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On Lyapunov-type inequalities for (p,q)-Laplacian systems

Mohamed Jleli and Bessem Samet*

*Correspondence:
bsamet@ksu.edu.sa
Department of Mathematics, College of Science, King Saud University, P.O. Box 2455, Riyadh, 11451, Saudi Arabia

Abstract
We establish Lyapunov-type inequalities for a system involving one-dimensional (p_i,q_i)-Laplacian operators ($i=1,2$). Next, the obtained inequalities are used to derive some geometric properties of the generalized spectrum associated to the considered problem.

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1 Introduction

In this paper, we are concerned with the following system involving one-dimensional (p_i,q_i)-Laplacian operators ($i=1,2$):

$$(\mathrm{S}):\quad \begin{cases} -(|u'(x)|^{p_1-2}u'(x))' - (|u'(x)|^{q_1-2}u'(x))' = f(x)|u(x)|^{\alpha-2}|v(x)|^{\beta}u(x), \\ -(|v'(x)|^{p_2-2}v'(x))' - (|v'(x)|^{q_2-2}v'(x))' = g(x)|u(x)|^{\alpha}|v(x)|^{\beta-2}v(x) \end{cases}$$

on the interval (a,b), under Dirichlet boundary conditions

$$(\mathrm{DBC}):\quad u(a)=u(b)=v(a)=v(b)=0.$$

System (S) is investigated under the assumptions

$$\alpha\geq 2,\qquad \beta\geq 2,\qquad p_i\geq 2,\qquad q_i\geq 2,\quad i=1,2,$$

and

$$\frac{2\alpha}{p_1+q_1}+\frac{2\beta}{p_2+q_2}=1. \tag{1}$$

We suppose also that f and g are two nonnegative real-valued functions such that $(f,g)\in L^1(a,b)\times L^1(a,b)$. We establish a Lyapunov-type inequality for problem (S)-(DBC). Next, we use the obtained inequality to derive some geometric properties of the generalized spectrum associated to the considered problem.

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The standard Lyapunov inequality [1] (see also [2]) states that if the boundary value problem

$$\begin{cases} u''(t) + q(t)u(t) = 0, & a < t < b, \\ u(a) = u(b) = 0, \end{cases}$$

has a nontrivial solution, where $q : [a,b] \to \mathbb{R}$ is a continuous function, then

$$\int_a^b \left|q(t)\right| dt > \frac{4}{b-a}. \tag{2}$$

Inequality (2) was successfully applied to oscillation theory, stability criteria for periodic differential equations, estimates for intervals of disconjugacy, and eigenvalue bounds for ordinary differential equations. In [3] (see also [4]), Elbert extended inequality (2) to the one-dimensional p-Laplacian equation. More precisely, he proved that, if u is a nontrivial solution of the problem

$$\begin{cases} (|u'|^{p-2}u')' + h(t)|u|^{p-2}u = 0, & a < t < b, \\ u(a) = u(b) = 0, \end{cases}$$

where $1 < p < \infty$ and $h \in L^1(a,b)$, then

$$\int_a^b \left|h(t)\right| dt > \frac{2^p}{(b-a)^{p-1}}. \tag{3}$$

Observe that for $p = 2$, (3) reduces to (2). Inequality (3) was extended in [5] to the following problem involving the φ-Laplacian operator:

$$\begin{cases} (\varphi(u'))' + w(t)\varphi(u) = 0, & a < t < b, \\ u(a) = u(b) = 0, \end{cases}$$

where $\varphi : \mathbb{R} \to \mathbb{R}$ is a convex nondecreasing function satisfying a Δ_2 condition. In [6], Nápoli and Pinasco considered the quasilinear system of resonant type

$$\begin{cases} -(|u'(x)|^{p-2}u'(x))' = f(x)|u(x)|^{\alpha-2}|v(x)|^{\beta}u(x), \\ -(|v'(x)|^{q-2}v'(x))' = g(x)|u(x)|^{\alpha}|v(x)|^{\beta-2}v(x) \end{cases} \tag{4}$$

on the interval (a,b), with Dirichlet boundary conditions

$$u(a) = u(b) = v(a) = v(b) = 0. \tag{5}$$

Under the assumptions $p,q > 1$, $f,g \in L^1(a,b)$, $f,g \geq 0$, $\alpha,\beta \geq 0$, and

$$\frac{\alpha}{p} + \frac{\beta}{q} = 1,$$

it was proved (see [6], Theorem 1.5) that if (4)-(5) has a nontrivial solution, then

$$2^{\alpha+\beta} \leq (b-a)^{\frac{\alpha}{p'}+\frac{\beta}{q'}} \left(\int_a^b f(x)\,dx\right)^{\frac{\alpha}{p}} \left(\int_a^b g(x)\,dx\right)^{\frac{\beta}{q}}, \tag{6}$$

where $p' = \frac{p}{p-1}$ and $q' = \frac{q}{q-1}$. Some nice applications to generalized eigenvalues are also presented in [6]. Different generalizations and extensions of inequality (6) were obtained by many authors. In this direction, we refer the reader to [7–16] and the references therein. For other results concerning Lyapunov-type inequalities, we refer the reader to [17–29] and the references therein.

2 Lyapunov-type inequalities

A Lyapunov-type inequality for problem (S)-(DBC) is established in this section, and some particular cases are discussed.

Theorem 2.1 *If* (S)-(DBC) *admits a nontrivial solution* $(u,v) \in C^2[a,b] \times C^2[a,b]$, *then*

$$\begin{aligned}
&\left[\min\left\{\frac{2^{p_1}}{(b-a)^{p_1-1}}, \frac{2^{q_1}}{(b-a)^{q_1-1}}\right\}\right]^{\frac{2\alpha}{p_1+q_1}} \left[\min\left\{\frac{2^{p_2}}{(b-a)^{p_2-1}}, \frac{2^{q_2}}{(b-a)^{q_2-1}}\right\}\right]^{\frac{2\beta}{p_2+q_2}} \\
&\quad \leq \left(\frac{1}{2}\int_a^b f(x)\,dx\right)^{\frac{2\alpha}{p_1+q_1}} \left(\frac{1}{2}\int_a^b g(x)\,dx\right)^{\frac{2\beta}{p_2+q_2}}.
\end{aligned} \tag{7}$$

Proof Let $(u,v) \in C^2[a,b] \times C^2[a,b]$ be a nontrivial solution to (S)-(DBC). Let $(x_0,y_0) \in (a,b) \times (a,b)$ be such that

$$\left|u(x_0)\right| = \max\left\{\left|u(x)\right| : a \leq x \leq b\right\}$$

and

$$\left|v(y_0)\right| = \max\left\{\left|v(x)\right| : a \leq x \leq b\right\}.$$

From the boundary conditions (DBC), we can write that

$$2u(x_0) = \int_a^{x_0} u'(x)\,dx - \int_{x_0}^b u'(x)\,dx,$$

which yields

$$2\left|u(x_0)\right| \leq \int_a^b \left|u'(x)\right|\,dx.$$

Using Hölder's inequality with parameters p_1 and $p_1' = \frac{p_1}{p_1-1}$, we get

$$2\left|u(x_0)\right| \leq (b-a)^{\frac{1}{p_1'}} \left(\int_a^b \left|u'(x)\right|^{p_1}\,dx\right)^{\frac{1}{p_1}},$$

that is,

$$\frac{2^{p_1}}{(b-a)^{p_1-1}}\left|u(x_0)\right|^{p_1} \le \int_a^b \left|u'(x)\right|^{p_1}\,dx. \tag{8}$$

Similarly, using Hölder's inequality with parameters q_1 and $q_1' = \frac{q_1}{q_1-1}$, we get

$$\frac{2^{q_1}}{(b-a)^{q_1-1}}\left|u(x_0)\right|^{q_1} \le \int_a^b \left|u'(x)\right|^{q_1}\,dx. \tag{9}$$

By repeating the same argument for the function v, we obtain

$$\frac{2^{p_2}}{(b-a)^{p_2-1}}\left|v(y_0)\right|^{p_2} \le \int_a^b \left|v'(x)\right|^{p_2}\,dx \tag{10}$$

and

$$\frac{2^{q_2}}{(b-a)^{q_2-1}}\left|v(y_0)\right|^{q_2} \le \int_a^b \left|v'(x)\right|^{q_2}\,dx. \tag{11}$$

Now, multiplying the first equation of (S) by u and integrating over (a,b), we obtain

$$\int_a^b \left|u'(x)\right|^{p_1}\,dx + \int_a^b \left|u'(x)\right|^{q_1}\,dx = \int_a^b f(x)\left|u(x)\right|^{\alpha}\left|v(x)\right|^{\beta}\,dx. \tag{12}$$

Multiplying the second equation of (S) by v and integrating over (a,b), we obtain

$$\int_a^b \left|v'(x)\right|^{p_2}\,dx + \int_a^b \left|v'(x)\right|^{q_2}\,dx = \int_a^b g(x)\left|u(x)\right|^{\alpha}\left|v(x)\right|^{\beta}\,dx. \tag{13}$$

Using (8), (9) and (12), we obtain

$$\left|u(x_0)\right|^{\alpha}\left|v(y_0)\right|^{\beta}\int_a^b f(x)\,dx \ge \frac{2^{p_1}}{(b-a)^{p_1-1}}\left|u(x_0)\right|^{p_1} + \frac{2^{q_1}}{(b-a)^{q_1-1}}\left|u(x_0)\right|^{q_1},$$

which yields

$$\left|u(x_0)\right|^{\alpha}\left|v(y_0)\right|^{\beta}\int_a^b f(x)\,dx \ge \min\left\{\frac{2^{p_1}}{(b-a)^{p_1-1}}, \frac{2^{q_1}}{(b-a)^{q_1-1}}\right\}\left(\left|u(x_0)\right|^{p_1} + \left|u(x_0)\right|^{q_1}\right).$$

Using the inequality

$$A + B \ge 2\sqrt{A}\sqrt{B}$$

with $A = |u(x_0)|^{p_1}$ and $B = |u(x_0)|^{q_1}$, we get

$$\min\left\{\frac{2^{p_1+1}}{(b-a)^{p_1-1}}, \frac{2^{q_1+1}}{(b-a)^{q_1-1}}\right\} \le \left|u(x_0)\right|^{\alpha-\frac{p_1+q_1}{2}}\left|v(y_0)\right|^{\beta}\int_a^b f(x)\,dx. \tag{14}$$

Similarly, using (10), (11) and (13), we obtain

$$\min\left\{\frac{2^{p_2+1}}{(b-a)^{p_2-1}}, \frac{2^{q_2+1}}{(b-a)^{q_2-1}}\right\} \le \left|u(x_0)\right|^{\alpha}\left|v(y_0)\right|^{\beta-\frac{p_2+q_2}{2}}\int_a^b g(x)\,dx. \tag{15}$$

Raising inequality (14) to a power $e_1 > 0$, inequality (15) to a power $e_2 > 0$, and multiplying the resulting inequalities, we obtain

$$\begin{aligned}&\left[\min\left\{\frac{2^{p_1+1}}{(b-a)^{p_1-1}},\frac{2^{q_1+1}}{(b-a)^{q_1-1}}\right\}\right]^{e_1}\left[\min\left\{\frac{2^{p_2+1}}{(b-a)^{p_2-1}},\frac{2^{q_2+1}}{(b-a)^{q_2-1}}\right\}\right]^{e_2}\\&\quad\le\left|u(x_0)\right|^{(\alpha-\frac{p_1+q_1}{2})e_1+\alpha e_2}\left|v(y_0)\right|^{\beta e_1+(\beta-\frac{p_2+q_2}{2})e_2}\left(\int_a^b f(x)\,dx\right)^{e_1}\left(\int_a^b g(x)\,dx\right)^{e_2}.\end{aligned}$$

Next, we take (e_1, e_2) any solution of the homogeneous linear system

$$\begin{cases}(\alpha-\frac{p_1+q_1}{2})e_1+\alpha e_2=0,\\ \beta e_1+(\beta-\frac{p_2+q_2}{2})e_2=0.\end{cases}$$

Using (1), we may take

$$\begin{cases}e_1=\alpha,\\ e_2=\frac{\beta(p_1+q_1)}{p_2+q_2}.\end{cases}$$

Therefore, we obtain

$$\begin{aligned}&2^{\alpha+\frac{\beta(p_1+q_1)}{p_2+q_2}}\left[\min\left\{\frac{2^{p_1}}{(b-a)^{p_1-1}},\frac{2^{q_1}}{(b-a)^{q_1-1}}\right\}\right]^{\alpha}\left[\min\left\{\frac{2^{p_2}}{(b-a)^{p_2-1}},\frac{2^{q_2}}{(b-a)^{q_2-1}}\right\}\right]^{\frac{\beta(p_1+q_1)}{p_2+q_2}}\\&\quad\le\left(\int_a^b f(x)\,dx\right)^{\alpha}\left(\int_a^b g(x)\,dx\right)^{\frac{\beta(p_1+q_1)}{p_2+q_2}}.\end{aligned}$$

Using again (1), we get

$$\begin{aligned}&2\left[\min\left\{\frac{2^{p_1}}{(b-a)^{p_1-1}},\frac{2^{q_1}}{(b-a)^{q_1-1}}\right\}\right]^{\frac{2\alpha}{p_1+q_1}}\left[\min\left\{\frac{2^{p_2}}{(b-a)^{p_2-1}},\frac{2^{q_2}}{(b-a)^{q_2-1}}\right\}\right]^{\frac{2\beta}{p_2+q_2}}\\&\quad\le\left(\int_a^b f(x)\,dx\right)^{\frac{2\alpha}{p_1+q_1}}\left(\int_a^b g(x)\,dx\right)^{\frac{2\beta}{p_2+q_2}},\end{aligned}$$

which proves Theorem 2.1. □

As a consequence of Theorem 2.1, we deduce the following result for the case of a single equation.

Corollary 1 *Let us assume that there exists a nontrivial solution of*

$$\begin{cases}-(|u'(x)|^{p-2}u'(x))'-(|u'(x)|^{q-2}u'(x))'=f(x)|u(x)|^{\frac{p+q}{2}-2}u(x),\quad x\in(a,b),\\ u(a)=u(b)=0,\end{cases}$$

where $p > 1$, $q > 1$, $f \ge 0$, *and* $f \in L^1(a,b)$. *Then*

$$\min\left\{\frac{2^p}{(b-a)^{p-1}},\frac{2^q}{(b-a)^{q-1}}\right\}\le\frac12\int_a^b f(x)\,dx.$$

Proof An application of Theorem 2.1 with

$$p_1 = p_2 = p, \qquad q_1 = q_2 = q, \qquad \alpha = \frac{p+q}{2}, \qquad \beta = 0, \qquad v = u, \qquad g = f,$$

yields the desired result. □

Remark 1 Taking $f = 2h$ and $q = p$ in Corollary 1, we obtain Lyapunov-type inequality (3) for the one-dimensional p-Laplacian equation.

Remark 2 Taking $p_1 = q_1 = p$ and $p_2 = q_2 = q$ in Theorem 2.1, we obtain Lyapunov-type inequality (6).

3 Generalized eigenvalues

The concept of generalized eigenvalues was introduced by Protter [30] for a system of linear elliptic operators. The first work dealing with generalized eigenvalues for p-Laplacian systems is due to Nápoli and Pinasco [6]. Inspired by that work, we present in this section some applications to generalized eigenvalues related to problem (S)-(DBC).

Let us consider the generalized eigenvalue problem

$$(\mathrm{S})_{\lambda,\mu}: \quad \begin{cases} -(|u'(x)|^{p_1-2}u'(x))' - (|u'(x)|^{q_1-2}u'(x))' = \lambda\alpha w(x)|u(x)|^{\alpha-2}|v(x)|^{\beta}u(x), \\ -(|v'(x)|^{p_2-2}v'(x))' - (|v'(x)|^{q_2-2}v'(x))' = \mu\beta w(x)|u(x)|^{\alpha}|v(x)|^{\beta-2}v(x), \end{cases}$$

on the interval (a,b), with Dirichlet boundary conditions (DBC). If problem $(\mathrm{S})_{\lambda,\mu}$-(DBC) admits a nontrivial solution $(u,v) \in C^2[a,b] \times C^2[a,b]$, we say that (λ,μ) is a generalized eigenvalue of $(\mathrm{S})_{\lambda,\mu}$-(DBC). The set of generalized eigenvalues is called generalized spectrum, and it is denoted by σ.

We assume that

$$\alpha \ge 2, \qquad \beta \ge 2, \qquad p_i \ge 2, \qquad q_i \ge 2, \quad i = 1,2, \qquad w \ge 0, \qquad w \in L^1(a,b),$$

and (1) is satisfied.

The following result provides lower bounds of the generalized eigenvalues of $(\mathrm{S})_{\lambda,\mu}$-(DBC).

Theorem 3.1 *Let* (λ,μ) *be a generalized eigenvalue of* $(\mathrm{S})_{\lambda,\mu}$-(DBC). *Then*

$$\mu \ge h(\lambda), \tag{16}$$

where $h : (0,\infty) \to (0,\infty)$ *is the function defined by*

$$h(t) = \frac{1}{\beta}\left(\frac{C}{t^{\frac{2\alpha}{p_1+q_1}} \int_a^b w(x)\,dx}\right)^{\frac{p_2+q_2}{2\beta}}, \quad t > 0,$$

with

$$\alpha^{\frac{2\alpha}{p_1+q_1}} C = 2\left[\min\left\{\frac{2^{p_1}}{(b-a)^{p_1-1}}, \frac{2^{q_1}}{(b-a)^{q_1-1}}\right\}\right]^{\frac{2\alpha}{p_1+q_1}} \times \left[\min\left\{\frac{2^{p_2}}{(b-a)^{p_2-1}}, \frac{2^{q_2}}{(b-a)^{q_2-1}}\right\}\right]^{\frac{2\beta}{p_2+q_2}}.$$

Proof Let (λ, μ) be a generalized eigenpair, and let u, v be the corresponding nontrivial solutions. By replacing in Lyapunov-type inequality (7) the functions

$$f(x) = \alpha\lambda w(x), \qquad g(x) = \beta\mu w(x),$$

and using condition (1), we obtain

$$2M \leq \alpha^{\frac{2\alpha}{p_1+q_1}} \lambda^{\frac{2\alpha}{p_1+q_1}} \beta^{\frac{2\beta}{p_2+q_2}} \mu^{\frac{2\beta}{p_2+q_2}} \int_a^b w(x)\,dx,$$

where

$$M = \left[\min\left\{\frac{2^{p_1}}{(b-a)^{p_1-1}}, \frac{2^{q_1}}{(b-a)^{q_1-1}}\right\}\right]^{\frac{2\alpha}{p_1+q_1}} \left[\min\left\{\frac{2^{p_2}}{(b-a)^{p_2-1}}, \frac{2^{q_2}}{(b-a)^{q_2-1}}\right\}\right]^{\frac{2\beta}{p_2+q_2}}.$$

Hence, we have

$$\mu^{\frac{2\beta}{p_2+q_2}} \geq \frac{C}{\lambda^{\frac{2\alpha}{p_1+q_1}} \beta^{\frac{2\beta}{p_2+q_2}} \int_a^b w(x)\,dx},$$

which yields

$$\mu \geq \frac{1}{\beta}\left(\frac{C}{\lambda^{\frac{2\alpha}{p_1+q_1}} \int_a^b w(x)\,dx}\right)^{\frac{p_2+q_2}{2\beta}},$$

and the proof is finished. □

As consequences of the previous obtained result, we deduce the following Protter's type results for the generalized spectrum.

Corollary 2 *There exists a constant $c_{a,b} > 0$ that depends on a and b such that no point of the generalized spectrum σ is contained in the ball $B(0, c_{a,b})$, where*

$$B(0, c_{a,b}) = \left\{x = (x_1, x_2) \in \mathbb{R}^2 : \|x\|_\infty < c_{a,b}\right\},$$

and $\|\cdot\|_\infty$ is the Chebyshev norm in $\mathbb{R}^2$.

Proof Let $(\lambda, \mu) \in \sigma$. From (16), we obtain easily that

$$\lambda^{\frac{2\alpha}{p_1+q_1}} \mu^{\frac{2\beta}{p_2+q_2}} \geq \frac{C}{\beta^{\frac{2\beta}{p_2+q_2}} \int_a^b w(x)\,dx}. \tag{17}$$

On the other hand, using condition (1), we have

$$\lambda^{\frac{2\alpha}{p_1+q_1}} \mu^{\frac{2\beta}{p_2+q_2}} \leq \left\|(\lambda, \mu)\right\|_\infty^{\frac{2\alpha}{p_1+q_1}+\frac{2\beta}{p_2+q_2}} = \left\|(\lambda, \mu)\right\|_\infty.$$

Therefore, we obtain

$$\left\|(\lambda, \mu)\right\|_\infty \geq c_{a,b},$$

where

$$c_{a,b} = \frac{C}{\beta^{\frac{2\beta}{p_2+q_2}} \int_a^b w(x)\,dx}.$$

The proof is finished. □

Corollary 3 *Let* (λ, μ) *be fixed. There exists an interval* J *of sufficiently small measure such that, if* $I = [a,b] \subset J$*, then there are no nontrivial solutions of* $(\mathrm{S})_{\lambda,\mu}$-(DBC).

Proof Suppose that $(\mathrm{S})_{\lambda,\mu}$-(DBC) admits a nontrivial solution. Since $C \to +\infty$ as $b - a \to 0^+$, where C is defined in Theorem 3.1, there exists $\delta > 0$ such that

$$b - a < \delta \quad \Longrightarrow \quad \frac{C}{\int_a^b w(x)\,dx} > \lambda^{\frac{2\alpha}{p_1+q_1}} \mu^{\frac{2\beta}{p_2+q_2}} \beta^{\frac{2\beta}{p_2+q_2}}.$$

Let $J = [a, a + \delta]$. Hence, if $I \subset J$, we have

$$\frac{C}{\beta^{\frac{2\beta}{p_2+q_2}} \int_a^b w(x)\,dx} > \lambda^{\frac{2\alpha}{p_1+q_1}} \mu^{\frac{2\beta}{p_2+q_2}},$$

which is a contradiction with (17). Therefore, if $I \subset J$, there are no nontrivial solutions of $(\mathrm{S})_{\lambda,\mu}$-(DBC). □

4 Conclusion

Lyapunov-type inequalities for a system of differential equations involving one-dimensional (p_i, q_i)-Laplacian operators $(i = 1, 2)$ are derived. It was shown that such inequalities are very useful to obtain geometric characterizations of the generalized spectrum associated to the considered problem.

Competing interests
The authors declare to have no competing interests.

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Authors' contributions
All authors contributed equally in writing this paper.

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