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Coefficient inequalities for a comprehensive class of bi-univalent functions related with bounded boundary variation

Huo Tang[1*], N. Magesh[2], V.K. Balaji[3] and C. Abirami[4]

*Correspondence:
thth2009@tom.com
[1]School of Mathematics and Statistics, Chifeng University, Chifeng, P.R. China
Full list of author information is available at the end of the article

Abstract

In the current work, we delineate a comprehensive class of bi-univalent functions connected with a bounded boundary variation to get the estimates of the first two Taylor–Maclaurin coefficients. In addition, certain special cases and some appealing interpretation of the results presented here are pointed out.

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1 Introduction and definitions

Let $\mathcal{A}$ be the class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disc $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. The Koebe one-quarter theorem [4] ensures that the image of Δ under every univalent function $f \in \mathcal{A}$ contains the disc with the center in the origin and the radius 1/4. Thus, every univalent function $f \in \mathcal{A}$ has an inverse $f^{-1} : f(\Delta) \to \Delta$, satisfying $f^{-1}(f(z)) = z$, $z \in \Delta$, and

$$f\bigl(f^{-1}(w)\bigr) = w, \qquad |w| < r_0(f), \qquad r_0(f) \ge \frac{1}{4}.$$

In addition, it is straightforward to witness that the inverse function has the series expansion

$$f^{-1}(w) = w - a_2 w^2 + \bigl(2a_2^2 - a_3\bigr)w^3 - \bigl(5a_2^3 - 5a_2 a_3 + a_4\bigr)w^4 + \cdots, \quad w \in f(\Delta). \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent, if both f and f^{-1} are univalent in Δ, in the sense that f^{-1} has a univalent analytic continuation to Δ and we denote by Σ this class of bi-univalent functions. Actually, the study of the Taylor–Maclaurin coefficient inequalities for various classes of bi-univalent functions was recently revived by Srivastava et al. [16]. The huge flood of papers (for example) [1, 3, 5, 6, 9, 10, 12–15, 17–19] which emerged essentially from the pioneering work of Srivastava et al. [16]. One could refer [16], the above-mentioned work and the references therein for history, examples and different classes and its subclasses of bi-univalent functions. Recently, Çağlar et al. [2] found the upper bounds for the second Hankel determinant for certain subclasses of analytic and bi-univalent functions and Srivastava et al. [11] used the Faber polynomial expansions to address a new subclass of Σ and obtained bounds for their nth ($n \geq 3$) coefficients subject to a given gap series condition.

Definition 1.1 ([7]) Let $\mathcal{P}_k(\alpha)$, with $k \geq 2$ and $0 \leq \alpha < 1$, denote the class of univalent analytic functions P, normalized with $P(0) = 1$, and satisfying

$$\int_0^{2\pi} \left| \frac{\operatorname{Re} P(z) - \alpha}{1 - \alpha} \right| \, \mathrm{d}\theta \leq k\pi,$$

where $z = re^{i\theta} \in \Delta$.

For $\alpha = 0$, we denote $\mathcal{P}_k := \mathcal{P}_k(0)$, hence the class $\mathcal{P}_k$ corresponds to the class of functions p analytic in Δ, normalized with $p(0) = 1$, and having the expression

$$p(z) = \int_0^{2\pi} \frac{1 - ze^{it}}{1 + ze^{it}} \, \mathrm{d}\,\mu(t), \tag{1.3}$$

where λ is a real-valued function with bounded variation, which ensures

$$\int_0^{2\pi} d\mu(t) = 2\pi \quad \text{and} \quad \int_0^{2\pi} \left| d\mu(t) \right| \leq k, \quad k \geq 2. \tag{1.4}$$

Obviously, $\mathcal{P} := \mathcal{P}_2$ is the celebrated class of Carathéodory functions, that is, the normalized functions with positive real part in the open unit disc Δ.

Definition 1.2 A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

belongs to the class $\mathcal{B}_{\Sigma}^{\lambda,\eta,\delta}(k;\alpha)$, $\lambda \geq 0$, $\delta \geq 1$, $\eta \geq 0$, $k \geq 2$ and $0 \leq \alpha < 1$, if the subsequent conditions are fulfilled:

$$(1 - \delta)\left(\frac{f(z)}{z}\right)^{\lambda} + \delta f'(z)\left(\frac{f(z)}{z}\right)^{\lambda-1} + \nu\eta z f''(z) \in \mathcal{P}_k(\alpha), \quad z \in \Delta, \tag{1.5}$$

and

$$(1 - \delta)\left(\frac{g(w)}{w}\right)^{\lambda} + \delta g'(w)\left(\frac{g(w)}{w}\right)^{\lambda-1} + \nu\eta w g''(w) \in \mathcal{P}_k(\alpha), \quad w \in \Delta, \tag{1.6}$$

where the function $g(w) = f^{-1}(w)$ is defined by (1.2) and $\nu = \frac{2\delta+\lambda}{2\delta+1}$.

It is remarkable that the particular values of λ, δ, η, α and m direct the class $\mathcal{B}_{\Sigma}^{\lambda,\eta,\delta}(k;\alpha)$ to different subclasses, we exhibit the following subclasses:

(1) For $\eta = 0$, we obtain the class $\mathcal{B}_{\Sigma}^{1,0,\delta}(k;\alpha) \equiv \mathcal{N}_{\Sigma}^{\lambda,\delta}(k;\alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{N}_{\Sigma}^{\lambda,\delta}(k;\alpha)$, if

$$(1-\delta)\left(\frac{f(z)}{z}\right)^{\lambda} + \delta f'(z)\left(\frac{f(z)}{z}\right)^{\lambda-1} \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$(1-\delta)\left(\frac{g(w)}{w}\right)^{\lambda} + \delta g'(w)\left(\frac{g(w)}{w}\right)^{\lambda-1} \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

Remark 1.3 For $k = 2$, the class $\mathcal{N}_{\Sigma}^{\lambda,\delta}(2;\alpha) \equiv \mathcal{N}_{\Sigma}^{\lambda,\delta}(\alpha)$ was considered by Çağlar et al. [3].

(2) For $\delta = 1$ and $\eta = 0$, we observe the class $\mathcal{B}_{\Sigma}^{\lambda,0,1}(k;\alpha) \equiv \mathcal{R}_{\Sigma}^{\lambda}(k;\alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{R}_{\Sigma}^{\lambda}(k;\alpha)$, if

$$f'(z)\left(\frac{f(z)}{z}\right)^{\lambda-1} \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$g'(w)\left(\frac{g(w)}{w}\right)^{\lambda-1} \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

Remark 1.4 For $k = 2$, the class $\mathcal{R}_{\Sigma}^{\lambda}(2;\alpha) \equiv \mathcal{R}_{\Sigma}^{\lambda}(\alpha)$ was considered in [8].

(3) For $\lambda = 0$; $\delta = 1$ and $\eta = 0$, we have $\mathcal{B}_{\Sigma}^{1,0,1}(k;\alpha) \equiv \mathcal{S}_{\Sigma}^{*}(k;\alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{S}_{\Sigma}^{*}(k;\alpha)$, if

$$\frac{zf'(z)}{f(z)} \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$\frac{wg'(w)}{g(w)} \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

Remark 1.5 For $k = 2$, we attain the class $\mathcal{S}^*_\Sigma(2;\alpha) \equiv \mathcal{S}^*_\Sigma(\alpha)$.

(4) For $\lambda = 1$, we have the class $\mathcal{B}^{1,\eta,\delta}_\Sigma(k;\alpha) \equiv \mathcal{B}^{\eta,\delta}_\Sigma(k;\alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{B}^{\eta,\delta}_\Sigma(k;\alpha)$, if

$$(1-\delta)\frac{f(z)}{z} + \delta f'(z) + \eta z f''(z) \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$(1-\delta)\frac{g(w)}{w} + \delta g'(w) + \eta w g''(w) \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

(5) For $\delta = \lambda = 1$, we obtain the class $\mathcal{B}^{1,\eta,1}_\Sigma(k;\alpha) \equiv \mathcal{F}_\Sigma(\eta, k;\alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{F}_\Sigma(\eta, k;\alpha)$, if

$$f'(z) + \eta z f''(z) \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$g'(w) + \eta w g''(w) \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

(6) For $\lambda = 1$ and $\eta = 0$, we obtain the class $\mathcal{B}^{1,0,\delta}_\Sigma(k;\alpha), \equiv \mathcal{B}_\Sigma(\delta, k;\alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{B}_\Sigma(\delta, m;\alpha)$, if

$$(1-\delta)\frac{f(z)}{z} + \delta f'(z) \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$(1-\delta)\frac{g(w)}{w} + \delta g'(w) \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

Remark 1.6 For $k = 2$, the class $\mathcal{B}_\Sigma(\delta, 2; \alpha) \equiv \mathcal{B}_\Sigma(\delta; \alpha)$ was considered by Frasin and Aouf [5].

(7) For $\delta = 1$, $\lambda = 1$ and $\eta = 0$, we have the class $\mathcal{B}_\Sigma^{1,0,1}(k; \alpha) \equiv \mathcal{P}_\Sigma(k; \alpha)$. A function $f \in \Sigma$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

is said to be in $\mathcal{P}_\Sigma(m; \alpha)$, if

$$f'(z) \in \mathcal{P}_k(\alpha), \quad z \in \Delta,$$

and for $g(w) = f^{-1}(w)$

$$g'(w) \in \mathcal{P}_k(\alpha), \quad w \in \Delta,$$

holds.

Remark 1.7 For $k = 2$, the class $\mathcal{P}_\Sigma(2; \alpha) \equiv \mathcal{P}_\Sigma(\alpha)$ was introduced and studied by Srivastava et al. [16].

To prove the results discussed in this article, we need the following lemma.

Lemma 1.8 *Let the function $\Phi(z) = 1 + \sum_{n=1}^{\infty} h_n z^n$, $z \in \Delta$, such that $\Phi \in \mathcal{P}_m(\alpha)$. Then*

$$|h_n| \leq k(1-\alpha), \quad n \geq 1.$$

In this study, we stumble on the estimates for the coefficients $|a_2|$ and $|a_3|$ for functions in the subclass $\mathcal{B}_\Sigma^{\lambda,\eta,\delta}(k; \alpha)$. Also, we attain the upper bounds of the Fekete–Szegö inequality by means of the results of $|a_2|$ and $|a_3|$.

2 Main results

In the subsequent theorem, we find the coefficient estimates for functions in $\mathcal{B}_\Sigma^{\lambda,\eta,\delta}(k; \alpha)$.

Theorem 2.1 *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be in the class $\mathcal{B}_\Sigma^{\lambda,\eta,\delta}(k; \alpha)$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{2k(1-\alpha)}{(2\delta+\lambda)(\lambda+1) + 12\nu\eta}}; \frac{k(1-\alpha)}{\delta+\lambda+2\nu\eta}\right\},$$

$$|a_3| \leq \min\left\{\frac{k(1-\alpha)}{2\delta+\lambda+6\nu\eta} + \frac{2k(1-\alpha)}{(2\delta+\lambda)(\lambda+1)+12\nu\eta}, \frac{k(1-\alpha)}{2\delta+\lambda+6\nu\eta} + \frac{k^2(1-\alpha)^2}{(\delta+\lambda+2\nu\eta)^2}\right\},$$

and

$$\left|a_3 - \mu a_2^2\right| \le \frac{k(1-\alpha)}{2\delta + \lambda + 6\nu\eta},$$

where

$$\mu = \frac{(2\delta + \lambda)(\lambda + 3) + 24\nu\eta}{2(2\delta + \lambda + 6\nu\eta)}.$$

Proof Since $f \in \mathcal{B}_{\Sigma}^{\lambda,\eta,\delta}(k;\alpha)$, from Definition 1.2 we have

$$(1-\delta)\left(\frac{f(z)}{z}\right)^{\lambda} + \delta f'(z)\left(\frac{f(z)}{z}\right)^{\lambda-1} + \nu\eta z f''(z) = p(z) \tag{2.1}$$

and

$$(1-\delta)\left(\frac{g(w)}{w}\right)^{\lambda} + \delta g'(w)\left(\frac{g(w)}{w}\right)^{\lambda-1} + \nu\eta w g''(w) = q(w), \tag{2.2}$$

where $p, q \in \mathcal{P}_m(\alpha)$ and $g = f^{-1}$. Using the fact that the functions p and q have the following Taylor expansions:

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots, \quad z \in \Delta, \tag{2.3}$$

$$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \cdots, \quad w \in \Delta, \tag{2.4}$$

and equating the coefficients in (2.1) and (2.2), from (1.2) we obtain

$$(\delta + \lambda + 2\nu\eta)a_2 = p_1, \tag{2.5}$$

$$(2\delta + \lambda)\left[\left(\frac{\lambda-1}{2}\right)a_2^2 + \left(1 + \frac{6\eta}{2\delta+1}\right)a_3\right] = p_2, \tag{2.6}$$

$$-(\delta + \lambda + 2\nu\eta)a_2 = q_1, \tag{2.7}$$

$$(2\delta + \lambda)\left[\left(\frac{\lambda+3}{2} + \frac{12\eta}{2\delta+1}\right)a_2^2 - \left(1 + \frac{6\eta}{2\delta+1}\right)a_3\right] = q_2. \tag{2.8}$$

In view of the fact that $p, q \in \mathcal{P}_m(\alpha)$ and Lemma 1.8, the following inequalities hold:

$$|p_k| \le k(1-\alpha), \qquad |q_k| \le k(1-\alpha), \quad k \ge 1. \tag{2.9}$$

It follows from (2.6) and (2.8), additionally, by means of the inequalities (2.9), that

$$|a_2| \le \sqrt{\frac{2k(1-\alpha)}{(2\delta+\lambda)(\lambda+1) + 12\nu\eta}}. \tag{2.10}$$

From (2.5) and (2.7), we have

$$p_1 = -q_1$$

and

$$a_2^2 = \frac{p_1^2}{(\delta + \lambda + 2\nu\eta)^2}, \tag{2.11}$$

which, by applying (2.9), shows

$$|a_2| \le \frac{k(1-\alpha)}{\delta + \lambda + 2\nu\eta}.$$

Next, combining the above inequality with (2.10), the first inequality of the conclusion is proved.

On the other hand, by subtracting (2.8) from (2.6), we have

$$a_3 = \frac{p_2 - q_2}{2(2\delta + \lambda + 6\nu\eta)} + a_2^2. \tag{2.12}$$

By using (2.10) in (2.12), we show

$$|a_3| \le \frac{k(1-\alpha)}{2\delta + \lambda + 6\nu\eta} + \frac{2k(1-\alpha)}{(2\delta + \lambda)(\lambda + 1) + 12\nu\eta}$$

and using (2.11) in (2.12), we get

$$|a_3| \le \frac{k(1-\alpha)}{2\delta + \lambda + 6\nu\eta} + \frac{k^2(1-\alpha)^2}{(\delta + \lambda + 2\nu\eta)^2}.$$

From (2.8), we have

$$\frac{(2\delta + \lambda)(\lambda + 3) + 24\nu\eta}{2(2\delta + \lambda + 6\nu\eta)} a_2^2 - a_3 = \frac{q_2}{2\delta + \lambda + 6\nu\eta}.$$

Furthermore, using (2.9), we finally deduce

$$\left|a_3 - \mu a_2^2\right| \le \frac{|q_2|}{2\delta + \lambda + 6\nu\eta} \le \frac{k(1-\alpha)}{2\delta + \lambda + 6\nu\eta},$$

where

$$\mu = \frac{(2\delta + \lambda)(\lambda + 3) + 24\nu\eta}{2(2\delta + \lambda + 6\nu\eta)},$$

which completes our proof. □

Remark 2.2 For $k = 2$, the results obtained in Theorem 2.1 improves the results of Yousef et al. [20, Theorem 4.1].

Corollary 2.3 *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be in the class $\mathcal{B}_{\Sigma}^{\lambda,\eta,\delta}(\alpha)$. Then*

$$|a_2| \le \min\left\{\sqrt{\frac{4(1-\alpha)}{(2\delta + \lambda)(\lambda + 1) + 12\nu\eta}}; \frac{2(1-\alpha)}{\delta + \lambda + 2\nu\eta}\right\},$$

$$|a_3| \leq \min\left\{\frac{2(1-\alpha)}{2\delta+\lambda+6\nu\eta} + \frac{4(1-\alpha)}{(2\delta+\lambda)(\lambda+1)+12\nu\eta}, \frac{2(1-\alpha)}{2\delta+\lambda+6\nu\eta} + \frac{4(1-\alpha)^2}{(\delta+\lambda+2\nu\eta)^2}\right\},$$

and

$$\left|a_3 - \frac{(2\delta+\lambda)(\lambda+3)+24\nu\eta}{2(2\delta+\lambda+6\nu\eta)}a_2^2\right| \leq \frac{2(1-\alpha)}{2\delta+\lambda+6\nu\eta}.$$

Corollary 2.4 *Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be in the class $\mathcal{N}_{\Sigma}^{\lambda,\delta}(k;\alpha)$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{2k(1-\alpha)}{(2\delta+\lambda)(\lambda+1)}}; \frac{k(1-\alpha)}{\delta+\lambda}\right\},$$

$$|a_3| \leq \min\left\{\frac{k(1-\alpha)}{2\delta+\lambda} + \frac{2k(1-\alpha)}{(2\delta+\lambda)(\lambda+1)}, \frac{k(1-\alpha)}{2\delta+\lambda} + \frac{k^2(1-\alpha)^2}{(\delta+\lambda)^2}\right\},$$

and

$$\left|a_3 - \frac{\lambda+3}{2}a_2^2\right| \leq \frac{k(1-\alpha)}{2\delta+\lambda}.$$

3 Concluding remarks and observations

In this paper, we investigate the estimates of second and third Taylor–Maclaurin coefficients for a comprehensive class $\mathcal{B}_{\Sigma}^{\lambda,\eta,\delta}(k;\alpha)$ of bi-univalent functions. Also, the corresponding coefficient estimates for functions in the subclasses $\mathcal{R}_{\Sigma}^{\lambda}(k;\alpha)$, $\mathcal{S}_{\Sigma}^{*}(k;\alpha)$, $\mathcal{B}_{\Sigma}^{\eta,\delta}(k;\alpha)$, $\mathcal{F}_{\Sigma}(\eta,k;\alpha)$, $\mathcal{B}_{\Sigma}(\delta,k;\alpha)$ and $\mathcal{P}_{\Sigma}(k;\alpha)$ as mentioned above can be derived easily and so we omit the details. Also, some interesting remarks on the results presented here are given.

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The authors declare that they have no competing interests.

Authors' contributions

All authors equally worked on the results and they read and approved the final manuscript.

Author details

[1]School of Mathematics and Statistics, Chifeng University, Chifeng, P.R. China. [2]Post-Graduate and Research Department of Mathematics, Government Arts College for Men, Krishnagiri, India. [3]Department of Mathematics, L.N. Govt College, Ponneri, India. [4]Faculty of Engineering and Technology, SRM University, Kattankulathur, India.

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