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Hausdorff measure of noncompactness of matrix operators on some new difference sequence spaces

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Abstract

The new sequence spaces $X(r, s, t; \Delta)$ for $X \in \{l_\infty, c, c_0\}$ have been defined by using generalized means and difference operator. In this work, we establish identities or estimates for the operator norms and the Hausdorff measure of noncompactness of certain matrix operators on some new difference sequence spaces $X(r, s, t; \Delta)$ where $X \in \{l_\infty, c, c_0, l_p\}$ ($1 \leq p < \infty$), as derived by using generalized means. Further, we find the necessary and sufficient conditions for such operators to be compact by applying the Hausdorff measure of noncompactness. Finally, as applications we characterize some classes of compact operators between these new difference sequence spaces and some other BK -spaces.

Keywords: sequence space; difference operators; matrix transformation; generalized means; compact operators; Hausdorff measure of noncompactness

1 Preliminaries and background

The study of sequence spaces has been very useful in many branches of analysis. Recently, some new sequence spaces have been defined by using matrix domain of a suitable matrix. Beside this, the Hausdorff measure of noncompactness is very useful in the classification of compact operators between Banach spaces.

The difference sequence spaces were introduced for the first time by Kizmaz in [1]. Afterwards, many authors have introduced and studied some new sequence spaces defined by using the difference operator. For example, in [2] Ahmad and Mursaleen, in [3] Colak and Et, in [4] Basar and Altay, in [5] Orhan, in [6] Polat and Altay, in [7] Aydin and Basar, and in [8] Basar and Altay have introduced and studied some new sequence spaces defined by using a difference operator. Some authors like Malkowsky and Savas [9], Altay and Basar [10], Mursaleen and Noman [11] and Basarir and Kara [12] have defined the sequence spaces by using the generalized weighted means. For the first time, in 2011 Polat *et al.* [13] have introduced the new sequence spaces by combining both the weighted means and the difference operator. Then in 2015 Manna *et al.* [14] have introduced new sequence spaces defined by using both the generalized means and the difference operator.

In this paper we obtain some identities or estimates for the operator norms and the Hausdorff measure of noncompactness of certain matrix operators on new difference sequence spaces defined by Manna *et al.* Further, we find the necessary and sufficient condi-

tions for such operators to be compact and we consider some matrix mappings between the spaces of the generalized means (these have been defined in [15]) and difference sequence spaces derived by using the generalized means. Finally, as applications we characterize some classes of compact operators between these new difference sequence spaces and some other *BK*-spaces.

In this section, we give some related definitions and preliminary results.

We shall write w for the set of all complex sequences $x = (x_k)_{k=0}^\infty$. Any vector subspace of w is called a sequence space. Let l_∞ , c , and c_0 denote the sets of all bounded, convergent and null sequences, respectively, and $l_p = \{x \in w : \sum_{k=0}^\infty |x_k|^p < \infty, 1 \leq p < \infty\}$. For an infinite matrix A and a sequence space X , the matrix domain of A is defined by $X_A = \{x \in w : Ax \in X\}$. By e and e^n ($n \in \mathbb{N}$), we denote the sequences such that $e_k = 1$ for $k = 0, 1, \dots$ and $e_n^{(n)} = 1$ and $e_k^n = 0$ ($k \neq n$). Let bs and cs be the sequence spaces of all bounded and convergent series, respectively. A sequence (b_n) in a normed linear space $(X, \|\cdot\|)$ is called a Schauder basis for X if for every $x \in X$, there is a unique sequence $(\delta_n)_{n=0}^\infty$ of scalars such that $x = \sum_{n=0}^\infty \delta_n b_n$. For definitions of *K*-space, *FK*-space, *BK*-space and *AK*-property we refer the reader to [16].

Throughout, the matrices are infinite matrices of complex number. Let $A = (a_{nk})_{n,k}$ be an infinite matrix with real or complex entries a_{nk} . We write A_n as the sequence of the n th row of A , that is, $A_n = (a_{nk})_k$ for every n . In addition, if $x = (x_k) \in w$ then we define the A -transform of x as the sequence $Ax = (A_n(x))_{n=0}^\infty$, that is,

$$A_n(x) = \sum_{k=0}^{\infty} a_{nk} x_k \quad (n \in \mathbb{N}), \quad (1)$$

provided the series on the right converges for each $n \in \mathbb{N}$. For any two sequence spaces X and Y , we denote by (X, Y) the class of all infinite matrices A that map X into Y . As is well known the β -dual a subset X^β of w is defined by

$$X^\beta = \{a = (a_k) \in w : ax = (a_k x_k) \in cs \text{ for all } x = (x_k) \in X\}.$$

Thus $A \in (X, Y)$ if and only if $A_n \in X^\beta$ for all $n \in \mathbb{N}$ and $Ax \in Y$ for all $x \in X$. An infinite matrix $T = (t_{nk})$ is said to be triangle if $t_{nk} = 0$ for $k > n$ and $t_{nn} \neq 0$, $n \in \mathbb{N}_0$ (where $\mathbb{N}_0 = \{0, 1, 2, \dots\}$).

If $X \supset \phi$ is a *BK*-space and $a = (a_{nk}) \in w$, then we write

$$\|a\|_X^* = \sup_{x \in S_X} \left| \sum_{k=0}^{\infty} a_k x_k \right|, \quad (2)$$

provided the expression on the right is defined and finite which is the case whenever $a \in X^\beta$ [17].

2 The Hausdorff measure of noncompactness

As is well known, one of the best methods in the characterization of compact operators between the Banach spaces depends on applying the Hausdorff measure of noncompactness. Therefore, many authors like Kara and Basarir in [18, 19], Basarir and Kara in [20, 21], Kara *et al.* in [22] and Alotaibi *et al.* in [23] characterized some classes of compact operators on the spaces in their papers by using the Hausdorff measure of noncompactness.

Let S and M be subsets of a metric space (X, d) and $\epsilon > 0$. We say that S is an ϵ -net of M if for every $x \in M$ there exists $s \in S$ such that $d(x, s) < \epsilon$. The ϵ -net S of M is a finite ϵ -net if S is a finite set.

We denote by M_X the collection of all bounded subsets of a metric space (X, d) . If $Q \in M_X$, we denote by $\chi(Q)$ the Hausdorff measure of noncompactness of the set Q and define it by

$$\chi(Q) = \inf\{\epsilon > 0 : Q \text{ has a finite } \epsilon\text{-net in } X\}.$$

The function $\chi : M_X \rightarrow [0, \infty)$ is called the Hausdorff measure of noncompactness [24]. We can find the basic properties of the Hausdorff measure of noncompactness in [25].

We give an estimate for the Hausdorff measure of noncompactness in Banach spaces with a Schauder basis with the following theorem.

Theorem 2.1 ([25]) *Let X be a Banach space with a Schauder basis $(b_k)_{k=0}^\infty$, $P_n : X \rightarrow X$ ($n \in \mathbb{N}$) be the projector onto the linear span of $\{b_0, b_1, \dots, b_n\}$ and $Q \in M_X$. Then we have*

$$\frac{1}{a} \cdot \limsup \left(\sup_{x \in Q} \|(I - P_n)(x)\| \right) \leq \chi(Q) \leq \limsup \left(\sup_{x \in Q} \|(I - P_n)(x)\| \right),$$

where $a = \limsup_{n \rightarrow \infty} \|I - P_n\|$ and I denotes the identity operator on X .

Now, we show how to compute the Hausdorff measure of noncompactness in the spaces c_0 and l_p ($1 \leq p < \infty$).

Theorem 2.2 ([26]) *Let Q be a bounded subset of the normed space X and X is c_0 or l_p ($1 \leq p < \infty$). If $P_n : X \rightarrow X$ ($n \in \mathbb{N}$) is the operator defined by $P_n(x) = x^{[n]} = (x_0, x_1, \dots, x_n, 0, 0, 0, \dots)$ for all $x = (x_k)_{k=0}^\infty \in X$, then*

$$\chi(Q) = \lim_{n \rightarrow \infty} \left(\sup_{x \in Q} \|(I - P_n)(x)\| \right).$$

We have the following result concerning with the Hausdorff measure of noncompactness in the matrix domain of triangles in normed sequence spaces.

Theorem 2.3 ([27]) *Let X be a normed sequence space, T a triangle, and χ_T and χ denote the Hausdorff measure of noncompactness on M_{X_T} and M_X . Then $\chi_T(Q) = \chi(T(Q))$ for all $Q \in M_{X_T}$.*

Let X and Y be Banach spaces and $L \in B(X, Y)$. Then we denote by $\|L\|_\chi$ the Hausdorff measure of noncompactness of L and define it by

$$\|L\|_\chi = \chi(L(S_X)) = \chi(L(\overline{B}_X)), \quad (3)$$

where $S_X = \{x \in X : \|x\| = 1\}$ is the unit sphere and $\overline{B}_X = \{x \in X : \|x\| \leq 1\}$ is the unit ball in X , and

$$L \text{ is compact if and only if } \|L\|_\chi = 0 \quad [27]. \quad (4)$$

3 The sequence spaces $X(r, s, t; \Delta)$ for $X \in \{l_\infty, c, c_0, l_p\}$ ($1 \leq p < \infty$)

In this subsection we first introduce the generalized means that given by Mursaleen and Noman in [15].

We first define the sets \bar{U} and \bar{U}_0 as follows:

$$\bar{U} = \{u = (u_n)_{n=0}^\infty \in w : u_n \neq 0 \ \forall n\},$$

$$\bar{U}_0 = \{u = (u_n)_{n=0}^\infty \in w : u_0 \neq 0\}.$$

Let $r = (r_n) \in \bar{U}$, $t = (t_n) \in \bar{U}$, and $s = (s_n) \in \bar{U}_0$. For any sequence $x = (x_n) \in w$, we define the sequence $y = (y_n)$ of generalized means of x by:

$$y_n = \frac{1}{r_n} \sum_{k=0}^n s_{n-k} t_k x_k \quad (n \in \mathbb{N}_0). \quad (5)$$

Further, we define the infinite matrix $A(r, s, t)$ of generalized means by

$$(A(r, s, t)_{nk}) = \begin{cases} s_{n-k} t_k / r_n, & 0 \leq k \leq n, \\ 0, & k > n \end{cases}$$

for all $n, k \in \mathbb{N}$. By using the notation (1) and (3), we see that y is the $A(r, s, t)$ -transform of x , that is, $y = (A(r, s, t))x$ for all $x \in w$. Let $D_0^{(s)} = \frac{1}{s_0}$ and

$$D_n^{(s)} = \frac{1}{s_0^{n+1}} \begin{vmatrix} s_1 & s_0 & 0 & 0 & \cdots & 0 \\ s_2 & s_1 & s_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ s_{n-1} & s_{n-2} & s_{n-3} & s_{n-4} & \cdots & s_0 \\ s_n & s_{n-1} & s_{n-2} & s_{n-3} & \cdots & s_1 \end{vmatrix} \quad (n = 1, 2, 3, \dots),$$

so the inverse of $A(r, s, t)$ is the triangle $B = (b_{nk})_{nk}$, which is defined by

$$b_{nk} = \begin{cases} (-1)^{n-k} \frac{D_{n-k}^{(s)}}{t_n} r_k, & 0 \leq k \leq n, \\ 0, & k > n. \end{cases}$$

We now ready to introduce the sequence space $X(r, s, t; \Delta)$ where $X \in \{l_\infty, c, c_0\}$ as the matrix domain of triangle $A(r, s, t; \Delta)$ as follows:

$$X(r, s, t; \Delta) = \left\{ x = (x_n) \in w : \left(\frac{1}{r_n} \sum_{k=0}^n s_{n-k} t_k \Delta x_k \right)_n \in X \right\},$$

which is a combination of the generalized means and the difference operator Δ ($\Delta x_k = x_k - x_{k-1}$, $x_{-1} = 0$) and by using notation (1) we have $X(r, s, t; \Delta) = X_{A(r, s, t; \Delta)}$ where $A(r, s, t; \Delta) = A(r, s, t) \cdot \Delta$. It is clear that if X is a sequence space, then $X(r, s, t; \Delta)$ is a sequence space too and we call that the difference sequence space derived by using generalized means.

Lemma 3.1 ([14]) *The sequence spaces $X(r, s, t; \Delta)$ where $X \in \{l_\infty, c, c_0\}$ are BK-spaces with the norm given by*

$$\begin{aligned}\|x\|_{X(r,s,t;\Delta)} &= \sup_n \left| \frac{1}{r_n} \sum_{k=0}^n s_{n-k} t_k \Delta x_k \right| \\ &= \sup_n \left| (A(r, s, t; \Delta)x)_n \right|\end{aligned}$$

and so they are linearly isomorphic to the spaces $X \in \{l_\infty, c, c_0\}$, respectively.

We mention the following basic lemmas, which will be used in proving our results.

Lemma 3.2 *Let X be a BK-space with AK or $X = l_\infty$ and $\overline{\Delta X} = X(r, s, t; \Delta)$. If $a = (a_k) \in (\overline{\Delta X})^\beta$, then $\tilde{a} = (\tilde{a}_k) \in X^\beta$ and we have*

$$\sum_{k=0}^{\infty} a_k x_k = \sum_{k=0}^{\infty} \tilde{a}_k y_k, \quad (6)$$

which holds for every $x = (x_k) \in X(r, s, t; \Delta)$ with $y = A(r, s, t) \Delta x$ where

$$\tilde{a}_k = \sum_{j=0}^n \sum_{k=0}^{n-j} (-1)^k \frac{D_k^{(s)}}{t_{k+j}} a_j r_j \quad (k \in \mathbb{N}). \quad (7)$$

Proof It is immediate by the same technique as ([11], Theorem 4.5). \square

Lemma 3.3 *Let X be a BK-space with AK property or $X = l_\infty$ and $\overline{\Delta X} = X(r, s, t; \Delta)$. Then we have*

$$\|a\|_{\overline{\Delta X}}^* = \|\tilde{a}\|_{X^\beta}$$

for all $a = (a_k) \in (\overline{\Delta X})^\beta$, where $\tilde{a} = (\tilde{a}_k)$ is the sequence defined by (7).

Proof Let $a = (a_k) \in (\overline{\Delta X})^\beta$. By applying Lemma 3.2 we have $\tilde{a} = (\tilde{a}_k) \in X^\beta$ and equality (6) holds for all sequences $x = (x_k) \in \overline{\Delta X}$ and $y = (y_k) \in X$. Further, it follows by the equality in Lemma 3.1 that $x \in S_{\overline{\Delta X}}$ if and only if $y \in S_X$. So, we derive from (2) and (6) that

$$\begin{aligned}\|a\|_{\overline{\Delta X}}^* &= \sup_{x \in S_{\overline{\Delta X}}} \left| \sum_{k=0}^{\infty} a_k \Delta x_k \right| \\ &= \sup_{y \in S_X} \left| \sum_{k=0}^{\infty} \tilde{a}_k y_k \right| \\ &= \|\tilde{a}\|_{X^\beta}.\end{aligned}$$

This completes the proof. \square

Throughout this paper we assume $A = (a_{nk})$ is an infinite matrix and $\tilde{A} = (\tilde{a}_{nk})$ is the associated matrix which is defined by

$$\tilde{a}_{nk} = \sum_{j=0}^n \sum_{k=0}^{n-j} (-1)^k \frac{D_k^{(s)}}{t_{k+j}} a_{nj} r_j \quad (n, k \in \mathbb{N}), \quad (8)$$

and provided the series on the right converge for all $n, k \in \mathbb{N}$. Then we have the following.

Lemma 3.4 *Let X be a BK-space with AK property or $X = l_\infty$ and $\overline{\Delta X} = X(r, s, t; \Delta)$, Y be any sequence space and $A = (a_{nk})$ be an infinite matrix. If $A \in (\overline{\Delta X}, Y)$, then $\tilde{A} \in (X, Y)$ such that $Ax = \tilde{A}y$ for all $x \in X$ and connected sequence $y = A(r, s, t) \cdot \Delta x$, where $\tilde{A} = (\tilde{a}_{nk})$ is the associated matrix defined by (8).*

Proof Let $x \in \overline{\Delta X}$ and $y \in Y$ be connected by equation (5) and $A \in (\overline{\Delta X}, Y)$. By applying Lemma 3.3, we have $\tilde{A}_n \in X^\beta$ for all $n \in \mathbb{N}$ and the equality $Ax = \tilde{A}y$ holds, hence $\tilde{A}y \in Y$. Because every $y \in Y$ is the associated sequence of some $x \in \overline{\Delta X}$, we conclude that $\tilde{A} \in (X, Y)$. This completes the proof. \square

Theorem 3.1 *Let $\overline{\Delta X} = X(r, s, t; \Delta)$ where X be a BK-space with AK or $X = l_\infty$, $A = (a_{nk})$ is an infinite matrix and $\tilde{A} = (\tilde{a}_{nk})$ is the associated matrix. If A is in any of the classes $(\overline{\Delta X}, l_\infty)$, $(\overline{\Delta X}, c)$, or $(\overline{\Delta X}, c_0)$, then we have*

$$\|L_A\| = \|A\|_{(\overline{\Delta X}, l_\infty)} = \sup_n \|\tilde{A}_n\|_X^* < \infty.$$

Proof By using Lemma 3.1 we know that the spaces $\overline{\Delta X}$ and X are linearly isomorphic. Now by combining ([16], Remark 1.2(a)), and Lemma 3.3 the proof is complete. \square

Theorem 3.2 *Let X be a BK-space with AK or $X = l_\infty$ and $\overline{\Delta X} = X(r, s, t; \Delta)$. If A is in $(\overline{\Delta X}, l_1)$, then*

$$\|A\|_{(\overline{\Delta X}, l_1)} \leq \|L_A\| \leq 4 \cdot \|A\|_{(\overline{\Delta X}, l_1)},$$

where

$$\|A\|_{(\overline{\Delta X}, l_1)} = \sup_{N \in F} \left\| \sum_{n \in N} \tilde{A}_n \right\|_X^* < \infty.$$

Proof By using Lemma 3.1 we know the spaces $\overline{\Delta X}$ and X are linearly isomorphic. Now by combining ([16], Remark 1.2(b)) and Lemma 3.3 the proof is complete. \square

Theorem 3.3 *Let $\overline{\Delta l_1} = l_1(r, s, t; \Delta)$ and $1 \leq p < \infty$. If $A \in (\overline{\Delta l_1}, l_p)$, then*

$$\|L_A\| = \|A\|_{(\overline{\Delta l_1}, l_p)} = \sup_k \left(\sum_{n=0}^{\infty} |\tilde{a}_{nk}|^p \right)^{\frac{1}{p}} < \infty.$$

Proof By using [14] and [11] we deduce $A \in (\overline{\Delta l_1}, l_p)$ if and only if the expression on the right of (8) converges and

$$\sup_k \left(\sum_{n=0}^{\infty} |\tilde{a}_{nk}|^p \right) < \infty$$

and

$$\sup_k \left(\max_{0 \leq k \leq m} |a_{mk}^{(n)}| \right) < \infty \quad (\forall n \in \mathbb{N}),$$

where

$$a_{mk}^{(n)} = \sum_{j=0}^n \sum_{k=0}^{n-j} (-1)^k \frac{D_k^{(s)}}{t_{k+j}} a_{nj} r_j \quad (0 \leq k \leq m, m \in \mathbb{N}).$$

□

By using Theorem 3.2, we immediately obtain the following result.

Corollary 3.1 Let $\overline{\Delta l_p} = l_p(r, s, t; \Delta)$ ($1 \leq p < \infty$) and $A \in (\overline{\Delta l_p}, l_1)$. Then we have

$$\|A\|_{(\overline{\Delta l_p}, l_1)}^{(r)} \leq \|L_A\| \leq 4 \cdot \|A\|_{(\overline{\Delta l_p}, l_1)}^{(r)},$$

where

$$\|A\|_{(\overline{\Delta l_p}, l_1)}^{(r)} = \sup_{N \in F_r} \left(\sum_{k=0}^{\infty} \left| \sum_{n \in N} \tilde{a}_{nk} \right|^q \right)^{\frac{1}{q}} \quad (r \in \mathbb{N}).$$

Proof Let $X = l_p$ in Theorem 3.2. □

Now, as a consequence of Corollary 3.1 we have the following lemma.

Lemma 3.5 Let $\overline{\Delta l_1} = l_1(r, s, t; \Delta)$ and $1 \leq p < \infty$. If $A \in (l_p, \overline{\Delta l_1})$, then

$$\|A\|_{(l_p, \overline{\Delta l_1})}^{(r)} \leq \|L_A\| \leq 4 \cdot \|A\|_{(l_p, \overline{\Delta l_1})}^{(r)},$$

where

$$\|A\|_{(l_p, \overline{\Delta l_1})}^{(r)} = \sup_{N \in F_r} \left(\sum_{k=0}^{\infty} \left| \sum_{n \in N} \tilde{a}_{nk} \right|^q \right)^{\frac{1}{q}} \quad (r \in \mathbb{N}),$$

and \tilde{a}_{nk} is defined by (8).

Proof Because the sequence spaces $\overline{\Delta l_1}$ and l_1 are norm-isomorphic,

$$A \in (l_p, \overline{\Delta l_1}) \cong A \in (l_p, l_1).$$

Now this is a special case of ([16], Remark 1.2(b)) when $X = l_p$.

This completes the proof. □

4 Compact operators on the spaces $X(r, s, t; \Delta)$ for $X \in \{l_\infty, c, c_0, l_p\}$ ($1 \leq p < \infty$)

In this section, we apply our results to obtaining some identities or estimates for the Hausdorff measure of noncompactness of certain matrix operators on the spaces $X(r, s, t; \Delta)$ (where $X \in \{l_\infty, c, c_0, l_p\}$ ($1 \leq p < \infty$)). Also, we consider the necessary and sufficient conditions for such operators to be compact.

Lemma 4.1 ([28]) *Let $X \supset \phi$ be a BK-space. Then we have:*

(a) *If $A \in (X, l_\infty)$, then*

$$0 \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \|A_n\|_X^*.$$

(b) *If $A \in (X, c_0)$, then*

$$\|L_A\|_X = \limsup_{n \rightarrow \infty} \|A_n\|_X^*.$$

(c) *If X has AK or $X = l_\infty$ and $A \in (X, c)$, then*

$$\frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \|A_n - \alpha\|_X^* \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \|A_n - \alpha\|_X^*,$$

where $\alpha = (\alpha_k)$ with $\alpha_k = \lim_{n \rightarrow \infty} a_{nk}$ for all $k \in \mathbb{N}$.

Now, by combining Lemmas 3.3, 3.4, and 4.1 we have the next result.

Theorem 4.1 *Let X be a BK-space with AK or $X = l_\infty$ and $\overline{\Delta X} = X(r, s, t; \Delta)$. Then we have:*

(a) *If $A \in (\overline{\Delta X}, l_\infty)$, then*

$$0 \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n| \right), \quad (9)$$

and

$$L_A \text{ is compact if } \lim_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n| \right) = 0. \quad (10)$$

(b) *If $A \in (\overline{\Delta X}, c_0)$, then*

$$\|L_A\|_X = \limsup_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n| \right), \quad (11)$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n| \right) = 0. \quad (12)$$

(c) *If $A \in (\overline{\Delta X}, c)$, then*

$$\frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n - \tilde{\alpha}| \right) \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n - \tilde{\alpha}| \right) \quad (13)$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n - \tilde{\alpha}| \right) = 0, \quad (14)$$

where $\tilde{\alpha} = (\tilde{\alpha}_k)$ with $\tilde{\alpha}_k = \lim_{n \rightarrow \infty} \tilde{a}_{nk}$ for all $k \in \mathbb{N}$.

Proof It is clear that (10), (12), and (14) are, respectively, obtained from (9), (11), and (13) by using (4). So, to prove (9), (11), and (13) we have the following.

Because $\overline{\Delta X}$ is a BK -space, hence by combining Lemma 3.3 and parts (a) and (b) of Lemma 4.1 we obtain (9) and (11).

To prove (13), if $A \in (\overline{\Delta X}, c)$, then by Lemma 3.4 we have $\tilde{A} \in (X, c)$. So, by part (c) of Lemma 4.1 we have

$$\frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n - \tilde{\alpha}| \right) \leq \|L_{\tilde{A}}\|_X \leq \limsup_{n \rightarrow \infty} \left(\sum_n |\tilde{A}_n - \tilde{\alpha}| \right), \quad (15)$$

where $\tilde{\alpha} = (\tilde{\alpha}_k)$ and $\tilde{\alpha}_k = \lim_{n \rightarrow \infty} \tilde{a}_{nk}$ for all $k \in \mathbb{N}$. Now, we write $S = S_X$ and $\overline{\Delta S} = S_{\overline{\Delta X}}$ for short. Then we obtain by (3) and ([16], Remark 1.2(c))

$$\|L_A\|_X = \chi(L_A(\overline{\Delta S})) = \chi(A\overline{\Delta S}) \quad (16)$$

and

$$\|L_{\tilde{A}}\|_X = \chi(L_{\tilde{A}}(S)) = \chi(\tilde{A}S). \quad (17)$$

Also, by using the equality in Lemma 3.1 we have $x \in \overline{\Delta S}$ if and only if $y \in S$, and since $Ax = \tilde{A}y$ by Lemma 3.4, we derive that $A\overline{\Delta S} = \tilde{A}S$. This result together (16) and (17) leads to $\|L_A\|_X = \|L_{\tilde{A}}\|_X$. So, we get (13) from (15). This completes the proof. \square

The following example will show that it is possible for L_A in (10) to be compact but $\lim_{n \rightarrow \infty} (\sum_n |\tilde{A}_n|) \neq 0$. Hence, in general, we have just ‘if’ in (10) of Theorem 4.1(a).

Example 4.1 Let $\overline{\Delta X} = X(r, s, t; \Delta)$ where $X \in \{l_\infty, c, c_0\}$. Then let for $n = 1$ the sequence $s = (s_n) = 0$ and for every $n \neq 1$, $s = (s_n) = 1$. Also, let the sequences $r = (r_n) = 1$, $t = (t_n) = 1$ are constant. Define the matrix $A = (a_{nk})$ by

$$a_{nk} = \begin{cases} \frac{s_{n-k}t_k}{r_n} & (0 \leq k \leq 2), \\ 0 & (k \geq 3). \end{cases}$$

Then, for every $x = (x_n) \in \overline{\Delta X}$, we have $Ax = [\frac{s_2t_0}{r_2}x_0e^{(0)} + \frac{s_1t_1}{r_2}x_1e^{(1)} + \frac{s_0t_2}{r_2}x_2e^{(2)}]$. So $A \in (\overline{\Delta X}, l_\infty)$. It is clear that L_A is of finite rank so is compact. By using (8) we obtain $\tilde{A}_n = 1$, for all $n \in \mathbb{N}$. Thus, we have that $\sum_n |\tilde{A}_n| = 1$ for all $n \in \mathbb{N}$, which implies that $\lim_{n \rightarrow \infty} \sum_n |\tilde{A}_n| = 1 \neq 0$.

Let F_r ($r \in \mathbb{N}$) be the subcollection of F consisting of all nonempty and finite subsets of \mathbb{N} with elements that are greater than r , that is,

$$F_r = \{N \in F : n > r \text{ for all } n \in N\} \quad (r \in \mathbb{N}).$$

Lemma 4.2 ([11]) *Let $X \supset \phi$ be a BK -space. If $A \in (X, l_1)$, then*

$$\lim_{r \rightarrow \infty} \left(\sup_{N \in F_r} \left\| \sum_{n \in N} A_n \right\|_X^* \right) \leq \|L_A\|_X \leq 4 \cdot \lim_{r \rightarrow \infty} \left(\sup_{N \in F_r} \left\| \sum_{n \in N} A_n \right\|_X^* \right).$$

By using Lemma 4.2, we have the next result.

Theorem 4.2 *Let X be a BK-space with AK and $\overline{\Delta X} = X(r, s, t; \Delta)$. If $A \in (\overline{\Delta X}, l_1)$, then*

$$\lim_{r \rightarrow \infty} \|A\|_{(\overline{\Delta X}, l_1)}^{(r)} \leq \|L_A\|_X \leq 4 \cdot \lim_{r \rightarrow \infty} \|A\|_{(\overline{\Delta X}, l_1)}^{(r)}, \quad (18)$$

where

$$\|A\|_{(\overline{\Delta X}, l_1)}^{(r)} = \sup_{N \in F_r} \left\| \sum_{n \in N} \tilde{A}_n \right\|_X^* \quad (r \in \mathbb{N})$$

and

$$L_A \text{ is compact if and only if } \lim_{r \rightarrow \infty} \|A\|_{(\overline{\Delta X}, l_1)}^{(r)} = 0. \quad (19)$$

Proof By combining Lemma 3.3 and Lemma 4.2 we obtain (18). By using (4), we get (19) from (18). \square

By using the previous results we have the next result.

Theorem 4.3 *Let $\overline{\Delta l_1} = l_1(r, s, t; \Delta)$ and $1 \leq p \leq \infty$. If $A \in (\overline{\Delta l_1}, l_p)$, then*

$$\|L_A\|_X = \lim_{r \rightarrow \infty} \left(\sup_k \left(\sum_{n=r}^{\infty} |\tilde{a}_{nk}|^p \right)^{\frac{1}{p}} \right) \quad (20)$$

and

$$L_A \text{ is compact if and only if } \lim_{r \rightarrow \infty} \left(\sup_k \left(\sum_{n=r}^{\infty} |\tilde{a}_{nk}|^p \right)^{\frac{1}{p}} \right) = 0. \quad (21)$$

Proof It can be similarly proved by the same technique as Theorem 3.9 in [15]. \square

In the following example we show that there exist matrix operators in the class $B(\overline{\Delta l_1}, l_p)$ which are not compact ($1 \leq p < \infty$), that is, the limit in (20) may not be zero.

Example 4.2 Let $A = (a_{nk})$ be the infinite matrix defined by

$$a_{nk} = \begin{cases} s_{n-k} t_k / r_n, & 0 \leq k \leq 1, \\ 0, & k > 2, \end{cases}$$

where $s = (s_n) = 1$ if $n = 0$ and $s = (s_n) = 0$ for every $n \neq 0$ and $r = (r_n) = 1 = t = (t_n)$. Then $A \in (\overline{\Delta l_1}, l_1)$ because $\overline{\Delta l_1}$ is the matrix domain of A in l_1 and hence $A \in (\overline{\Delta l_1}, l_p)$ ($1 \leq p < \infty$). Further, it is easy to show that the associated matrix \tilde{A} is the identity matrix, that is, $\tilde{a}_{nn} = 1$ and $\tilde{a}_{nk} = 0$ for $k \neq n$ ($n \in \mathbb{N}$). Now, let $r \in \mathbb{N}$ be given. Then we have, for every $k \in \mathbb{N}$,

$$\sum_{n=r}^{\infty} |\tilde{a}_{nk}|^p = \begin{cases} 1, & k \geq r, \\ 0, & k < r. \end{cases}$$

This implies that

$$\sup_k \left(\sum_{n=r}^{\infty} |\tilde{a}_{nk}|^p \right)^{\frac{1}{p}} = 1 \quad (r \in \mathbb{N}),$$

which shows that by (20) that $\|L_A\|_X = 1 \neq 0$ and hence L_A is not compact.

By using Lemmas 3.5 and 4.2 we have the following result on the Hausdorff measure of noncompactness.

Corollary 4.1 *Let $\overline{\Delta l_1} = l_1(r, s, t; \Delta)$ and $1 \leq p \leq \infty$. If $A \in (l_p, \overline{\Delta l_1})$, then*

$$\|A\|_{(l_p, \overline{\Delta l_1})}^{(r)} \leq \|L_A\|_X \leq 4 \cdot \|A\|_{(l_p, \overline{\Delta l_1})}^{(r)}, \quad (22)$$

where

$$\|A\|_{(l_p, \overline{\Delta l_1})}^{(r)} = \sup_k \left(\sum_{n=r}^{\infty} |\tilde{a}_{nk}|^q \right)^{\frac{1}{q}} \quad (r \in \mathbb{N})$$

and

$$L_A \text{ is compact if and only if } \lim_{r \rightarrow \infty} \|A\|_{(l_p, \overline{\Delta l_1})}^{(r)} = 0. \quad (23)$$

Proof We know the spaces l_1 and $\overline{\Delta l_1}$ are norm-isomorphic. So, the proof is complete by Lemmas 3.5 and 4.2. \square

5 Some applications

In this section, by applying our results in the previous sections we consider some relations between the spaces of generalized means and difference sequence spaces derived by using the generalized means and we characterize some classes of compact operators between these new difference sequence spaces and some other BK -spaces. Also, we derive some identities and estimates for the operator norms and the Hausdorff measure of noncompactness on these spaces.

Throughout, $X(r, s, t)$ is the sequence space of the generalized means and for more details we refer the reader to [11]; further, for an arbitrary BK -space X , we put $\overline{X} = X(r, s, t)$, and $\overline{\Delta X} = X(r, s, t; \Delta)$.

The first result is a consequence of Theorem 3.1 and Theorem 4.1.

Corollary 5.1 *Let X be a BK -space with AK or $X = l_{\infty}$. Then:*

If A is in any of the classes $(\overline{\Delta X}, \overline{l_{\infty}})$, $(\overline{\Delta X}, \overline{c})$, or $(\overline{\Delta X}, \overline{c_0})$ then

$$\|L_A\| = \|A\|_{(\overline{\Delta X}, \overline{l_{\infty}})} = \sup_n \|\tilde{A}_n\|_X^* < \infty,$$

where $\tilde{A}_n = (\tilde{a}_{nk})$ is the associated matrix defined by

$$\tilde{a}_{nk} = \sum_{j=k}^{\infty} (-1)^{j-k} D_{j-k}^{(s)} r_k a_{nj} / t_j \quad (n, k \in \mathbb{N}). \quad (24)$$

Further:

(a) If $A \in (\overline{\Delta X}, \overline{l_\infty})$, then

$$0 \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \|\tilde{A}_n\|_X^*$$

and

$$L_A \text{ is compact if } \lim_{n \rightarrow \infty} \|\tilde{A}_n\|_X^* = 0.$$

(b) If $A \in (\overline{\Delta X}, \overline{c})$, then

$$\frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \|\tilde{A}_n - \tilde{\alpha}\|_X^* \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \|\tilde{A}_n - \tilde{\alpha}\|_X^*$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \|\tilde{A}_n - \tilde{\alpha}\|_X^* = 0,$$

where $\tilde{\alpha} = (\tilde{\alpha}_k)$ with $\tilde{\alpha}_k = \lim_{n \rightarrow \infty} \tilde{a}_{nk}$ for all $k \in \mathbb{N}$.

(c) If $A \in (\overline{\Delta X}, \overline{c_0})$, then

$$\|L_A\|_X = \limsup_{n \rightarrow \infty} \|\tilde{A}_n\|_X^*$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \|\tilde{A}_n\|_X^* = 0.$$

Proof We know that the spaces $\overline{l_\infty}$, \overline{c} , $\overline{c_0}$ and l_∞ , c , c_0 are norm-isomorphic, respectively. Now by Theorems 3.1 and 4.1 the proof is completed. \square

The next result is a consequence of Theorem 3.2 and Theorem 4.2.

Corollary 5.2 Let X be a BK-space with AK or $X = l_\infty$. If $A \in (\overline{\Delta X}, \overline{l_1})$, then

$$\|A\|_{(\overline{\Delta X}, \overline{l_1})} \leq \|L_A\| \leq 4 \cdot \|A\|_{(\overline{\Delta X}, \overline{l_1})},$$

where

$$\|A\|_{(\overline{\Delta X}, \overline{l_1})} = \sup_{N \in F} \left\| \sum_{n \in N} \tilde{A}_n \right\|_X^* < \infty,$$

and $\tilde{A}_n = (\tilde{a}_{nk})$ is similarly defined by (24).

Further:

If $A \in (\overline{\Delta X}, \overline{l_1})$, then

$$\|A\|_{(\overline{\Delta X}, \overline{l_1})}^{(r)} \leq \|L_A\|_X \leq 4 \cdot \|A\|_{(\overline{\Delta X}, \overline{l_1})}^{(r)},$$

where

$$\|A\|_{(\overline{\Delta X}, \overline{l_1})}^{(r)} = \sup_{N \in F_r} \left\| \sum \tilde{A}_n \right\|_X^* < \infty$$

and

$$L_A \text{ is compact if and only if } \lim_{r \rightarrow \infty} \|A\|_{(\overline{\Delta X}, \overline{l_1})}^{(r)} = 0.$$

Proof We know that the spaces $\overline{l_1}$ and l_1 are norm-isomorphic. Now by Theorems 3.2 and 4.2 the proof is complete. \square

Now we have a special case of Theorem 3.3 in [28] in the new spaces $\overline{\Delta X}$ where $X \in \{l_\infty, c, c_0\}$.

Corollary 5.3 *Let X be a BK-space with AK or $X = l_\infty$ and \overline{Y} denote any of the spaces $\overline{l_\infty}$, \overline{c} , or $\overline{c_0}$ we and let A be an infinite matrix. Then:*

If $A \in (\overline{Y}, \overline{\Delta X})$ then we have

$$\|L_A\| = \|A\|_{(\overline{Y}, \overline{\Delta l_\infty})} = \sup_n \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk}| \right) < \infty,$$

where $\tilde{A}_n = (\tilde{a}_{nk})$ is the associated matrix defined by (8).

Further:

(a) *If $A \in (\overline{l_\infty}, \overline{\Delta l_\infty})$, then*

$$0 \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk}| \right)$$

and

$$L_A \text{ is compact if } \lim_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk}| \right) = 0.$$

(b) *If $A \in (\overline{c}, \overline{\Delta l_\infty})$, then*

$$\frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk} - \tilde{\alpha}_k| \right) \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk} - \tilde{\alpha}_k| \right)$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk} - \tilde{\alpha}_k| \right) = 0,$$

where $\tilde{\alpha} = (\tilde{\alpha}_k)$ with $\tilde{\alpha}_k = \lim_{n \rightarrow \infty} \tilde{a}_{nk}$ for all $k \in \mathbb{N}$.

(c) *If $A \in (\overline{c}, \overline{\Delta l_\infty})$, then*

$$\|L_A\|_X = \limsup_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk}| \right)$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \left(\sum_{k=0}^{\infty} |\tilde{a}_{nk}| \right) = 0.$$

The spaces bs , cs , and cs_0 are BK -spaces with the same norm given by $\|x\|_{bs} = \sup_n |\sum_{k=0}^n x_k|$. Now, by using Theorems 3.1, 4.1 and 3.2, 4.2 we derive the next consequence.

Corollary 5.4 *Let $X \supset \phi$ be a BK -space and A an infinite matrix. If A is in any of the classes $(\overline{\Delta X}, cs_0)$, $(\overline{\Delta X}, cs)$, or $(\overline{\Delta X}, bs)$, then*

$$\|L_A\| = \sup_n \|\tilde{A}\|_{(\overline{\Delta X}, bs)}^{(n)} < \infty, \quad (25)$$

where

$$\|\tilde{A}\|_{(\overline{\Delta X}, bs)}^{(n)} = \sup_n \left(\sum_{k=0}^{\infty} \left| \sum_{m=0}^n \tilde{a}_{mk} \right| \right) \quad (n \in \mathbb{N}).$$

Furthermore, we have:

(a) If $A \in (\overline{\Delta X}, cs_0)$, then

$$\|L_A\|_X = \limsup_{n \rightarrow \infty} \|\tilde{A}\|_{(\overline{\Delta X}, bs)}^{(n)} \quad (26)$$

and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \|\tilde{A}\|_{(\overline{\Delta X}, bs)}^{(n)} = 0. \quad (27)$$

(b) If $A \in (\overline{\Delta X}, bs)$, then

$$0 \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \|\tilde{A}\|_{(\overline{\Delta X}, bs)}^{(n)} \quad (28)$$

and

$$L_A \text{ is compact if } \lim_{n \rightarrow \infty} \|\tilde{A}\|_{(\overline{\Delta X}, bs)}^{(n)} = 0. \quad (29)$$

(c) If X has AK and $A \in (\overline{\Delta X}, cs)$, then

$$\frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \|\tilde{A} - \tilde{\alpha}\|_{(\overline{\Delta X}, bs)}^{(n)} \leq \|L_A\|_X \leq \limsup_{n \rightarrow \infty} \|\tilde{A} - \tilde{\alpha}\|_{(\overline{\Delta X}, bs)}^{(n)}, \quad (30)$$

where $\tilde{\alpha} = (\tilde{\alpha}_k)$ with $\tilde{\alpha}_k = \lim_{m \rightarrow \infty} \tilde{a}_{mk}$ for all $k \in \mathbb{N}$ and

$$L_A \text{ is compact if and only if } \lim_{n \rightarrow \infty} \limsup_{n \rightarrow \infty} \|\tilde{A} - \tilde{\alpha}\|_{(\overline{\Delta X}, bs)}^{(n)} = 0. \quad (31)$$

Proof The proof is quite similar to Corollary 5.1 in [16] for the new spaces $\overline{\Delta X}$ where $X \in \{l_\infty, c, c_0\}$. \square

We denote the space of all sequences of bounded variation by bv and define

$$bv = \{x = (x_k) \in w : (x_k - x_{k-1}) \in l_1\}.$$

So, the space bv is a BK -space with

$$\|x\|_{bv} = \sum_{k=0}^{\infty} |x_k - x_{k-1}|.$$

Now, we have the following result.

Corollary 5.5 *Let $X \supset \phi$ be a BK -space. If $A \in (\overline{\Delta X}, bv)$, then*

$$\|A\|_{(\overline{\Delta X}, bv)} \leq \|L_A\| \leq 4 \cdot \|A\|_{(\overline{\Delta X}, bv)},$$

where

$$\|A\|_{(\overline{\Delta X}, bv)} = \sup_{N \in F} \left(\sum_{n \in N} \left| \sum_{k=0}^n (\tilde{A}_k - \tilde{A}_{k-1}) \right| \right) < \infty.$$

Furthermore:

If $A \in (\overline{\Delta X}, bv)$, then

$$\|A\|_{(\overline{\Delta X}, bv)}^{(r)} \leq \|L_A\|_X \leq 4 \cdot \|A\|_{(\overline{\Delta X}, bv)}^{(r)},$$

where

$$\|A\|_{(\overline{\Delta X}, bv)}^{(r)} = \sup_{N \in F_r} \left(\sum_{n \in N} \left| \sum_{k=0}^n (\tilde{A}_k - \tilde{A}_{k-1}) \right| \right) < \infty \quad (r \in \mathbb{N})$$

and

$$L_A \text{ is compact if and only if } \lim_{r \rightarrow \infty} \|A\|_{(\overline{\Delta X}, bv)}^{(r)} = 0.$$

Proof We proceed by Theorem 4.4 in [28] and Theorem 4.2. □

We denote bv^p for the space of all sequences of p -bounded variation and define it by

$$bv^p = \{x = (x_k) \in w : (x_k - x_{k-1}) \in l_p\} \quad (1 < p < \infty).$$

bv^p is a BK -space with its natural norm (cf. [17]). For every $a = (a_k) \in (bv^p)^\beta$, we have

$$\|a\|_{bv^p}^* = \left(\sum_{k=0}^{\infty} \left| \sum_{j=k}^{\infty} a_j \right|^q \right)^{\frac{1}{q}}. \quad (32)$$

Corollary 5.6 *Let X denote any of the spaces l_∞ or c_0 , and let A be an infinite matrix, $1 < p < \infty$, and $q = p/(p-1)$. If $A \in (bv^p, \overline{\Delta X})$, then*

$$\|L_A\| = \|A\|_{(bv^p, \overline{\Delta X})}^{(r)} = \sup_{n > r} \left(\sum_{k=0}^{\infty} \left| \sum_{j=k}^{\infty} \tilde{a}_{nj} \right|^q \right)^{\frac{1}{q}} \quad (r \in \mathbb{N}).$$

Further:

(a) If $A \in (bv^p, \overline{\Delta I_\infty})$, then

$$0 \leq \|L_A\|_X \leq \lim_{r \rightarrow \infty} \|A\|_{(bv^p, \overline{\Delta I_\infty})}^{(r)}$$

and

$$L_A \text{ is compact if } \lim_{r \rightarrow \infty} \|A\|_{(bv^p, \overline{\Delta I_\infty})}^{(r)} = 0.$$

(b) If $A \in (bv^p, \overline{\Delta c_0})$, then

$$\|L_A\|_X = \lim_{r \rightarrow \infty} \|A\|_{(bv^p, \overline{\Delta I_\infty})}^{(r)}$$

and

$$L_A \text{ is compact if and only if } \lim_{r \rightarrow \infty} \|A\|_{(bv^p, \overline{\Delta I_\infty})}^{(r)} = 0.$$

Proof The proof is a special case of ([28], Theorem 3.3) when $X = bv^p$. Then this completes the proof Lemma 3.1. \square

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors of the manuscript have read and agreed to its content and are accountable for all aspects of the accuracy and integrity of the manuscript in accordance with ICMJE criteria.

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