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Analysis of a stochastic predator–prey population model with Allee effect and jumps

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Abstract

This paper is concerned with a stochastic predator–prey model with Allee effect and Lévy noise. First, by the comparison theorem of stochastic differential equations, we prove that the model has a unique global positive solution starting from the positive initial value. Then we investigate the asymptotic pathwise behavior of the model by the generalized exponential martingale inequality and the Borel–Cantelli lemma. Next, we establish the conditions under which predator and prey populations are extinct. Furthermore, we show that the global positive solution is stochastically ultimate bounded under some conditions by using the Bernoulli equation and Chebyshev's inequality. At last, we introduce some numerical simulations to support the main results obtained. The results in this paper generalize and improve the previous related results.

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Keywords: Allee effect; Lévy noise; Exponential martingale inequality; Chebyshev's inequality; Predator–prey

1 Introduction

The dynamic relationship between predators and their preys has been universal in both ecology and mathematical ecology [1, 2]. The classic predator–prey population model is the Lotka–Volterra model established by Alfred James Lotka and Vito Volterra in the 1920s. There are many extensive studies in the literature concerned with the dynamics of the predator–prey models and we here do not mention them in detail. However, for the last decade, the importance of the Allee effect has been recognized. Because of the difficulties in finding mates when the prey population density becomes low, the Allee effect may occur in prey species [3]. For example, this might correspond to the density below which it is so difficult to find a mate that reproduction does not compensate for mortality. In [4], the authors studied the following deterministic predator–prey population with Allee effect on prey:

$$\begin{cases} \frac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)[\frac{bx(t)}{A_1+x(t)} - d_1 - \alpha x(t) - \frac{sy(t)}{1+sh_1x(t)}], \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[\frac{c_1sx(t)}{1+sh_1x(t)} - d_2], \end{cases} \tag{1.1}$$

with initial values $x(0) = x_0$, $y(0) = y_0$. Here $x(t)$ and $y(t)$ represent, respectively, the size of prey and predator population at time t; b is the per capita maximum fertility rate of prey population; d_i ($i = 1,2$) are the per capita death rates of prey and predators, respectively; α denotes the strength of the intra-competition of the prey population; s denotes the effective search rate; h_1 denotes the handling time of predators; and c_1 denotes the conversion efficiency of ingested prey into new predators. The product, $\frac{sx(t)}{1+sh_1x(t)}$, represents the predator's functional response. All the parameters of the model are supposed to be positive constants.

It is well known that populations in nature are inevitably subjected to various types of environmental noise. During the past decades, a great deal of attention has been paid to the study of stochastic biological models (see [5–9]). In [10], the authors perturb the death rates d_1 of prey and d_2 of the predator population in (1.1) with Gaussian white noise and obtain the following stochastic predator–prey population model:

$$\begin{cases} \mathrm{d}x(t) = x(t)[\frac{bx(t)}{A_1+x(t)} - d_1 - \alpha x(t) - \frac{sy(t)}{1+sh_1x(t)}]\,\mathrm{d}t - \sigma_1 x(t)\,\mathrm{d}w_1(t), \\ \mathrm{d}y(t) = y(t)[\frac{c_1sx(t)}{1+sh_1x(t)} - d_2]\,\mathrm{d}t - \sigma_2 y(t)\,\mathrm{d}w_2(t). \end{cases} \tag{1.2}$$

However, biological systems may suffer sudden environmental shocks: such as earthquakes, toxic pollutants, hurricanes and so on. Note that these sudden environmental shocks will cause jumps in the population dynamics. As mentioned in [11], here a jump means a sudden shift on the size of biological population, and the mathematical explanation is that sample paths are not continuous almost surely. It is recognized that stochastic differential equations with Lévy noise are quite suitable to describe such a discontinuous system. Recently, stochastic differential equations with Lévy processes have been researched widely both in theory and their applications [12–16]. References [13] and [14] are devoted to the stochastic Lotka–Volterra population dynamics with jumps, and the former with a stochastic competitive Lotka–Volterra model, and the latter with a general Lotka–Volterra model. In [15], the authors studied the asymptotic behavior of a stochastic population model with Allee effect and Lévy jumps.

Motivated by the above discussion, in this paper, we consider the following stochastic predator–prey model with Allee effect and Lévy jumps:

$$\begin{cases} \mathrm{d}x(t) = x(t-)[\frac{bx(t-)}{A_1+x(t-)} - d_1 - \alpha x(t-) - \frac{sy(t-)}{1+sh_1x(t-)}]\,\mathrm{d}t \\ \qquad - \sigma_1 x(t-)\,\mathrm{d}w_1(t) - \int_\Gamma \gamma_1(u)x(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}y(t) = y(t-)[\frac{c_1sx(t-)}{1+sh_1x(t-)} - d_2]\,\mathrm{d}t - \sigma_2 y(t-)\,\mathrm{d}w_2(t) \\ \qquad - \int_\Gamma \gamma_2(u)y(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \end{cases} \tag{1.3}$$

with initial values $x(0) = x_0$, $y(0) = y_0$. Here $x(t-)$ is the left limit of $x(t)$ and $y(t-)$ is the left limit of $y(t)$. All meanings of the parameters are exact as or similar to those for (1.1) except the following. $w(t) = \{w_1(t), w_2(t)\}$ represents a standard two-dimensional Brownian motion defined on a compete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all P-null sets). $N(\cdot,\cdot)$ is a Poisson counting process with characteristic measure λ on measurable subset Γ of $[0,\infty)$ with $\lambda(\Gamma) < \infty$. Then the compensated Poisson random measure $\widetilde{N}(\mathrm{d}t,\mathrm{d}u) = N(\mathrm{d}t,\mathrm{d}u) - \lambda(\mathrm{d}u)\,\mathrm{d}t$ is a martingale, which is independent of $w(t)$. Usually, the pair (w,N) is called Lévy noise. Throughout this paper, we denote $\mathbb{R}^2_+ = \{(x,y) \in \mathbb{R}^2 : x > 0, y > 0\}$ and assume that $\gamma_i(u) < 1$ are bounded functions on Γ, $i = 1,2$.

The remainder of the present paper is organized as follows. First, in Sect. 2, we prove that this model has a unique global positive solution starting from the positive initial value by the comparison theorem of stochastic differential equations. In Sect. 3, we investigate the asymptotic pathwise behavior of the model by using the generalized exponential martingale inequality and the Borel–Cantelli lemma. Extinction conditions of the population will be established in Sect. 4. In Sect. 5, we show that the global positive solution is stochastically ultimate bounded under some conditions. Section 6 contains numerical results, which are used to demonstrate the effectiveness of the theoretical results in this paper. The paper ends with a conclusion.

2 Existence and uniqueness of positive solution

In this section, by using comparison theorem of stochastic differential equations, we show that system (1.3) has a unique positive global solution with positive initial value. For simplicity, we introduce the following notation:

$$\beta_i = \frac{\sigma_i^2}{2} - \int_\Gamma \bigl[\ln\bigl(1-\gamma_i(u)\bigr) + \gamma_i(u)\bigr]\lambda(\mathrm{d}u), \quad i = 1, 2.$$

Note that $\gamma_i(u) < 1$ on Γ. Thus, for any $u \in \Gamma$, we have $1 - \gamma_i(u) > 0$. By the basic inequality $x - 1 - \ln x \geq 0$ for $x > 0$, we have

$$-\bigl[\gamma_i(u) + \ln\bigl(1-\gamma_i(u)\bigr)\bigr] = \bigl(1-\gamma_i(u)\bigr) - 1 - \ln\bigl(1-\gamma_i(u)\bigr) \geq 0 \quad \text{for } u \in \Gamma.$$

Thus, $\beta_i = \frac{\sigma_i^2}{2} - \int_\Gamma [\ln(1-\gamma_i(u)) + \gamma_i(u)]\lambda(\mathrm{d}u) \geq \frac{\sigma_i^2}{2} > 0$, $i = 1, 2$. Moreover, we assume that there is a constant $K > 0$ such that

$$\int_\Gamma \bigl[\ln\bigl(1-\gamma_i(u)\bigr)\bigr]^2 \lambda(\mathrm{d}u) < K, \quad i = 1, 2. \tag{2.1}$$

Theorem 2.1 *For any given initial value* $(x_0, y_0) \in \mathbb{R}_+^2$, *system* (1.3) *has a unique global positive solution* $(x(t), y(t))$ *on* $[0, \infty)$; *that is*, $(x(t), y(t)) \in \mathbb{R}_+^2$ *with probability one for* $t \in [0, \infty)$.

Proof In order to prove Theorem 2.1, we first consider the following system:

$$\begin{cases} \mathrm{d}X(t) = [\frac{be^{X(t-)}}{A_1 + e^{X(t-)}} - d_1 - \alpha e^{X(t-)} - \frac{se^{Y(t-)}}{1+sh_1e^{X(t-)}} - \beta_1]\,\mathrm{d}t - \sigma_1\,\mathrm{d}w_1(t) \\ \qquad\qquad + \int_\Gamma \ln(1-\gamma_1(u))\tilde{N}(\mathrm{d}t, \mathrm{d}u), \\ \mathrm{d}Y(t) = [\frac{c_1se^{X(t-)}}{1+sh_1e^{X(t-)}} - d_2 - \beta_2]\,\mathrm{d}t - \sigma_2\,\mathrm{d}w_2(t) \\ \qquad\qquad + \int_\Gamma \ln(1-\gamma_2(u))\tilde{N}(\mathrm{d}t, \mathrm{d}u), \end{cases} \tag{2.2}$$

with initial values $X(0) = \ln x_0$, $Y(0) = \ln y_0$. Obviously, the coefficients of (2.2) are locally Lipschitz continuous, and, hence, there is a unique maximal local solution $(X(t), Y(t))$ of system (2.2) on $[0, \tau_e)$, where τ_e represents the explosion time. By $x(t) = e^{X(t)}$, $y(t) = e^{Y(t)}$ and using Itô formula, it follows that $(x(t), y(t)) = (e^{X(t)}, e^{Y(t)})$ is the unique positive local solution of model (1.3) with initial value (x_0, y_0) on $[0, \tau_e)$.

Next, we use comparison theorem of stochastic differential equations to show that $(X(t), Y(t))$ is a global solution to system (2.2), that is $\tau_e = \infty$. Let us consider the following two stochastic differential systems:

$$\begin{cases} \mathrm{d}\Phi(t) = \Phi(t-)[b-\alpha\Phi(t-)]\,\mathrm{d}t - \sigma_1\Phi(t-)\,\mathrm{d}w_1(t) - \int_\Gamma \gamma_1(u)\Phi(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}\Psi(t) = (\frac{c_1}{h_1}-d_2)\Psi(t-)\,\mathrm{d}t - \sigma_2\Psi(t-)\,\mathrm{d}w_2(t) - \int_\Gamma \gamma_2(u)\Psi(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \end{cases} \tag{2.3}$$

with initial value $(\Phi(0),\Psi(0)) = (x_0,y_0)\in\mathbb{R}_+^2$ and

$$\begin{cases} \mathrm{d}\phi(t) = \phi(t-)[-d_1 - s\Psi(t-) - \alpha\phi(t-)]\,\mathrm{d}t - \sigma_1\phi(t-)\,\mathrm{d}w_1(t) \\ \qquad - \int_\Gamma \gamma_1(u)\phi(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}\psi(t) = -d_2\psi(t-)\,\mathrm{d}t - \sigma_2\psi(t-)\,\mathrm{d}w_2(t) - \int_\Gamma \gamma_2(u)\psi(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \end{cases} \tag{2.4}$$

with initial value $(\phi(0),\psi(0)) = (x_0,y_0)\in\mathbb{R}_+^2$.

Thanks to Lemmas 4.1 and 4.2 in [13], systems (2.3) and (2.4) can be explicitly solved as follows:

$$\begin{cases} \Phi(t) = \frac{\exp\{(b-\beta_1)t-\sigma_1 w_1(t)+\int_0^t\int_\Gamma \ln[1-\gamma_1(u)]\tilde{N}(\mathrm{d}r,\mathrm{d}u)\}}{\frac{1}{x_0}+\alpha\int_0^t \exp\{(b-\beta_1)z-\sigma_1 w_1(z)+\int_0^z\int_\Gamma \ln[1-\gamma_1(z)]\tilde{N}(\mathrm{d}r,\mathrm{d}u)\}\,\mathrm{d}z}, \\ \Psi(t) = y_0\exp\{(\frac{c_1}{h_1}-d_2-\beta_2)t - \sigma_2 w_2(t) + \int_0^t\int_\Gamma \ln[1-\gamma_2(u)]\tilde{N}(\mathrm{d}r,\mathrm{d}u)\}, \end{cases}$$

and

$$\begin{cases} \phi(t) = \frac{\exp\{(-d_1-\beta_1)t-s\int_0^t\Psi(r)\,\mathrm{d}r-\sigma_1 w_1(t)+\int_0^t\int_\Gamma \ln[1-\gamma_1(u)]\tilde{N}(\mathrm{d}r,\mathrm{d}u)\}}{\frac{1}{x_0}+\alpha\int_0^t \exp\{(-d_1-\beta_1)z-s\int_0^z\Psi(r)\,\mathrm{d}r-\sigma_1 w_1(z)+\int_0^z\int_\Gamma \ln[1-\gamma_1(u)]\tilde{N}(\mathrm{d}r,\mathrm{d}u)\}\,\mathrm{d}z}, \\ \psi(t) = y_0\exp\{(-d_2-\beta_2)t - \sigma_2 w_2(t) + \int_0^t\int_\Gamma \ln[1-\gamma_2(u)]\tilde{N}(\mathrm{d}r,\mathrm{d}u)\}. \end{cases}$$

Note that the local solution $(x(t),y(t))$ is positive on $[0,\tau_e)$. Then, on the basis of comparison theorem for stochastic differential equations (see Theorem 3.1 in [17]), we have

$$0 < \phi(t) \le x(t) \le \Phi(t), \qquad 0 < \psi(t) \le y(t) \le \Psi(t), \quad \text{a.s. for } t\in[0,\tau_e).$$

Thus,

$$\ln\phi(t) \le X(t) \le \ln\Phi(t), \qquad \ln\psi(t) \le Y(t) \le \ln\Psi(t), \quad \text{a.s. for } t\in[0,\tau_e).$$

Since $\ln\phi(t)$, $\ln\Phi(t)$, $\ln\psi(t)$ and $\ln\Psi(t)$ exist on $[0,\infty)$, it follows that $\tau_e=\infty$. This means that, for any initial value $(X(0),Y(0)) = (\ln x_0,\ln y_0)\in\mathbb{R}^2$, system (2.2) has a unique global solution $(X(t),Y(t))$ on $[0,\infty)$ a.s. Therefore, for any initial value $(x_0,y_0)\in\mathbb{R}_+^2$, system (1.3) has a unique global positive solution $(x(t),y(t)) = (e^{X(t)},e^{Y(t)})$ on $[0,\infty)$ a.s. The proof is therefore complete. □

3 Asymptotic pathwise estimation

The pathwise properties of the solutions are the subject of the present section. For later applications, we first give a useful lemma, which is a generalization of an exponential martingale inequality with jumps.

Lemma 3.1 ([18, 19]) *Let $f:[0,\infty)\to\mathbb{R}$ and $h:[0,\infty)\times\Gamma\to\mathbb{R}$ be both predictable $\{\mathcal{F}_t\}$-adapted processes such that, for any $T>0$,*

$$\int_0^T |f(t)|^2\,\mathrm{d}t<\infty \quad \textit{a.s.} \quad \textit{and} \quad \int_0^T\int_\Gamma |h(t,u)|\lambda(\mathrm{d}u)\,\mathrm{d}t<\infty \quad \textit{a.s.}$$

Then, for any positive constants α, β,

$$\mathbb{P}\biggl\{\sup_{0\le t\le T}\biggl[\int_0^t f(s)\,\mathrm{d}w(s)-\frac{\alpha}{2}\int_0^t|f(s)|^2\,\mathrm{d}s+\int_0^t\int_\Gamma h(s,u)\tilde{N}(\mathrm{d}s,\mathrm{d}u)$$
$$-\frac{1}{\alpha}\int_0^t\int_\Gamma\bigl[e^{\alpha h(s,u)}-1-\alpha h(s,u)\bigr]\lambda(\mathrm{d}u)\,\mathrm{d}s\biggr]>\beta\biggr\}\le e^{-\alpha\beta}.$$

Theorem 3.2 *Let Assumption* (2.1) *hold. For any initial value $(x_0,y_0)\in\mathbb{R}_+^2$, the solution $(x(t),y(t))$ of system* (1.3) *has the property*

$$\limsup_{t\to\infty}\frac{\ln x(t)}{\ln t}\le 1 \quad \textit{a.s.} \quad \textit{and} \quad \limsup_{t\to\infty}\frac{\ln y(t)}{t}\le\frac{c_1}{h_1}-d_2-\beta_2 \quad \textit{a.s.}$$

Proof For the prey population, applying the Itô formula to $[e^t\ln x(t)]$ leads to

$$\begin{aligned} e^t\ln x(t) = {}& \ln x_0+\int_0^t e^r\biggl[\ln x(r)+\frac{bx(r)}{A_1+x(r)}-d_1-\alpha x(r)-\frac{sy(r)}{1+sh_1x(r)}\biggr]\mathrm{d}r \\ &-\frac{1}{2}\int_0^t\sigma_1^2e^r\,\mathrm{d}r+\int_0^t\int_\Gamma e^r\bigl[\ln\bigl(1-\gamma_1(u)\bigr)+\gamma_1(u)\bigr]\lambda(\mathrm{d}u)\,\mathrm{d}r \\ &-\int_0^t\sigma_1e^r\,\mathrm{d}w_1(r)+\int_0^t\int_\Gamma e^r\ln\bigl(1-\gamma_1(u)\bigr)\tilde{N}(\mathrm{d}r,\mathrm{d}u). \end{aligned}$$

Then, using the fundamental inequality $\ln x\le x-1$ for $x>0$, we have

$$\begin{aligned} e^t\ln x(t)\le{}& \ln x_0+\int_0^t e^r\bigl[\ln x(r)+b-\alpha x(r)\bigr]\mathrm{d}r-\frac{1}{2}\int_0^t\sigma_1^2e^r\,\mathrm{d}r \\ &-\int_0^t\sigma_1e^r\,\mathrm{d}w_1(r)+\int_0^t\int_\Gamma e^r\ln\bigl(1-\gamma_1(u)\bigr)\tilde{N}(\mathrm{d}r,\mathrm{d}u). \end{aligned} \tag{3.1}$$

Let $n=1,2,\ldots$, $\gamma>0$, $\theta>1$ and $0<\varepsilon<1$. Choose $T=n\gamma$, $\alpha=\varepsilon e^{-n\gamma}$ and $\beta=(\theta e^{n\gamma}\ln n)/\varepsilon$. By Lemma 3.1, we deduce that

$$\begin{aligned} \mathbb{P}\biggl\{&\sup_{0\le t\le n\gamma}\biggl[\int_0^t-\sigma_1e^r\,\mathrm{d}w_1(r)+\int_0^t\int_\Gamma e^r\ln\bigl(1-\gamma_1(u)\bigr)\tilde{N}(\mathrm{d}r,\mathrm{d}u) \\ &-\frac{e^{n\gamma}}{\varepsilon}\int_0^t\int_\Gamma\bigl[\bigl(1-\gamma_1(u)\bigr)^{\varepsilon e^{r-n\gamma}}-1-\varepsilon e^{r-n\gamma}\ln\bigl(1-\gamma_1(u)\bigr)\bigr]\lambda(\mathrm{d}u)\,\mathrm{d}r \\ &-\frac{\varepsilon e^{-n\gamma}}{2}\int_0^t\sigma_1^2e^{2r}\,\mathrm{d}r\biggr]>\frac{\theta e^{n\gamma}\ln n}{\varepsilon}\biggr\}\le\frac{1}{n^\theta}. \end{aligned}$$

Since $\sum_{n=0}^\infty\frac{1}{n^\theta}<\infty$ for $\theta>1$, the Borel–Cantelli lemma (see Lemma 1.2.4 in [20]) shows that there exists a set $\Omega_0\in\mathcal{F}$ with $\mathbb{P}(\Omega_0)=1$ and an integer-valued random variable $n_0=$

$n_0(\omega)$ such that, for every $\omega \in \Omega_0$ and $n \geq n_0$, we have

$$\begin{aligned}
&-\int_0^t \sigma_1 e^r \, dw_1(r) + \int_0^t \int_\Gamma e^r \ln\bigl(1-\gamma_1(u)\bigr)\tilde{N}(dr, du) \\
&\quad\leq \frac{\theta e^{n\gamma} \ln n}{\varepsilon} + \frac{\varepsilon e^{-n\gamma}}{2} \int_0^t \sigma_1^2 e^{2r} \, dr \\
&\qquad + \frac{e^{n\gamma}}{\varepsilon} \int_0^t \int_\Gamma \bigl[\bigl(1-\gamma_1(u)\bigr)^{\varepsilon e^{r-n\gamma}} - 1 - \varepsilon e^{r-n\gamma} \ln\bigl(1-\gamma_1(u)\bigr)\bigr]\lambda(du)\, dr
\end{aligned}$$

for all $0 \leq t \leq n\gamma$. Substituting the above inequality into (3.1), we have

$$\begin{aligned}
e^t \ln x(t) \leq{}& \ln x_0 + \int_0^t e^r\bigl[\ln x(r) + b - \alpha x(r)\bigr]\, dr - \frac{1}{2}\int_0^t \sigma_1^2 e^r \, dr \\
&+ \frac{\varepsilon e^{-n\gamma}}{2}\int_0^t \sigma_1^2 e^{2r}\, dr + \frac{\theta e^{n\gamma}\ln n}{\varepsilon} \\
&+ \frac{e^{n\gamma}}{\varepsilon}\int_0^t \int_\Gamma \bigl[\bigl(1-\gamma_1(u)\bigr)^{\varepsilon e^{r-n\gamma}} - 1 - \varepsilon e^{r-n\gamma}\ln\bigl(1-\gamma_1(u)\bigr)\bigr]\lambda(du)\, dr
\end{aligned} \tag{3.2}$$

for all $0 \leq t \leq n\gamma$, $n \geq n_0$. Note that, for $0 \leq r \leq t \leq n\gamma$,

$$\frac{1}{2}\varepsilon e^{-n\gamma}\sigma_1^2 e^{2r} - \frac{1}{2}\sigma_1^2 e^r = \frac{1}{2}\sigma_1^2 e^r\bigl(\varepsilon e^{r-n\gamma} - 1\bigr) \leq \frac{1}{2}\sigma_1^2 e^r(\varepsilon - 1) < 0$$

and

$$\begin{aligned}
&\bigl(1-\gamma_1(u)\bigr)^{\varepsilon e^{r-n\gamma}} - 1 - \varepsilon e^{r-n\gamma}\ln\bigl(1-\gamma_1(u)\bigr) \\
&\quad\leq 1 - \varepsilon e^{r-n\gamma}\gamma_1(u) - 1 - \varepsilon e^{r-n\gamma}\ln\bigl(1-\gamma_1(u)\bigr) \\
&\quad= -\varepsilon e^{r-n\gamma}\bigl[\gamma_1(u) + \ln\bigl(1-\gamma_1(u)\bigr)\bigr],
\end{aligned}$$

where in the second inequality, we use the inequality $x^r \leq 1 + r(x-1)$, $x \geq 0$, $1 \geq r \geq 0$.

Substituting these into (3.2) yields

$$\begin{aligned}
e^t \ln x(t) \leq{}& \int_0^t e^r\Bigl[\ln x(r) + b - \alpha x(r) - \int_\Gamma \bigl[\gamma_1(u) + \ln\bigl(1-\gamma_1(u)\bigr)\bigr]\lambda(du)\Bigr]\, dr \\
&+ \frac{\theta e^{n\gamma}\ln n}{\varepsilon} + \ln x_0.
\end{aligned} \tag{3.3}$$

Let us consider function $f(x) = \ln x + b - \alpha x - \int_\Gamma[\gamma_1(u) + \ln(1-\gamma_1(u))]\lambda(du)$ on $[0,\infty)$. It is easy to show that the function f has a maximum value for $x = \frac{1}{\alpha} > 0$ and a maximum value of the function f is $f_{\max} = \ln\frac{1}{\alpha} + b - 1 - \int_\Gamma[\gamma_1(u) + \ln(1-\gamma_1(u))]\lambda(du)$. Thus, for almost all $0 \leq r \leq n\gamma$, there exists a positive constant K_1 such that $f(x) \leq K_1$. This, together with (3.3), yields

$$e^t \ln x(t) \leq \ln x_0 + K_1 e^t + \frac{\theta e^{n\gamma}\ln n}{\varepsilon}$$

for all $0 \leq t \leq n\gamma$, $n \geq n_0$. Therefore, for all $0 \leq (n-1)\gamma \leq t \leq n\gamma$, $n \geq n_0$, we have

$$\frac{\ln x(t)}{\ln t} \leq \frac{\ln x_0}{e^t \ln t} + \frac{K_1}{\ln t} + \frac{\theta e^{\gamma}\ln n}{\varepsilon \ln[(n-1)\gamma]}.$$

Letting $n \to \infty$ (and so $t \to \infty$), we obtain $\limsup_{t\to\infty} \frac{\ln x(t)}{\ln t} \le \frac{\theta e^{\gamma}}{\varepsilon}$. Letting $\theta \downarrow 1$, $\gamma \downarrow 0$ and $\varepsilon \uparrow 1$, we can get

$$\limsup_{t\to\infty} \frac{\ln x(t)}{\ln t} \le 1 \quad \text{a.s.}$$

For the predator population, applying the generalized Itô formula, we obtain

$$\begin{aligned} \ln y(t) = {}& \ln y_0 + \int_0^t \left[\frac{c_1 s x(r)}{1 + s h_1 x(r)} - d_2 - \beta_2 \right] \mathrm{d}r - \sigma_2 w_2(t) \\ & + \int_0^t \int_{\Gamma} \ln\bigl(1 - \gamma_2(u)\bigr) \tilde{N}(\mathrm{d}r, \mathrm{d}u). \end{aligned} \tag{3.4}$$

Clearly, Brownian motion $w_i(t)$ is a real-valued continuous local martingale vanishing at time 0. Then, from the strong law of large numbers (see [20]), it follows that

$$\lim_{t\to\infty} \frac{w_i(t)}{t} = 0, \quad i = 1, 2. \tag{3.5}$$

In addition, denote $M_i(t) = \int_0^t \int_{\Gamma} \ln(1 - \gamma_i(u)) \tilde{N}(\mathrm{d}r, \mathrm{d}u)$. Then $M_i(t)$ is a local martingale vanishing at time 0 and

$$\langle M_i \rangle(t) \doteq \langle M_i, M_i \rangle_t = \int_0^t \int_{\Gamma} \bigl[\ln\bigl(1 - \gamma_i(u)\bigr)\bigr]^2 \lambda(\mathrm{d}u)\, \mathrm{d}r, \quad i = 1, 2.$$

It follows from (2.1) that

$$\begin{aligned} \rho_{M_i}(t) &\doteq \int_0^t \frac{\mathrm{d}\langle M_i \rangle(r)}{(1+r)^2} = \int_0^t \frac{\int_{\Gamma} [\ln(1 - \gamma_i(u))]^2 \lambda(\mathrm{d}u)}{(1+r)^2} \mathrm{d}r \\ &\le K \int_0^t \frac{1}{(1+r)^2} \mathrm{d}r = K\left(1 - \frac{1}{1+t}\right), \quad i = 1, 2. \end{aligned}$$

Hence, $\lim_{t\to\infty} \rho_{M_i}(t) \le K < \infty$, $i = 1, 2$. Then, by the strong law of large numbers for local martingales (see [21]), we have

$$\lim_{t\to\infty} \frac{M_i(t)}{t} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \int_{\Gamma} \ln\bigl(1 - \gamma_i(u)\bigr) \tilde{N}(\mathrm{d}r, \mathrm{d}u) = 0, \quad i = 1, 2. \tag{3.6}$$

Thus, from (3.4), it follows that

$$\frac{\ln y(t)}{t} \le \left(\frac{c_1}{h_1} - d_2 - \beta_2 \right) - \frac{\sigma_2 w_2(t)}{t} + \frac{M_2(t)}{t} + \frac{\ln y_0}{t},$$

which, together with (3.5) and (3.6), yields

$$\begin{aligned} \limsup_{t\to\infty} \frac{\ln y(t)}{t} &\le \lim_{t\to\infty} \left[\left(\frac{c_1}{h_1} - d_2 - \beta_2 \right) - \frac{\sigma_2 w_2(t)}{t} + \frac{M_2(t)}{t} + \frac{\ln y_0}{t} \right] \\ &= \frac{c_1}{h_1} - d_2 - \beta_2. \end{aligned}$$

The proof is therefore complete. □

Remark 3.3 Note $\lim_{t\to\infty}\frac{\ln t}{t}=0$. Hence, $\limsup_{t\to\infty}\frac{\ln x(t)}{\ln t}\le 1$ a.s. yields

$$\limsup_{t\to\infty}\frac{\ln x(t)}{t}\le 0 \quad \text{a.s.}$$

4 Extinction

For a stochastic equation, we are interested in the long time behavior. In this section, we investigate how the intensities of noises and the Allee effect affect extermination of the predator population and the prey population. From Theorem 3.2, it is easy to see that the following theorem holds.

Theorem 4.1 *For any* $(x_0,y_0)\in\mathbb{R}^2_+$, *let* $(x(t),y(t))$ *be the solution of system* (1.3) *with initial value* (x_0,y_0). *If* $\frac{c_1}{h_1}-d_2-\beta_2<0$, *then* $\lim_{t\to\infty}y(t)=0$ *a.s., that is, the predator population becomes extinct with probability one*.

Remark 4.2 When $\gamma_1(u)=\gamma_2(u)=0$, we can conclude that, for any $(x_0,y_0)\in\mathbb{R}^2_+$, the solution $(x(t),y(t))$ of system (1.2) has the property: if $\sigma_2^2>2[\frac{c_1}{h_1}-d_2]$, then $\lim_{t\to\infty}y(t)=0$ a.s. This is consistent with Theorem 3.3 in [10]. Hence, Theorem 4.1 generalizes Theorem 3.3 in [10].

Now, we investigate how Allee effect and the intensities of noises affect the prey population and then the predator population.

Theorem 4.3 *For any* $(x_0,y_0)\in\mathbb{R}^2_+$, *let* $(x(t),y(t))$ *be the solution of system* (1.3) *with initial value* (x_0,y_0). *If one of the following conditions holds*:

(i) $\alpha A_1\ge b-(d_1+\beta_1)$;

(ii) $(\sqrt{b}-\sqrt{d_1+\beta_1})^2<\alpha A_1<b-(d_1+\beta_1)$,

then $\lim_{t\to\infty}x(t)=0$, $\lim_{t\to\infty}y(t)=0$ *a.s., that is, prey and predator populations become extinct with probability one*.

Proof For the prey population, applying the generalized Itô formula, we obtain

$$\begin{aligned}\mathrm{d}\ln x(t)={}&\left[\frac{bx(t-)}{A_1+x(t-)}-d_1-\alpha x(t-)-\frac{sy(t-)}{1+sh_1x(t-)}-\beta_1\right]\mathrm{d}t-\sigma_1\,\mathrm{d}w_1(t)\\&+\int_\Gamma\ln\bigl(1-\gamma_1(u)\bigr)\tilde{N}(\mathrm{d}t,\mathrm{d}u).\end{aligned}$$

Integrating both sides of the above equation from 0 to t and using the positivity of $x(t)$ and $y(t)$ yield

$$\ln x(t)\le\ln x_0+\int_0^t\left[\frac{bx(r)}{A_1+x(r)}-d_1-\alpha x(r)-\beta_1\right]\mathrm{d}r-\sigma_1w_1(t)+M_1(t). \tag{4.1}$$

Let us consider function

$$\begin{aligned}f(x)&=\frac{bx}{A_1+x}-\alpha x-d_1-\beta_1\\&=\frac{-\alpha x^2+[b-(d_1+\beta_1)-\alpha A_1]x-(d_1+\beta_1)A_1}{A_1+x},\end{aligned}$$

on $[0,\infty)$. Denote $g(x)=-\alpha x^2+[b-(d_1+\beta_1)-\alpha A_1]x-(d_1+\beta_1)A_1$ for $x\in[0,\infty)$.

(i) If $\alpha A_1 \geq b - (d_1 + \beta_1)$, then $g(x) \leq g(0) = -(d_1 + \beta_1)A_1 < 0$ for any $x \in [0, +\infty)$.
(ii) If $\alpha A_1 < b - (d_1 + \beta_1)$, then we know that the maximum value of g is

$$g_{\max} = \frac{[b - (d_1 + \beta_1) + \alpha A_1]^2 - 4b\alpha A_1}{4\alpha}.$$

This, together with $(\sqrt{b} - \sqrt{d_1 + \beta_1})^2 < \alpha A_1$, yields, for any $x \in [0, \infty)$,

$$g(x) \leq g_{\max} = \frac{[b - (d_1 + \beta_1) + \alpha A_1]^2 - 4b\alpha A_1}{4\alpha} < 0.$$

Therefore, if condition (i) or condition (ii) holds, then $f(x) < 0$ for $x \in [0, +\infty)$. In addition, $\lim_{x\to\infty} f(x) = -\infty$. Hence, there is a constant $D < 0$ such that $f(x) < D$ for all $x \in [0, \infty)$. Therefore, from (4.1), it follows that

$$\frac{\ln x(t)}{t} \leq D - \frac{\sigma_1 w_1(t)}{t} + \frac{M_1(t)}{t} + \frac{\ln x_0}{t}.$$

Thus, from (3.5) and (3.6), it follows that

$$\limsup_{t\to\infty} \frac{\ln x(t)}{t} \leq \lim_{t\to\infty} \left[D - \frac{\sigma_1 w_1(t)}{t} + \frac{M_1(t)}{t} + \frac{\ln x_0}{t}\right] = D.$$

Then $D < 0$ implies $\lim_{t\to\infty} x(t) = 0$ a.s.

Let $\Omega_1 = \{\omega \in \Omega : \lim_{t\to\infty} x(t, \omega) = 0\}$, then $\lim_{t\to\infty} x(t) = 0$ a.s. implies $\mathbb{P}(\Omega_1) = 1$. Hence, for any $\omega \in \Omega_1$ and any constant $\varepsilon > 0$, there exists a constant $T(\omega, \varepsilon) > 0$ such that $\frac{c_1 sx(t)}{1+sh_1 x(t)} \leq \varepsilon$ for any $t \geq T$. Therefore, from (3.4), it follows that

$$\begin{aligned}
\frac{\ln y(t)}{t} &= \frac{1}{t}\int_0^t \left[\frac{c_1 sx(r)}{1 + sh_1 x(r)} - d_2 - \beta_2\right] \mathrm{d}r - \frac{\sigma_2 w_2(t)}{t} + \frac{M_2(t)}{t} + \frac{\ln y_0}{t} \\
&\leq \frac{1}{t}\int_0^T \left(\frac{c_1}{h_1} - d_2 - \beta_2\right) \mathrm{d}r + \frac{1}{t}\int_T^t (\varepsilon - d_2 - \beta_2)\, \mathrm{d}r - \frac{\sigma_2 w_2(t)}{t} \\
&\quad + \frac{M_2(t)}{t} + \frac{\ln y_0}{t} \\
&= \left(\frac{c_1}{h_1} - d_2 - \beta_2\right)\frac{T}{t} + (\varepsilon - d_2 - \beta_2)\frac{t - T}{t} - \frac{\sigma_2 w_2(t)}{t} + \frac{M_2(t)}{t} + \frac{\ln y_0}{t}
\end{aligned}$$

for every $t \geq T$ and $\omega \in \Omega_1$. Thus, from (3.5) and (3.6), it follows that

$$\limsup_{t\to\infty} \frac{\ln y(t)}{t} \leq \varepsilon - d_2 - \beta_2 \quad \text{a.s.},$$

and the required assertion $\lim_{t\to\infty} y(t) = 0$ a.s. follows since $\varepsilon > 0$ is arbitrary. The proof is complete. □

Form Theorem 4.3, we can get the following corollary with the proof being omitted.

Corollary 4.4 *For any initial value $(x_0, y_0) \in \mathbb{R}_+^2$, let $(x(t), y(t))$ be the solution of system* (1.2) *with initial value (x_0, y_0). If one of the following conditions holds*:

(C_1) $\alpha A_1 \geq b - (d_1 + \frac{\sigma_1^2}{2})$;
(C_2) $(\sqrt{b} - \sqrt{d_1 + \frac{\sigma_1^2}{2}})^2 < \alpha A_1 < b - (d_1 + \frac{\sigma_1^2}{2})$,
then $\lim_{t\to\infty} x(t) = 0$, $\lim_{t\to\infty} y(t) = 0$ *a.s.*

Remark 4.5 From Theorem 3.2 in [10], for any $(x_0, y_0) \in \mathbb{R}_+^2$, if one of the following conditions holds:
(B_1) $d_1 > b$ and $2\alpha A_1 < d_1 - b$;
(B_2) $d_1 \leq b$ and $\alpha A_1 \geq b - d_1$;
(B_3) $d_1 \leq b$, $\alpha A_1 < b - d_1$ and $(b - d_1 - \alpha A_1)^2(b - d_1 + 8\alpha A_1) - 27d_1(\alpha A_1)^2 < 0$;
(B_4) $\alpha A_1 < b$ and $\sigma_1^2 > 2[(\sqrt{b} - \sqrt{\alpha A_1})^2 - d_1]$,
then the solution $(x(t), y(t))$ of model (1.2) with any initial value (x_0, y_0) satisfies $\lim_{t\to\infty} x(t) = 0$ and $\lim_{t\to\infty} y(t) = 0$ a.s.

Obviously, if condition (B_1) or (B_2) holds, then condition (C_1) in Corollary 4.4 holds. If condition (B_3) holds, then condition (C_1) or (C_2) in Corollary 4.4 holds. If condition (B_4) holds, then condition (C_1) or (C_2) in Corollary 4.4 holds. Moreover, if $\alpha A_1 > b$ and $b < d_1 \leq 3b$, then condition (C_1) holds. However, if $\alpha A_1 > b$ and $b < d_1 \leq 3b$, then $0 < d_1 - b \leq 2b < 2\alpha A_1$, which means all the conditions of Theorem 3.2 in [10] are not satisfied. Therefore, Corollary 4.4 improves Theorem 3.2 in [10].

5 Stochastically ultimate boundness

In this section, we investigate the stochastically ultimate boundedness of system (1.3). Firstly, its definition will be given.

Definition 5.1 ([22]) The solutions of system (1.3) are called stochastically ultimately bounded, if for any $\varepsilon \in (0,1)$, there exist two positive constants $H_1 = H_1(\varepsilon)$ and $H_2 = H_2(\varepsilon)$ such that the solution $(x(t), y(t))$ of model (1.3) with any initial value in $\mathbb{R}_+^2$ has the property that

$$\limsup_{t\to\infty} \mathbb{P}\{x(t) > H_1\} < \varepsilon, \qquad \limsup_{t\to\infty} \mathbb{P}\{y(t) > H_2\} < \varepsilon.$$

We provide the following useful lemmas from which the stochastically ultimate boundedness follows directly. Denote $\mathbb{R}_+ = (0, \infty)$.

Lemma 5.2 *Let $\Phi(t)$ be the solution of the first equation in system* (2.3) *with the initial value $\Phi(0) = x_0 \in \mathbb{R}_+$, then*

$$\limsup_{t\to\infty} \mathbb{E}[\Phi(t)] \leq \frac{b}{\alpha}.$$

Proof Integrating both sides of the first equation in system (2.3) from 0 to t yields

$$\Phi(t) = x_0 + \int_0^t \Phi(s)[b - \alpha\Phi(s)]\,\mathrm{d}s - \int_0^t \sigma_1\Phi(s)\,\mathrm{d}w_1(s) - \int_0^t \int_\Gamma \gamma_1(u)\Phi(s)\tilde{N}(\mathrm{d}s, \mathrm{d}u). \tag{5.1}$$

Note that $\int_0^t \sigma_1 \Phi(s)\,\mathrm{d}w_1(s)$ is a real-valued continuous local martingale and $\int_0^t \int_\Gamma \gamma_1(u)\Phi(s)\tilde{N}(\mathrm{d}s,\mathrm{d}u)$ is a real-valued local martingale. Taking the expectation on both sides of (5.1), we have

$$\mathbb{E}\big[\Phi(t)\big] = x_0 + \mathbb{E}\int_0^t \Phi(s)\big[b - \alpha\Phi(s)\big]\,\mathrm{d}s. \tag{5.2}$$

Using the Hölder inequality, it follows that

$$\frac{\mathrm{d}\mathbb{E}[\Phi(t)]}{\mathrm{d}t} = b\mathbb{E}\big[\Phi(t)\big] - \alpha\mathbb{E}\big[\Phi^2(t)\big] \le b\mathbb{E}\big[\Phi(t)\big] - \alpha\mathbb{E}^2\big[\Phi(t)\big].$$

The Bernoulli equation

$$\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t} = b\varphi(t) - \alpha\varphi^2(t)$$

with the initial value $\varphi(0) = x_0$, has the solution

$$\varphi(t) = \frac{b}{\alpha(1 - e^{-bt} + x_0^{-1}\frac{\alpha}{b}e^{-bt})}.$$

Then, by the comparison theorem, we have

$$\mathbb{E}\big[\Phi(t)\big] \le \frac{b}{\alpha(1 - e^{-bt} + x_0^{-1}\frac{\alpha}{b}e^{-bt})}.$$

From $b > 0$, it follows that

$$\limsup_{t\to\infty} \mathbb{E}\big[\Phi(t)\big] \le \lim_{t\to\infty} \frac{b}{\alpha(1 - e^{-bt} + x_0^{-1}e^{-bt})} = \frac{b}{\alpha}.$$

The proof is therefore complete. □

Lemma 5.3 *Let $\Psi(t)$ be the solution of the second equation in system* (2.3) *with initial value $\Psi(0) = y_0 \in \mathbb{R}_+$. If $\frac{c_1}{h_1} - d_2 < 0$, then*

$$\lim_{t\to\infty} \mathbb{E}\big[\Psi(t)\big] = 0.$$

Proof Integrating both sides of the second equation in system (2.3) from 0 to t yields

$$\begin{aligned}\Psi(t) = y_0 &+ \int_0^t \left(\frac{c_1}{h_1} - d_2\right)\Psi(s)\,\mathrm{d}s - \int_0^t \sigma_2\Psi(s)\,\mathrm{d}w_2(s)\\ &- \int_0^t \int_\Gamma \gamma_2(u)\Psi(s)\tilde{N}(\mathrm{d}s,\mathrm{d}u).\end{aligned} \tag{5.3}$$

Note that $\int_0^t \sigma_2 \Psi(s)\,\mathrm{d}w_2(s)$ is a real-valued continuous local martingale and $\int_0^t \int_\Gamma \gamma_2(u)\Psi(s)\tilde{N}(\mathrm{d}s,\mathrm{d}u)$ is a real-valued local martingale. Taking the expectation on both sides of (5.3), we have

$$\mathbb{E}\big[\Psi(t)\big] = y_0 + \mathbb{E}\int_0^t \left(\frac{c_1}{h_1} - d_2\right)\Psi(s)\,\mathrm{d}s, \tag{5.4}$$

which implies the differentiability of $\mathbb{E}[\Psi(t)]$. Then

$$\frac{\mathrm{d}\mathbb{E}[\Psi(t)]}{\mathrm{d}t} = \left(\frac{c_1}{h_1} - d_2\right)\mathbb{E}\big[\Psi(t)\big]. \tag{5.5}$$

It is easy to show that the solution of Eq. (5.5) with initial $\mathbb{E}[\Psi(0)] = y_0$ is

$$\mathbb{E}\big[\Psi(t)\big] = y_0 \exp\left\{\left(\frac{c_1}{h_1} - d_2\right)t\right\}.$$

This, together with $\frac{c_1}{h_1} - d_2 < 0$, yields

$$\lim_{t\to\infty} \mathbb{E}\big[\Psi(t)\big] = \lim_{t\to\infty} y_0 \exp\left\{\left(\frac{c_1}{h_1} - d_2\right)t\right\} = 0.$$

The proof is therefore complete. □

According to Chebyshev's inequality and the application of Lemmas 5.2 and 5.3, we have the following result.

Theorem 5.4 *If* $\frac{c_1}{h_1} - d_2 < 0$, *then the solutions of model* (1.3) *are stochastically ultimately bounded.*

Proof Let $(x(t), y(t))$ be the solution of model (1.3) with any initial values $(x_0, y_0) \in \mathbb{R}^2_+$. Combining $x(t) \le \Phi(t)$, $y(t) \le \Psi(t)$ a.s. with Lemmas 5.2 and 5.3, it follows that

$$\limsup_{t\to\infty} \mathbb{E}\big[x(t)\big] \le \frac{b}{\alpha}, \qquad \lim_{t\to\infty} \mathbb{E}\big[y(t)\big] = 0.$$

Now, for any $\varepsilon \in (0,1)$, let $H_1 = \frac{b}{\alpha\varepsilon} + 1$ and $H_2 = 1$. Then by Chebyshev's inequality

$$\mathbb{P}\big\{x(t) > H_1\big\} \le \frac{\mathbb{E}[x(t)]}{H_1}, \qquad \mathbb{P}\big\{y(t) > H_2\big\} \le \frac{\mathbb{E}[y(t)]}{H_2}.$$

Hence,

$$\limsup_{t\to\infty} \mathbb{P}\big\{x(t) > H_1\big\} \le \limsup_{t\to\infty} \frac{\mathbb{E}[x_1(t)]}{H_1} < \varepsilon,$$
$$\limsup_{t\to\infty} \mathbb{P}\big\{y(t) > H_2\big\} \le \limsup_{t\to\infty} \frac{\mathbb{E}[y(t)]}{H_2} = 0.$$

The proof is therefore complete. □

6 Numerical simulations

In this section, we make numerical simulations to illustrate our results by using the method by stationary Poisson point processes [11]. Numerical experiments are made by using the following set of parameters: $\Gamma = (0, +\infty)$, $\lambda(\Gamma) = 1$, initial value $(30, 4)$ and some parameter values are shown in the table below (see [10]).

(i) Assume that $\alpha = 0.01$, $A_1 = 0.5$, $\sigma_1^2 = \sigma_2^2 = 0.005$, $\gamma_1(u) = \gamma_2(u) = 0.05$ and for the other parameter values see Table 1. By a simple computation, $\alpha A_1 = 0.005$, $\beta_i = 0.0038$, $i = 1, 2$.

Table 1 The parameters of the model

Parameters	Description	Values
b	per capita maximum reproduction rate of prey population	0.22 per year
d_1	per capita death rates of prey	0.008 per year
d_2	per capita death rates of predators	0.7 per year
s	effective search rate	0.05 per year
h_1	handling time of predators	25 days
c_1	conversion efficiency of ingested prey into new predators	0.005 per year

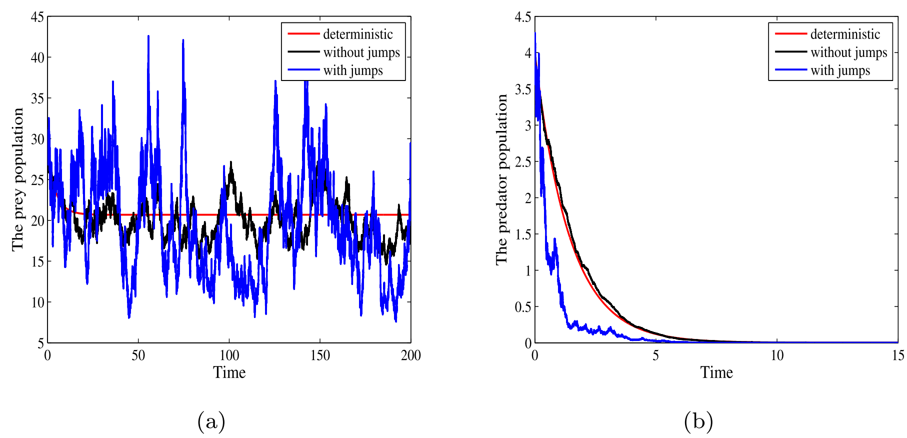

Figure 1 Deterministic and stochastic trajectories of prey population and predator population with parameters in Table 1. Moreover, $\sigma_1^2 = \sigma_2^2 = 0.005$ and $\gamma_1(u) = \gamma_2(u) = 0.05$. (**a**) Trajectories of prey population. (**b**) Trajectories of predator population. (Color figure online)

Thus, $b - (d_1 + \beta_1) = 0.2082$, $(\sqrt{b} - \sqrt{d_1 + \beta_1})^2 \approx 0.13$ and $\frac{c_1}{h_1} - d_2 - \beta_2 < 0$. Therefore, the conditions of Theorem 4.3 are not satisfied while the condition of Theorem 4.1 is fulfilled. From Theorem 4.1, it follows that the predator population becomes extinct with probability one. As can be seen from Fig. 1 that the prey population does not become extinct and predator population becomes extinct.

(ii) Moreover, for the same $\alpha = 0.01$, intensities of white noises $\sigma_1^2 = \sigma_2^2 = 0.005$, jumps $\gamma_1(u) = \gamma_2(u) = 0.05$ and greater Allee effect constant, $A_1 = 20$, we can obtain $\alpha A_1 = 0.2$, $b - (d_1 + \beta_1) = 0.2082$ and $(\sqrt{b} - \sqrt{d_1 + \beta_1})^2 = 0.1299$. This implies the condition (ii) of Theorem 4.3 is fulfilled. Then, from Theorem 4.3, it follows that both prey population and predator population become extinct. Figures 2(a) and 2(b) are the trajectories of prey population and predator population, respectively. From Fig. 2, we can see that both prey population and predator population become extinct. Comparing Figs. 1(a) and 2(a), we can conclude that stronger Allee effects can lead to the extinction of the prey population, even if intensity of noise is not high.

(iii) For the same α and A_1 taken as those in (i). If intensities of white noises $\sigma_1^2 = 0.31$, $\sigma_2^2 = 0.2$ and jumps $\gamma_1(u) = 0.1$, $\gamma_2(u) = 0.05$, then $\alpha A_1 = 0.005$, $b - (d_1 + \beta_1) = 0.0516$, $(\sqrt{b} - \sqrt{d_1 + \beta_1})^2 = 0.003481$, $b - (d_1 + \frac{\sigma_1^2}{2}) = 0.057$ and $(\sqrt{b} - \sqrt{d_1 + \frac{\sigma_1^2}{2}})^2 = 0.00426$. Then condition (ii) in Theorem 4.3 holds and condition (C_2) in Corollary 4.4 holds. Thus, from Theorem 4.3 and Corollary 4.4 the prey populations in (1.2) and (1.3) both become extinct. As can be seen from Fig. 3(a) that the prey population in system (1.2) and (1.3) both become extinct. Moreover, if intensity of white noise $\sigma_1^2 = 0.05$, $\sigma_2^2 = 0.2$ and jumps $\gamma_1(u) =$

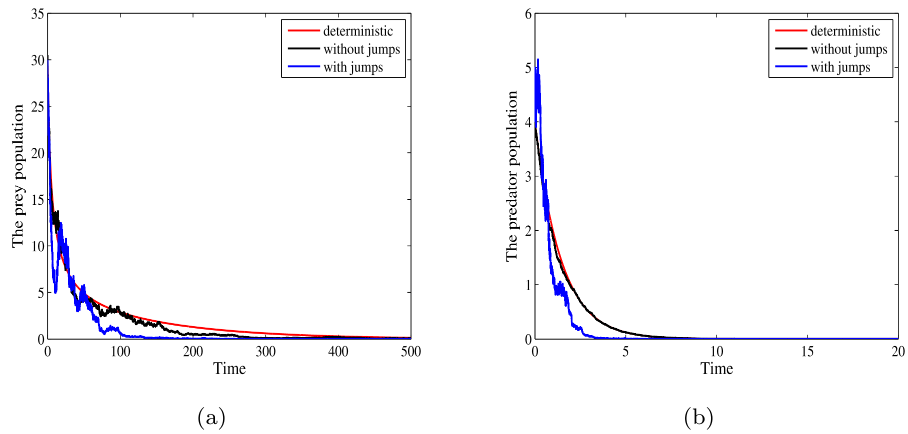

Figure 2 Deterministic and stochastic trajectories of prey population and predator population with parameters in Table 1. Moreover, $\alpha = 0.01$, $A_1 = 20$ $\sigma_1^2 = \sigma_2^2 = 0.005$ and $\gamma_1(u) = \gamma_2(u) = 0.05$. (**a**) Trajectories of prey population. (**b**) Trajectories of predator population. (Color figure online)

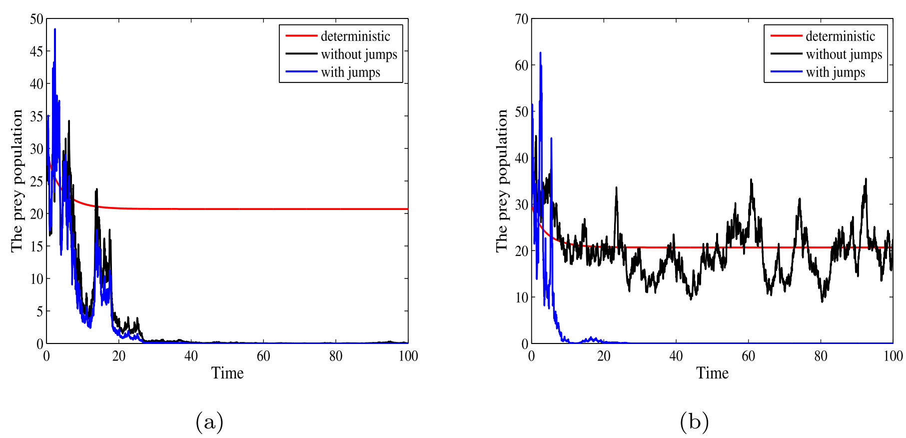

Figure 3 Deterministic and stochastic trajectories of prey population with parameters in Table 1, $\alpha = 0.01$, $A_1 = 0.5$ and different intensities of white noise and jumps. (**a**) $\sigma_1^2 = 0.31$, $\sigma_2^2 = 0.2$, $\gamma_1(u) = 0.1$, $\gamma_2(u) = 0.05$. (**b**) $\sigma_1^2 = 0.05$, $\sigma_2^2 = 0.2$, $\gamma_1(u) = 0.5$, $\gamma_2(u) = 0.2$. (Color figure online)

0.5, $\gamma_2(u) = 0.2$, then $\alpha A_1 = 0.005$, $b - (d_1 + \beta_1) = 0.0165$, $(\sqrt{b} - \sqrt{d_1 + \beta_1})^2 = 0.00032$, $b - (d_1 + \frac{\sigma_1^2}{2}) = 0.2095$ and $(\sqrt{b} - \sqrt{d_1 + \frac{\sigma_1^2}{2}})^2 = 0.1343$. Thus, condition (ii) in Theorem 4.3 holds while all the conditions in Corollary 4.4 are not satisfied. Therefore, from Theorem 4.3, it follows that the prey population in system (1.3) becomes extinct. As can be seen from Fig. 3(b) that the prey population in system (1.3) becomes extinct while the prey population in model (1.2) does not become extinct. Comparing Figs. 1(a) and 3(a), we conclude that white noise can lead to the extinction of prey population. Comparing Figs. 3(a) and 3(b), we conclude that jumps can lead to the extinction of prey population.

(iv) Choose $b = 0.02$, $d_1 = 0.04$, $\alpha = 0.01$, $A_1 = 3$, $\sigma_1^2 = 0.3$, $\sigma_2^2 = 0.2$, $\gamma_1(u) = \gamma_2(u) = 0.1$ and the other parameters are taken as in Table 1. By a simple computation, $\alpha A_1 = 0.03 > -0.175 = b - (d_1 + \beta_1)$. This means condition (i) in Theorem 4.3 holds. Hence, from Theorem 4.3, it follows that the prey population and predator both become extinct. Moreover, we have $\alpha A_1 > b$ and $b < d_1 \leq 3b$. However, from Remark 4.5, if $\alpha A_1 > b$ and $b < d_1 \leq 3b$,

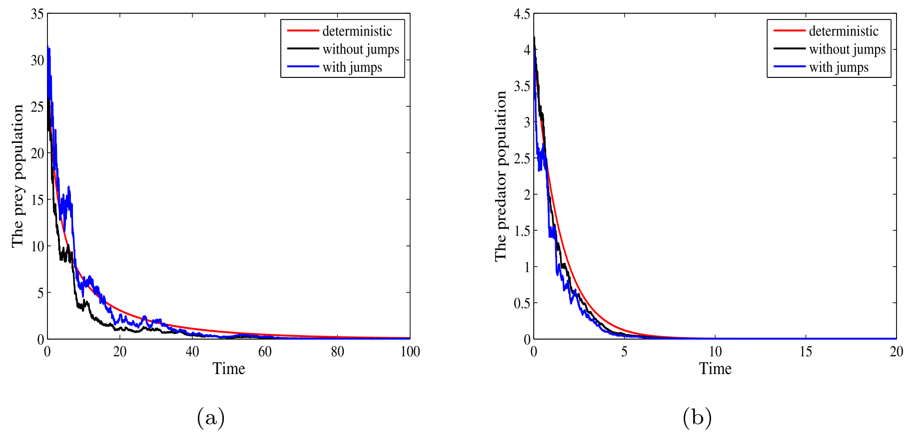

Figure 4 Deterministic and stochastic trajectories of prey population and predator population with $b = 0.02$, $d_1 = 0.04$, $\alpha = 0.01$, $A_1 = 3$, $\sigma_1^2 = 0.3$, $\sigma_2^2 = 0.2$, $\gamma_1(u) = \gamma_2(u) = 0.1$ and other parameters are taken as in Table 1. (**a**) Trajectories of prey population. (**b**) Trajectories of predator population. (Color figure online)

then all conditions of Theorem 3.2 in [10] are not hold. From Fig. 4, we can see that the prey population will become extinct with probability one and then the predator population.

7 Conclusions and discussions

In this paper, we consider a stochastic predator–prey population model with Allee effect and Lévy noise. First, by the comparison theorem of stochastic differential equations and the Itô formula, we prove that this model has a unique global positive solution starting from the positive initial value. Then we investigate the asymptotic pathwise behavior of the model by the generalized exponential martingale inequality and the Borel–Cantelli lemma. Next, we establish the conditions under which extinction of predator and prey populations occur. Furthermore, we show that the global positive solution is stochastically ultimate bounded under some conditions by using the Chebyshev's inequality. At last, we introduce some numerical simulations to support the main results obtained.

When $\gamma_1(u) = \gamma_2(u) = 0$, we can get the stochastic predator–prey population (1.2), which is studied in [10]. From Remark 4.5, it follows that Corollary 4.4 generalizes and improves Theorem 3.2 in [10]. Moreover, our investigation shows that the Allee effect, white noise and jumps may cause great influence on the survival of species. It can be seen from Figs. 1(a) and 2(a) that stronger Allee effects can lead to the extinction of prey population, even if intensity of noise is not high. From Figs. 1(a), 3(a) and 3(b), we can conclude that the high intensity noise and jumps can also lead to the extinction of prey population, even if the Allee effects is not large. Therefore, Allee effect is an important factor in population modeling. Moreover, considering the sudden environmental shocks, a stochastic model especially with jumps is better than a deterministic model in describing the population dynamics.

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Authors' contributions
The study was carried out in collaboration with equal responsibility. All authors read and approved the final manuscript.

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