

$$\frac{a_{n+p}}{a_{n+p-1}} = \frac{[n+1]_q\{2([n]_q - 1) + \delta_k(1+q)\}}{2\{\{[n+1]_q - 1\}[n+p]_q}.$$

Similarly, we deduce the following result:

$$a_{n+p-1} = \frac{a_{n+p-1}}{a_{n+p-2}} \cdot \frac{a_{n+p-2}}{a_{n+p-3}} \cdot \dots \cdot \frac{a_{p+2}}{a_{p+1}} \cdot \frac{a_{p+1}}{a_p} \cdot a_p.$$

This completes the proof of Lemma 2. □

3 Main results

In this section, we prove our main results. We assume that

$$k \geq 0 \quad \text{and} \quad q \in (0,1).$$

Theorem 1 *Let f be a p-valently analytic function of the form* (1.1). *Then f belongs to the class* $k\text{-}\mathcal{S}_q^*$ *if it satisfies the condition*

$$\sum_{n=1}^{\infty} \frac{[n+p]_q!}{[n+1]_q!} \Lambda_3 |a_{n+p}| < (1+q), \tag{3.1}$$

where

$$\Lambda_3 = 2(k+1)q\big([n+1]_q - 1\big) + \big|(q-1)[n+1]_q + 2\big|. \tag{3.2}$$

Proof If (3.1) holds, then it suffices to establish the inequality

$$k\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{D_q^{(p-1)}f(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{D_q^{(p-1)}f(z)}+2}-1\right|-\Re\left[\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{D_q^{(p-1)}f(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{D_q^{(p-1)}f(z)}+2}-1\right]<1.$$

Since

$$\begin{aligned}
&k\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}+2}-1\right|-\Re\left[\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}+2}-1\right]\\
&\leq(k+1)\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}+2}-1\right|\\
&=2(k+1)\left|\frac{z(D_q^{(p)}f)(z)-(D_q^{(p-1)}f)(z)}{(q-1)z(D_q^{(p)}f)(z)+2(D_q^{(p-1)}f)(z)}\right|\\
&=2(k+1)\left|\frac{\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}([n+1]_q-1)a_{n+p}z^{n+1}}{(q+1)[p]_q!+\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}\{(q-1)[n+1]_q+2\}a_{n+p}z^{n+1}}\right|\\
&\leq 2\frac{\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}([n+1]_q-1)|a_{n+p}|}{(q+1)+\sum_{n=2}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}|(q-1)[n+1]_q+2||a_{n+p}|},
\end{aligned}\tag{3.3}$$

the upper bound of the relation given by (3.3) is unity if

$$\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}\Lambda_3\cdot|a_{n+p}|<(1+q),$$

where Λ_3 is given by (3.2). Consequently, proof is completed. □

If in Theorem 3, we put $p=1$ and $q\longrightarrow 1-$, then we get the following result.

Corollary 1 (See [11]) *Any function $f\in\mathcal{A}$ of the form (1.1) belongs to the class k-$\mathcal{S}^*\mathcal{T}$ if it satisfies the inequality*

$$\sum_{n=1}^{\infty}\{(k+1)n+1\}|a_n|<1.$$

Theorem 2 *A function $f\in\mathcal{A}(p)$ of the form (1.1) belongs to the function class k-$\mathcal{C}_q(p)$ if*

$$\sum_{n=2}^{\infty}\frac{[n+p]_q!}{[n]_q!}\Lambda_3|a_{n+p}|<(1+q),$$

where Λ_3 is defined in (3.2).

Proof We omit the details of the proof, since it easily follows by applying Theorem 1 in conjunction with Definition 11. □

Theorem 3 *A function $f \in \mathcal{A}(p)$ having series expansion* (1.1) *belongs to the class k-$\mathcal{K}_q^*(p)$ if*

$$\sum_{n=2}^{\infty}\left\{2(k+1)\Lambda_1+\Lambda_2\right\}<(1+q), \tag{3.4}$$

where

$$\Lambda_1=\left|b_{n+p}-[n+1]_q a_{n+p}\right| \tag{3.5}$$

and

$$\Lambda_2=\left|(1-q)[n+1]_q a_{n+p}-2b_{n+p}\right|. \tag{3.6}$$

Proof Assuming that (3.4) holds, it suffices to check that

$$k\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}-1\right|-\Re\left[\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}-1\right]<1.$$

We have

$$\begin{aligned}
&k\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}-1\right|-\Re\left[\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}-1\right]\\
&\quad\le(k+1)\left|\frac{(1+q)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}}{(q-1)\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)}+2}-1\right|\\
&\quad=2(k+1)\left|\frac{z(D_q^{(p)}f)(z)-2(D_q^{(p-1)}g)(z)}{(q-1)z(D_q^{(p)}f)(z)+2(D_q^{(p-1)}g)(z)}\right|\\
&\quad=2(k+1)\left|\frac{\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}\{[n+1]_q a_{n+p}-b_{n+p}\}z^{n+1}}{(q+1)[p]_q!z+\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}\{(q-1)[n+1]_q a_{n+p}+2b_{n+p}\}z^{n+1}}\right|\\
&\quad\le\frac{2(k+1)\sum_{n=1}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}|[n+1]_q a_{n+p}-b_{n+p}|}{(q+1)[p]_q!-\sum_{n=2}^{\infty}\frac{[n+p]_q!}{[n+1]_q!}|(1-q)[n+1]_q a_{n+p}-2b_{n+p}|}.
\end{aligned}\tag{3.7}$$

The last expression in (3.7) is bounded above by 1 if

$$\sum_{n=1}^{\infty}\left\{2(k+1)\Lambda_1+\Lambda_2\right\}<(1+q),$$

where Λ_1 and Λ_2 are given by (3.5) and (3.6), respectively, which completes the proof. □

Theorem 4 *A function f from the class $\mathcal{A}(p)$ having series expansion* (1.1) *belongs to the class $k\text{-}\mathcal{C}_q^*(p)$ if*

$$\sum_{n=1}^{\infty}[n+1]_q\left\{2(k+1)\Lambda_1+\Lambda_2\right\}<(1+q),$$

where Λ_1 and Λ_2 are respectively presented in (3.5) *and* (3.6).

Proof The proof of Theorem 4 follows easily by using Theorem 3 and Definition 13. □

Theorem 5 *Let $f \in k\text{-}\mathcal{S}_q^*(p)$ be of the form* (1.1). *Then*

$$|a_{n+p-1}| \leq \prod_{j=2}^{n} \frac{[j]_q\{2([j-1]_q-1)+(q+1)\delta_k\}}{2\{[j]_q-1\}[j+p-1]_q} \quad \bigl(n \in \mathbb{N}\setminus\{1\}\bigr). \tag{3.8}$$

Proof For $f \in k\text{-}\mathcal{S}_q^*(p)$, by definition we obtain

$$\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}f)(z)}=\psi(z), \tag{3.9}$$

where

$$\psi(z) \prec \frac{2p_k(z)}{(1+q)+(1-q)p_k(z)}. \tag{3.10}$$

If

$$p_k(z)=1+\delta_k z+\cdots,$$

then after some appropriate computations, condition (3.10) can be written as

$$\psi(z) \prec 1+\frac{(1+q)}{2}\delta_k z+\cdots, \tag{3.11}$$

and if

$$\psi(z)=1+\sum_{n=1}^{\infty}\psi_n z^n, \tag{3.12}$$

then by application of Lemma 1 in conjunction with (3.11) and (3.12) we have

$$|\psi_n| \leq \frac{(1+q)}{2}|\delta_k| \quad (n \in \mathbb{N}). \tag{3.13}$$

Now from (3.9) we set

$$z\bigl(D_q^{(p)}f\bigr)(z)=\bigl(D_q^{(p-1)}f\bigr)(z)\psi(z),$$

which implies that

$$[p]_q!z + \sum_{n=1}^{\infty} \frac{[n+p]_q!}{[n+1]_q!} a_{n+p} z^{n+1} = \left([p]_q!z + \sum_{n=2}^{\infty} \frac{[n+p]_q!}{[n+1]_q!} a_{n+p} z^{n+1} \right) \cdot \left(1 + \sum_{n=1}^{\infty} \psi_n z^n \right).$$

Now the comparison of the corresponding coefficients of z^n gives

$$\frac{[n+p]_q!}{[n+1]_q!} \big([n+1]_q - 1 \big) a_{n+p} = \sum_{l=1}^{n} \frac{[n+p-l]_q!}{[n+1-l]_q!} a_{n+p-l} c_l, \quad a_p = 1.$$

Equivalently,

$$|a_{n+p}| \le \frac{[n+1]_q!}{[n+p]_q! ([n+1]_q - 1)} \sum_{l=1}^{n} \frac{[n+p-l]_q!}{[n+1-l]_q!} |a_{n+p-l}||c_l|, \quad a_p = 1.$$

Moreover, by (3.13) we have

$$|a_{n+p}| \le \frac{\delta_k (q+1)[n+1]_q!}{2[n+p]_q! ([n+1]_q - 1)} \sum_{l=1}^{n-1} \frac{[p+l]_q!}{[l+1]_q!} |a_{p+l}|, \quad a_p = 1. \tag{3.14}$$

Now, using Lemma 2, we have

$$|a_{n+p-1}| \le \prod_{j=2}^{n} \frac{[j]_q \{2([j]_q - 1) + \delta_k (q+1)\}}{2\{[j]_q - 1\}[j+p-1]_q}. \tag{3.15}$$

□

Specifically, for instance, setting $p = 1$ and letting $q \longrightarrow 1-$, we obtain the estimate on the nth coefficient of the class k-$\mathcal{S}^*\mathcal{T}$, settled by Wisniowska and Kanas as follows.

Corollary 2 (See [11]) *For an analytic function $f \in k$-$\mathcal{S}^*\mathcal{T}$, we have*

$$|a_n| \le \prod_{j=0}^{n-2} \frac{j(j-2) + \delta_k}{(j-1)(j)} \quad \big(n \in \mathbb{N} \setminus \{1\} \big).$$

Theorem 6 *Let $f \in k$-$\mathcal{C}_q(p)$ be of the form (1.1). Then*

$$|a_{n+p-1}| \le \frac{1}{[n+p]_q} \prod_{j=2}^{n} \frac{[j]_q \{2([j-1]_q - 1) + (q+1)\delta_k\}}{2\{[j]_q - 1\}[j+p-1]_q} \quad \big(n \in \mathbb{N} \setminus \{1\} \big).$$

Proof The proof of Theorem 6 follows easily by using Definition 11 and Theorem 5. □

Theorem 7 *Let $f \in k\text{-}\mathcal{K}_q^*$ be of the form (1.1). Then*

$$|a_{n+p-1}| \leq \frac{[n]_q!}{[n+p]_q!}\left(\prod_{j=0}^{n-2}\frac{[j]_q\{2([j-1]_q-1)+\delta_k(q+1)\}}{2\{[j]_q-1\}[j+p-1]_q}\right) + \frac{(q+1)|\delta_k|}{2[n]_q}\sum_{j=1}^{n-1}\prod_{j=0}^{n-2}\frac{|\delta_k(q+1)+2q[j]_q|}{2q[j+1]_q} \quad \bigl(n \in \mathbb{N}\setminus\{1\}\bigr). \tag{3.16}$$

Proof By definition, for a function f belonging to $k\text{-}\mathcal{K}_q(p)$, we have that

$$\frac{z(D_q^{(p)}f)(z)}{(D_q^{(p-1)}g)(z)} = \psi(z), \tag{3.17}$$

where

$$g(z) = z^p + \sum_{n=1}^{\infty} b_{n+p}z^{n+p},$$

and that

$$\psi(z) \prec \frac{2p_k(z)}{(1+q)+(1-q)p_k(z)}. \tag{3.18}$$

If

$$p_k(z) = 1 + \delta_k z + \cdots,$$

then after some convenient computations, condition (3.18) can be written as

$$\psi(z) \prec 1 + \frac{(1+q)}{2}\delta_k z + \cdots, \tag{3.19}$$

and if

$$\psi(z) = 1 + \sum_{n=1}^{\infty}\psi_n z^n, \tag{3.20}$$

then applying Lemma 1 in conjunction with (3.19) and (3.20), we get

$$|\psi_n| \leq \frac{(1+q)}{2}|\delta_k| \quad (n \in \mathbb{N}). \tag{3.21}$$

Next, equation (3.17) may be written as

$$z\bigl(D_q^{(p)}f\bigr)(z) = \bigl(D_q^{(p-1)}g\bigr)(z)\psi(z),$$

and using the series form, we get

$$[p]_q!z + \sum_{n=1}^{\infty} \frac{[n+p]_q!}{[n+1]_q!} a_{n+p} z^{n+1} = \left([p]_q!z + \sum_{n=2}^{\infty} \frac{[n+p]_q!}{[n+1]_q!} b_{n+p} z^{n+1} \right) \cdot \left(1 + \sum_{n=1}^{\infty} \psi_n z^n \right).$$

Next, the comparison of the corresponding coefficients of z^n yields

$$\frac{[n+p]_q!}{[n+1]_q!} \left([n+1]_q a_{n+p} - b_{n+p} \right) = \sum_{l=1}^{n} \frac{[n+p-l]_q!}{[n+1-l]_q!} a_{n+p-l} c_l \quad (a_p = 1).$$

This implies that

$$\frac{[n+p]_q!}{[n+1]_q!} [n+1]_q |a_{n+p}| \leq |b_{n+p}| + \sum_{j=1}^{n-1} |a_{n+p-l}||c_l| \quad (a_1 = 1).$$

Moreover, using Theorem 5 and (3.21), we get

$$|a_{n+p}| \leq \frac{[n]_q!}{[n+p]_q!} \left(\prod_{j=0}^{n-2} \frac{[j]_q \{2([j-1]_q - 1) + \delta_k (q+1)\}}{2\{[j]_q - 1\}[j+p-1]_q} \right) + \frac{[n+1]_q!(q+1)|\delta_k|}{2[n+1]_q [n+p]_q!} \sum_{j=1}^{n-1} \prod_{j=0}^{n-2} \frac{[j]_q \{2([j-1]_q - 1) + \delta_k (q+1)\}}{2\{[j]_q - 1\}[j+p-1]_q}.$$

Thus we have proved the statement of Theorem 7. □

Putting $p = 1$ and letting $q \longrightarrow 1-$ in Theorem 7, we obtain the estimates of the nth coefficients of the functions from the class $k\text{-}\mathcal{UK}$, given by Noor et al.

Corollary 3 (See [18]) *Let $f \in k\text{-}\mathcal{UK}$ be of the form (1.1). Then*

$$|a_n| \leq \frac{(|\delta_k|)_{n-1}}{n!} + \frac{|\delta_k|}{n} \sum_{j=1}^{n-1} \frac{(|\delta_k|)_{j-1}}{(j-1)!} \quad \big(n \in \mathbb{N} \setminus \{1\}\big).$$

Further, setting

$$k = 0 = p - 1$$

in Theorem 7, then $\delta_k = 2$, and letting $q \longrightarrow 1-$, we get the known result by Kaplan et al.

Corollary 4 ([12]) *Let $f \in \mathcal{K}$ be an analytic function. Then*

$$|a_n| \leq n \quad \big(n \in \mathbb{N} \setminus \{1\}\big).$$

Theorem 8 *Let $f \in k\text{-}\mathcal{C}_q^*(p)$ with series expansion (1.1). Then*

$$|a_n| \le \frac{[n]_q^2!}{[n+p]_q^2!}\left(\prod_{j=0}^{n-2}\frac{[j]_q\{2([j-1]_q-1)+\delta_k(q+1)\}}{2\{[j]_q-1\}[j+p-1]_q}\right)+\frac{(q+1)|\delta_k|}{2[n+p]_q!}$$
$$\cdot\sum_{j=1}^{n-1}\prod_{j=0}^{n-2}\frac{[j]_q\{2([j-1]_q-1)+\delta_k(q+1)\}}{2\{[j]_q-1\}[j+p-1]_q}\quad\bigl(n\in\mathbb{N}\setminus\{1\}\bigr). \tag{3.22}$$

Proof Using Theorem 7 and Definition 13 immediately yields the proof. □

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