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On blow-up criteria for a coupled chemotaxis fluid model

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Abstract
We consider a coupled chemotaxis fluid model and prove some blow-up criteria of the local strong solution.

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1 Introduction

We consider the following coupled chemotaxis fluid model [1]:

$$u_t + (u \cdot \nabla)u + \nabla\pi - \Delta u + n\nabla\phi = 0, \tag{1.1}$$

$$\operatorname{div} u = 0, \tag{1.2}$$

$$n_t + (u \cdot \nabla)n - \Delta n = -\nabla \cdot \big(n\chi(p)\nabla p\big), \tag{1.3}$$

$$p_t + (u \cdot \nabla)p = -nf(p), \tag{1.4}$$

$$(u, n, p)(x, 0) = (u_0, n_0, p_0)(x) \quad \text{in } \mathbb{R}^3. \tag{1.5}$$

Here u denotes the velocity vector field of the fluid and π is the pressure scalar, p and n denote the concentration of oxygen and bacteria, respectively. $\nabla\phi$ is the gravitation force. $f(p) \geq f(0) = 0$ and $\chi(p) \geq 0$ are two given smooth functions of p.

When $\phi = 0$, (1.1) and (1.2) are the well-known Navier-Stokes system. Kozono *et al.* [2] and Kozono and Shimada [3] proved the following blow-up criteria:

$$u \in L^2\big(0, T; \dot{B}^0_{\infty,\infty}\big), \tag{1.6}$$

$$u \in L^{\frac{2}{1-\theta}}\big(0, T; \dot{B}^{-\theta}_{\infty,\infty}\big) \quad \text{with } 0 < \theta < 1, \tag{1.7}$$

$$\omega := \operatorname{curl} u \in L^1\big(0, T; \dot{B}^0_{\infty,\infty}\big). \tag{1.8}$$

Here $\dot{B}^s_{p,q}$ denotes the homogeneous Besov space. Zhang *et al.* [4] showed the following blow-up criterion in terms of pressure:

$$\pi \in L^{\frac{2}{2+r}}\big(0, T; \dot{B}^r_{\infty,\infty}\big) \quad \text{with } -1 \leq r \leq 1. \tag{1.9}$$

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When $u = \nabla\phi = 0$, (1.3) and (1.4) are the Keller-Segel model which was studied in [5, 6].

Very recently, Chae *et al.*[7] showed the local well-posedness of smooth solutions to problem (1.1)-(1.5) and the following blow-up criterion:

$$u \in L^{\frac{2q}{q-3}}\big(0,T;L^q\big) \quad \text{and} \quad n \in L^2\big(0,T;L^\infty\big) \quad \text{with } 3 < q \le \infty. \tag{1.10}$$

The aim of this paper is to refine (1.10) further; we will prove the following.

Theorem 1.1 *Let the initial data (u_0, n_0, p_0) be given in $H^l \times H^{l-1} \times H^l$ for $l > \frac{5}{2}$ and n_0, $p_0 \ge 0$ in $\mathbb{R}^3$ and $\int_{\mathbb{R}^3} n_0\,dx < \infty$. Suppose that ϕ is a smooth function. Let (u,n,p) be a local smooth solution on $[0,\tilde{T})$ for some $\tilde{T} \le \infty$. If u satisfies* (1.6) *or* (1.7) *or* (1.8) *or π satisfies* (1.9) $(r = -1)$ *and n satisfies*

$$n \in L^2\big(0,T;L^\infty\big) \tag{1.11}$$

with $\tilde{T} \le T < \infty$, then the solution (u,n,p) can be extended beyond $T > 0$.

Corollary 1.1 *If u satisfies* (1.6) *or* (1.7) *or* (1.8) *or π satisfies* (1.9) *and ∇p satisfies*

$$\nabla p \in L^{\frac{2q}{q-3}}\big(0,T;L^q\big) \quad \textit{with } 3 < q \le \infty, \tag{1.12}$$

with $\tilde{T} \le T < \infty$, then the solution (u,n,p) can be extended beyond $T > 0$.

Remark 1.1 By the very same calculations as those in Zhou [8], we can prove the following blow-up criteria:

$$\pi \in L^{\frac{2q}{2q-3}}\big(0,T;L^q\big) \quad \text{with } 3/2 < q \le \infty, \tag{1.13}$$

or

$$\nabla\pi \in L^{\frac{2q}{3q-3}}\big(0,T;L^q\big) \quad \text{with } 1 < q \le \infty, \tag{1.14}$$

and n satisfies (1.11). We omit the details here.

2 Preliminary

Here we recall the definitions and some properties of spaces.

Let $\mathfrak{B} = \{\xi \in \mathbb{R}^d, |\xi| \le \frac{4}{3}\}$ and $\mathfrak{C} = \{\xi \in \mathbb{R}^d, \frac{3}{4} \le |\xi| \le \frac{8}{3}\}$. Choose two nonnegative smooth radial functions χ, φ supported, respectively, in $\mathfrak{B}$ and $\mathfrak{C}$ such that

$$\chi(\xi) + \sum_{j\ge 0} \varphi\big(2^{-j}\xi\big) = 1, \quad \xi \in \mathbb{R}^d,$$

$$\sum_{j\in\mathbb{Z}} \varphi\big(2^{-j}\xi\big) = 1, \quad \xi \in \mathbb{R}^d \setminus \{0\}.$$

We denote $\varphi_j = \varphi(2^{-j}\xi)$, $h = \mathfrak{F}^{-1}\varphi$ and $\tilde{h} = \mathfrak{F}^{-1}\chi$, where $\mathfrak{F}^{-1}$ stands for the inverse Fourier transform. Then the dyadic blocks Δ_j and S_j can be defined as follows:

$$\Delta_j f = \varphi(2^{-j}D)f = 2^{jd}\int_{\mathbb{R}^d} h(2^j y)f(x-y)\,dy,$$
$$S_j f = \sum_{k\le j-1}\Delta_k f = \chi(2^{-j}D)f = 2^{jd}\int_{\mathbb{R}^d} \tilde{h}(2^j y)f(x-y)\,dy.$$

Formally, $\Delta_j = S_j - S_{j-1}$ is a frequency projection to annulus $\{\xi : C_1 2^j \le |\xi| \le C_2 2^j\}$, and S_j is a frequency projection to the ball $\{\xi : |\xi| \le C2^j\}$. One can easily verify that, with our choice of φ,

$$\Delta_j\Delta_k f = 0 \quad \text{if } |j-k| \ge 2 \quad \text{and} \quad \Delta_j(S_{k-1}f\,\Delta_k f) = 0 \quad \text{if } |j-k| \ge 5.$$

With the introduction of Δ_j and S_j, let us recall the definition of the Besov space.

Definition 2.1 ([9, 10]) Let $s \in \mathbb{R}$, $(p,q) \in [1,\infty]^2$, the homogeneous space $\dot{B}^s_{p,q}$ is defined by

$$\dot{B}^s_{p,q} = \{f \in \mathfrak{S}'; \|f\|_{\dot{B}^s_{p,q}} < \infty\},$$

where

$$\|f\|_{\dot{B}^s_{p,q}} = \begin{cases} (\sum_{j\in\mathbb{Z}} 2^{sqj}\|\Delta_j f\|^q_{L^p})^{\frac{1}{q}}, & \text{for } 1 \le q < \infty, \\ \sup_{j\in\mathbb{Z}} 2^{sj}\|\Delta_j f\|_{L^p}, & \text{for } q = \infty, \end{cases}$$

and $\mathfrak{S}'$ denotes the dual space of $\mathfrak{S} = \{f \in \mathcal{S}(\mathbb{R}^d); \partial^\alpha \hat{f}(0) = 0; \forall\alpha \in \mathbb{N}^d \text{ multi-index}\}$ and can be identified by the quotient space of $\mathcal{S}'/\mathcal{P}$ with the polynomials space $\mathcal{P}$.

Lemma 2.1 ([4]) *Let a measurable function* π *satisfy*

$$\pi \in \dot{B}^r_{\infty,\infty}(\mathbb{R}^3)$$

for some r *with* $-1 \le r \le 1$, *then there exists a decomposition* $\pi := \pi_\ell + \pi_h$ *such that*

$$\nabla^2\pi_\ell \in L^\infty(\mathbb{R}^3) \quad \text{and} \quad \pi_h \in W^{-1,\infty}(\mathbb{R}^3),$$

and

$$\|\nabla^2\pi_\ell\|^{\frac{1}{2}}_{L^\infty} + \|\pi_h\|^2_{W^{-1,\infty}} \le C\big(e + \|\pi\|^{\frac{2}{2+r}}_{\dot{B}^r_{\infty,\infty}}\big),$$
$$\|\pi_\ell\|_{L^2} \le C\|\pi\|_{L^2}, \|\nabla\pi_h\|_{L^2} \le C\|\nabla\pi\|_{L^2}.$$

3 Proof of Theorem 1.1

This section is devoted to the proof of Theorem 1.1. Since local existence results have been proved in [7], we only need to prove *a priori* estimates.

To begin with, it is easy to see that

$$n \geq 0, \qquad 0 \leq p \leq C, \qquad \int_{\mathbb{R}^3} n\,dx = \int_{\mathbb{R}^3} n_0\,dx < \infty. \tag{3.1}$$

Case 1. Let (1.6) and (1.11) hold true.

Testing (1.1) by u and using (1.2), we infer that

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3} |u|^2\,dx + \int_{\mathbb{R}^3} |\nabla u|^2\,dx &= \int_{\mathbb{R}^3} n\nabla\phi u\,dx \\ &\leq \|n\|_{L^\infty}\|\nabla\phi\|_{L^2}\|u\|_{L^2},\end{aligned}$$

which leads to

$$\|u\|_{L^\infty(0,T;L^2)} + \|u\|_{L^2(0,T;H^1)} \leq C. \tag{3.2}$$

In the following calculations, we will use the following elegant inequality [11, 12]:

$$\|\nabla u\|_{L^4}^2 \leq C\|u\|_{\dot{B}^0_{\infty,\infty}}\|\Delta u\|_{L^2}.$$

Testing (1.1) by Δu, using (1.2) and the above inequality, we find that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3} |\nabla u|^2\,dx + \int_{\mathbb{R}^3} |\Delta u|^2\,dx \\ &\quad= \int_{\mathbb{R}^3} (u\cdot\nabla)u\cdot\Delta u\,dx + \int_{\mathbb{R}^3} n\nabla\phi\Delta u\,dx \\ &\quad= \sum_{i,j}\int_{\mathbb{R}^3} u_i\partial_i u\,\partial_j^2 u\,dx + \int_{\mathbb{R}^3} n\nabla\phi\Delta u\,dx \\ &\quad= -\sum_{i,j}\int_{\mathbb{R}^3} \partial_j u_i\,\partial_i u\partial_j u dx + \int_{\mathbb{R}^3} n\nabla\phi\Delta u\,dx \\ &\quad\leq C\|\nabla u\|_{L^4}^2\|\nabla u\|_{L^2} + \|n\|_{L^\infty}\|\nabla\phi\|_{L^2}\|\Delta u\|_{L^2} \\ &\quad\leq C\|u\|_{\dot{B}^0_{\infty,\infty}}\|\Delta u\|_{L^2}\|\nabla u\|_{L^2} + C\|n\|_{L^\infty}\|\Delta u\|_{L^2} \\ &\quad\leq \frac{1}{2}\|\Delta u\|_{L^2}^2 + C\|u\|_{\dot{B}^0_{\infty,\infty}}^2\|\nabla u\|_{L^2}^2 + C\|n\|_{L^\infty}^2,\end{aligned}$$

which gives

$$\|u\|_{L^\infty(0,T;H^1)} + \|u\|_{L^2(0,T;H^2)} \leq C. \tag{3.3}$$

By (1.10), this completes the proof of Case 1.

Case 2. Let (1.7) and (1.11) hold true.

Testing (1.1) by $-\Delta u$, using (1.2) and the following inequalities [3, 11]:

$$\|u\cdot\nabla u\|_{L^2} \leq C\|u\|_{\dot{B}^{-\theta}_{\infty,\infty}}\|u\|_{\dot{B}^{1+\theta}_{2,1}}, 0 < \theta < 1, \tag{3.4}$$

$$\|u\|_{\dot{B}^{\theta}_{2,1}} \leq C\|u\|_{L^2}^{1-\theta}\|\nabla u\|_{L^2}^{\theta}, 0 < \theta < 1, \tag{3.5}$$

we derive

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3}|\nabla u|^2\,dx+\int_{\mathbb{R}^3}|\Delta u|^2\,dx\\
&\quad=\int_{\mathbb{R}^3}(u\cdot\nabla)u\cdot\Delta u\,dx+\int_{\mathbb{R}^3}n\nabla\phi\Delta u\,dx\\
&\quad\le\|u\cdot\nabla u\|_{L^2}\|\Delta u\|_{L^2}+\|n\|_{L^\infty}\|\nabla\phi\|_{L^2}\|\Delta u\|_{L^2}\\
&\quad\le C\|u\|_{\dot{B}^{-\theta}_{\infty,\infty}}\|\nabla u\|_{L^2}^{1-\theta}\|\Delta u\|_{L^2}^{1+\theta}+C\|n\|_{L^\infty}\|\Delta u\|_{L^2}\\
&\quad\le\frac{1}{2}\|\Delta u\|_{L^2}^2+C\|u\|_{\dot{B}^{\frac{2}{1-\theta}}_{\infty,\infty}}\|\nabla u\|_{L^2}^2+C\|n\|_{L^\infty}^2,
\end{aligned}$$

which yields (3.3); this completes the proof of Case 2 again by (1.10).

Case 3. Let (1.8) and (1.11) hold true.

Testing (1.1) by $-\Delta u$, using (1.2), we deduce that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3}|\nabla u|^2\,dx+\int_{\mathbb{R}^3}|\Delta u|^2\,dx\\
&\quad=-\sum_{i,j}\int_{\mathbb{R}^3}\partial_j u_i\,\partial_i u\partial_j u dx+\int_{\mathbb{R}^3}n\nabla\phi\Delta u\,dx\\
&\quad=:I_1+\int_{\mathbb{R}^3}n\nabla\phi\Delta u\,dx.
\end{aligned}\tag{3.6}$$

By the very same calculations as those in [13], we get

$$I_1\le\frac{1}{8}\|\Delta u\|_{L^2}^2+C\|\nabla u\|_{L^2}^2+C\|\nabla u\|_{\dot{B}^0_{\infty,\infty}}\|\nabla u\|_{L^2}^2\log\bigl(e+\|\nabla u\|_{L^2}^2\bigr).\tag{3.7}$$

Inserting (3.7) into (3.6) and solving the resulting inequality, we arrive at (3.3). This completes the proof of Case 3.

Case 4. Let (1.9) ($r=-1$) and (1.11) hold true.

Testing (1.1) by $|u|^2u$ and using (1.2), we observe that

$$\begin{aligned}
&\frac{1}{4}\frac{d}{dt}\int_{\mathbb{R}^3}|u|^4\,dx+\int_{\mathbb{R}^3}|u|^2|\nabla u|^2\,dx+\frac{1}{2}\int_{\mathbb{R}^3}\bigl|\nabla|u|^2\bigr|^2\,dx\\
&\quad=-\int_{\mathbb{R}^3}(u\cdot\nabla)\pi|u|^2\,dx-\int_{\mathbb{R}^3}n\nabla\phi|u|^2u\,dx\\
&\quad=:I_2+I_3.
\end{aligned}\tag{3.8}$$

I_3 can be bounded as follows:

$$I_3\le\|n\|_{L^\infty}\|\nabla\phi\|_{L^4}\|u\|_{L^4}^3.\tag{3.9}$$

We bounded I_2 as follows:

$$\begin{aligned}
I_2&=\int_{\mathbb{R}^3}\pi\,u\cdot\nabla|u|^2\,dx\\
&\le\|\pi\|_{L^4}\|u\|_{L^4}\bigl\|\nabla|u|^2\bigr\|_{L^2}
\end{aligned}$$

$$
\begin{aligned}
&\le \|\pi\|_{\dot{B}^{-1}_{\infty,\infty}}^{\frac{1}{2}} \|\nabla\pi\|_{L^2}^{1/2} \|u\|_{L^4} \big\|\nabla|u|^2\big\|_{L^2} \\
&\le \|\pi\|_{\dot{B}^{-1}_{\infty,\infty}}^{\frac{1}{2}} \big(\|u\cdot\nabla u\|_{L^2} + \|n\nabla\phi\|_{L^2}\big)^{1/2} \|u\|_{L^4} \big\|\nabla|u|^2\big\|_{L^2} \\
&\le \|\pi\|_{\dot{B}^{-1}_{\infty,\infty}}^{\frac{1}{2}} \big(\big\||u|\nabla u|\big\|_{L^2} + \|n\|_{L^\infty}\big)^{1/2} \|u\|_{L^4} \big\|\nabla|u|^2\big\|_{L^2} \\
&\le \frac{1}{8}\big\|\nabla|u|^2\big\|_{L^2}^2 + \frac{1}{8}\big\||u|\nabla u|\big\|_{L^2}^2 + C\|\pi\|_{\dot{B}^{-1}_{\infty,\infty}}^2 \|u\|_{L^4}^4 + C\|n\|_{L^\infty}^2,
\end{aligned}
\tag{3.10}
$$

where we have used the elegant inequality [11, 12]

$$
\|\pi\|_{L^4}^2 \le C\|\pi\|_{\dot{B}^{-1}_{\infty,\infty}} \|\nabla\pi\|_{L^2},
\tag{3.11}
$$

and the pressure estimate

$$
\|\nabla\pi\|_{L^2} \le C\big(\|u\cdot\nabla u\|_{L^2} + \|n\nabla\phi\|_{L^2}\big).
\tag{3.12}
$$

Inserting (3.9) and (3.10) into (3.8) and using the Gronwall inequality, we conclude that

$$
\|u\|_{L^\infty(0,T;L^4)} \le C.
\tag{3.13}
$$

By (1.10), this completes the proof of Case 4.

4 Proof of Corollary 1.1

Testing (1.3) by n^{m-1} ($m \ge 2$), using (1.2) and (3.1) and denoting $w := n^{\frac{m}{2}}$, we have

$$
\begin{aligned}
&\frac{1}{m}\frac{d}{dt}\int_{\mathbb{R}^3} w^2\,dx + \frac{4(m-1)}{m^2}\int_{\mathbb{R}^3} |\nabla w|^2\,dx \\
&\le C\left|\int \chi(p)\nabla p\cdot w\nabla w\,dx\right| \\
&\le C\|\nabla p\|_{L^q}\|w\|_{L^{\frac{2q}{q-2}}}\|\nabla w\|_{L^2} \\
&\le C\|\nabla p\|_{L^q}\|w\|_{L^2}^{1-\frac{3}{q}}\|\nabla w\|_{L^2}^{1+\frac{3}{q}} \\
&\le \frac{m-1}{m^2}\|\nabla w\|_{L^2}^2 + C\|\nabla p\|_{L^q}^{\frac{2q}{q-3}}\|w\|_{L^2}^2,
\end{aligned}
$$

which implies

$$
\|n\|_{L^\infty(0,T;L^m)} \le C \quad \text{for } m > 2.
\tag{4.1}
$$

Here we used the Gagliardo-Nirenberg inequality

$$
\|w\|_{L^{\frac{2q}{q-2}}} \le C\|w\|_{L^2}^{1-\frac{3}{q}}\|\nabla w\|_{L^2}^{\frac{3}{q}} \quad \text{with } 3 < q \le \infty.
\tag{4.2}
$$

Now, since the proofs of other cases are very similar to those in Case 1, Case 2, Case 3 and Case 4, we only prove the following case: Let (1.9) ($-1 < r \le 1$) and (1.12) hold true.

We still have (3.8) and (3.9).

Using Lemma 2.1, (3.11), (3.12) and the pressure estimate

$$\begin{aligned}\|\pi\|_{L^2} &\le C\big(\|u\|_{L^4}^2 + \big\|(-\Delta)^{-\frac{1}{2}}(n\nabla\phi)\big\|_{L^2}\big)\\ &\le C\big(\|u\|_{L^4}^2 + \|n\nabla\phi\|_{L^{\frac{6}{5}}}\big)\\ &\le C\big(\|u\|_{L^4}^2 + 1\big),\end{aligned} \tag{4.3}$$

. we bound I_2 as follows:

$$\begin{aligned}I_2 &= -\int_{\mathbb{R}^3} u\nabla\pi_\ell|u|^2\,dx - \int_{\mathbb{R}^3} u\nabla\pi_h|u|^2\,dx\\ &\le \|\nabla\pi_\ell\|_{L^4}\|u\|_{L^4}^3 + \int_{\mathbb{R}^3} u\pi_h\nabla|u|^2\,dx\\ &\le \|\pi_\ell\|_{L^2}^{\frac{1}{2}}\big\|\nabla^2\pi_\ell\big\|_{L^\infty}^{\frac{1}{2}}\|u\|_{L^4}^3 + \|u\|_{L^4}\|\pi_h\|_{L^4}\big\|\nabla|u|^2\big\|_{L^2}\\ &\le \|\pi_\ell\|_{L^2}^{\frac{1}{2}}\big\|\nabla^2\pi_\ell\big\|_{L^\infty}^{\frac{1}{2}}\|u\|_{L^4}^4 + \|u\|_{L^4}\|\pi_h\|_{W^{-1,\infty}}^{\frac{1}{2}}\|\nabla\pi_h\|_{L^2}^{\frac{1}{2}}\big\|\nabla|u|^2\big\|_{L^2}\\ &\le \big\|\nabla^2\pi_\ell\big\|_{L^\infty}^{\frac{1}{2}}\big(\|u\|_{L^4}^4 + 1\big) + C\|\pi_h\|_{W^{-1,\infty}}^{\frac{1}{2}}\big(\|u\cdot\nabla u\|_{L^2} + 1\big)^{\frac{1}{2}}\|u\|_{L^4}\big\|\nabla|u|^2\big\|_{L^2}\\ &\le \frac{1}{8}\big\|\nabla|u|^2\big\|_{L^2}^2 + \frac{1}{8}\|u\cdot\nabla u\|_{L^2}^2 + C\big(e + \|\pi\|_{B^r_{\infty,\infty}}\big)^{\frac{2}{2+r}}\big(\|u\|_{L^4}^4 + 1\big) + C.\end{aligned} \tag{4.4}$$

Inserting (3.9) and (4.4) into (3.8), we obtain (3.13).

By the classical regularity theory of parabolic equations [14], it follows from (1.2), (1.3), (3.13) and (4.4) that

$$\begin{aligned}\|\nabla n\|_{L^2(0,T;L^{\tilde r})} &\le C\big(1 + \|un\|_{L^2(0,T;L^{\tilde r})} + \big\|n\chi(p)\nabla p\big\|_{L^2(0,T;L^{\tilde r})}\big)\\ &\le C\big(1 + \|u\|_{L^\infty(0,T;L^4)}\|n\|_{L^\infty(0,T;L^{\frac{4\tilde r}{4-\tilde r}})} + \|n\|_{L^\infty(0,T;L^{\frac{q\tilde r}{q-\tilde r}})}\|\nabla p\|_{L^2(0,T;L^q)}\big)\\ &\le C\end{aligned} \tag{4.5}$$

for some $3 < \tilde r < 4$ and $\tilde r < q$.

Therefore,

$$\|n\|_{L^2(0,T;L^\infty)} \le C. \tag{4.6}$$

This completes the proof.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors of the manuscript have read and agreed to its content and are accountable for all the aspects of accuracy and integrity of the manuscript.

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