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Convergence results of a matrix splitting algorithm for solving weakly nonlinear complementarity problems

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Abstract

In this paper, we consider a class of weakly nonlinear complementarity problems (WNCP) with large sparse matrix. We present an accelerated modulus-based matrix splitting algorithm by reformulating the WNCP as implicit fixed point equations based on two splittings of the system matrixes. We show that, if the system matrix is a *P*-matrix, then under some mild conditions the sequence generated by the algorithm is convergent to the solution of WNCP.

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Keywords: weakly nonlinear complementarity problems; modulus-based; matrix splitting algorithm; convergence analysis

1 Introduction

Consider the following weakly nonlinear complementarity problems, which is to find $z \in R^n$ such that

$$z \geq 0, \quad Az + \Psi(z) \geq 0, \quad z^T(Az + \Psi(z)) = 0, \tag{1}$$

abbreviated as WNCP, where $A = (a_{ij}) \in R^{n \times n}$ is a given large, sparse and real matrix, $\Psi(z): R^n \to R^n$ is a Lipschitz continuous nonlinear function, $z \geq 0$ means $z_i \geq 0$, $i = 1, 2, \ldots, n$, and T always denotes the transpose of a vector.

As is well known, the classical nonlinear complementarity problems (NCP) are very important and fundamental topics in optimization theory and they have been developed into a well-established and fruitful principle. See [1–3] for details as regards the basic theories, effective algorithms and important applications of NCP. Problem (1) is a special case of NCP, but it extends from linear complementarity problems. When $\Psi(z) = q$ is a constant vector, problem (1) reduces to a linear complementarity problem. Recently, lots of researchers [4–8] have paid close attention to feasible and efficient methods for solving linear complementarity problems. Especially, by reformulating linear complementarity problems as an implicit fixed point equation, Van Bokhoven [9] proposed a modulus iteration method, which is defined as the solution of system of linear equations at each iteration. In 2010, Bai [10] presented a modulus-based matrix splitting iteration method and showed the convergence when the system matrix is an H_+-matrix. Consequently, Zhang

[11] proposed two-step modulus-based matrix splitting iteration methods and considered the convergence theory when the system matrix is an H_+-matrix. Based on the above work, for solving problem (1), in [12] Sun and Zeng proposed a modified semismooth Newton method with A being an M-matrix and $\Psi(z)$ being a continuously differentiable monotone diagonal function on R^n.

In this paper, for satisfying the requirements of the application, more details can be found in [13–16], we present an accelerated modulus-based matrix splitting algorithm for dealing with WNCP. The organization of this paper is as follows: Some necessary notations and definitions are introduced in Section 2. In Section 3, we establish a class of accelerated modulus-based matrix splitting iteration algorithms. In Section 4, the convergence conditions are considered.

2 Preliminaries

Some necessary notations, definitions and lemmas used in the sequel discussions are introduced in this section. For $B \in R^{n \times n}$, we write B^{-1}, B^T, $\rho(B)$ to denote the inverse, the transpose, the spectral radius of the matrix B, respectively. For $x \in R^n$, we write $\|x\|$, $|x|$ to denote the norm of the vector x, $|x| = (|x_1|, \ldots, |x_n|)$, respectively. $\|A\|$ denotes any norm of matrix A. Especially, we use $\| \cdot \|_2$ to denote a spectral norm. $\lambda \in \lambda(A)$ denotes the eigenvalue of matrix A where $\lambda(A)$ is the set of all eigenvalues of matrix A.

Definition 1 [17] *If for any* $x := (x_1, x_2, \ldots, x_n) \neq 0$, *there exists an index* k, *such that* $x_k(Ax)_k = x_k(a_{k1}x_1 + \cdots + a_{kn}x_n) > 0$, *we call that matrix* A *is a P-matrix.*

Definition 2 *For the function* $f(x) : R^n \to R^n$, *if for any* $x, y \in R^n$, *there exists a constant* L *such that*

$$\left\| f(x) - f(y) \right\| \leq L \|x - y\|,$$

then we call f *a Lipschitz continuous function on* R^n, *and* L *is called a Lipschitz constant.*

Lemma 3 [18] *Let* $A = (a_{ij}) \in R^{n \times n}$ *be a P-matrix, for any nonnegative diagonal matrix* Ω, *the matrix* $A + \Omega$ *is nonsingular.*

3 Algorithm

Theorem 4 *Let* $M_1 - N_1 = M_2 - N_2 = A$ *be two splittings of the matrix* $A \in R^{n \times n}$ *and* Ω, Γ *be* $n \times n$ *positive diagonal matrices,* Ω_1, Ω_2 *be* $n \times n$ *nonnegative diagonal matrices such that* $\Omega = \Omega_1 + \Omega_2$, *then the following statements hold.*

(i) *If* z *is a solution of problem* (1), *then* $x = \frac{1}{2}(\Gamma^{-1}z - \Omega^{-1}(Az + \Psi(z)))$ *satisfies the implicit fixed point equation*

$$(M_1\Gamma + \Omega_1)x = (N_1\Gamma - \Omega_2)x + (\Omega - M_2\Gamma)|x| + N_2\Gamma|x| - \Psi\big(\Gamma(|x| + x)\big). \tag{2}$$

(ii) *If* x *satisfies the implicit fixed point equation* (2), *then*

$$z = \Gamma\big(|x| + x\big)$$

is a solution of problem (1).

Proof First we prove part (i). Since z is a solution of problem (1), we have $z \geq 0$. So there exists $x \in R^n$ such that

$$z = \Gamma(|x| + x).$$

Define another nonnegative vector,

$$\upsilon = \Omega(|x| - x).$$

It is easy to see that $\upsilon \geq 0$, $z^T \upsilon = 0$, and $\upsilon = Az + \Psi(z)$ if and only if

$$\Omega(|x| - x) = A\Gamma(|x| + x) + \Psi(\Gamma(|x| + x)).$$

Replace A with $M_1 - N_1$, we have

$$(M_1\Gamma + \Omega)x = N_1\Gamma x + (\Omega - (M_1 - N_1)\Gamma)|x| - \Psi(\Gamma(|x| + x)).$$

Taking $M_1 - N_1 = M_2 - N_2$ and $\Omega = \Omega_1 + \Omega_2$ into account, it follows that

$$(M_1\Gamma + \Omega_1)x = (N_1\Gamma - \Omega_2)x + (\Omega - M_2\Gamma)|x| + N_2\Gamma|x| - \Psi(\Gamma(|x| + x)).$$

This shows that equation (2) holds.

We now turn to prove part (ii). By some simple calculations, the implicit fixed point equation (2) is equivalent to

$$A\Gamma(|x| + x) + \Psi(\Gamma(|x| + x)) = \Omega(|x| - x).$$

Set $z = \Gamma(|x| + x)$ and $\upsilon = \Omega(|x| - x)$. Evidently, it yields $z \geq 0$, $\upsilon \geq 0$, $z^T \upsilon = 0$, and $\upsilon = Az + \Psi(z)$, which means that z is a solution of problem (1). This completes the proof. $\square$

Note that the implicit fixed point equation (2) includes many parameters that are quite complicated to be determined in a computation. For solving this problem, we will give the simple formulation as follows. Subsequently, a matrix splitting iteration algorithm will be given for solving WNCP. Let $\Omega_1 = \Omega$, $\Omega_2 = 0$, $\Gamma = \frac{1}{\gamma}I$, where $\gamma > 0$ is a real number, then the implicit fixed point equation reduces to

$$(M_1 + \gamma\Omega)x = N_1 x + (\gamma\Omega - M_2)|x| + N_2|x| - \gamma\Psi\left(\frac{(|x| + x)}{\gamma}\right).$$

In fact, $\gamma\Omega$ in the above equation denotes a positive diagonal parameter matrix, which can be replaced by Ω for simplicity. That is, the above equation is essentially equivalent to

$$(M_1 + \Omega)x = N_1 x + (\Omega - M_2)|x| + N_2|x| - \gamma\Psi\left(\frac{(|x| + x)}{\gamma}\right). \tag{3}$$

In the rest of the paper, we will use the fixed point equation above to give the algorithm and convergence analysis for solving WNCP.

Algorithm 5

Step 1. Choose two splittings of the matrix $A \in R^{n \times n}$ satisfying $A = M_1 - N_1 = M_2 - N_2$.

Step 2. Set $k = 0$. Give an initial vector $x^0 \in R^n$, compute $z^0 = \frac{1}{\gamma}(|x^0| + x^0)$.

Step 3. Choose x^{k+1} such that

$$(M_1 + \Omega)x^{k+1} = N_1 x^k + (\Omega - M_2)|x^k| + N_2|x^{k+1}| - \gamma \Psi(z^k), \tag{4}$$

and set

$$z^{k+1} = \frac{1}{\gamma}(|x^{k+1}| + x^{k+1}), \quad k = k + 1.$$

Here, $\Omega, \Gamma \in R^{n \times n}$ are positive diagonal matrices and γ is a positive constant.

Step 4. If the sequence $\{z^k\}_0^{+\infty}$ is convergent, stop. Otherwise, go to Step 3.

4 Convergence theorems

In this section, we will consider the conditions that ensure the convergence of $\{z^k\}_0^{+\infty}$ obtained by Algorithm 5.

Theorem 6 *Let $A \in R^{n \times n}$ be a P-matrix, $M_1 - N_1 = M_2 - N_2 = A$ be two splittings of the matrix A with $M_1 \in R^{n \times n}$ being a P-matrix. Assume that $\Omega \in R^{n \times n}$ is a positive diagonal matrix, γ is a positive constant and $\Psi(z): R^n \to R^n$ is a Lipschitz continuous function with the Lipschitz constant L. Set*

$$\rho(\Omega) = 2g(\Omega) + 2q(\Omega) + f(\Omega),$$

where $g(\Omega) = \|(M_1 + \Omega)^{-1} N_2\|$, $q(\Omega) = \|(M_1 + \Omega)^{-1} N_1\| + L\|(M_1 + \Omega)^{-1}\|$, $f(\Omega) = \|(M_1 + \Omega)^{-1}(\Omega - M_1)\|$. If the matrix Ω satisfies $\rho(\Omega) < 1$, then for any initial vector $x^0 \in R^n$, the iteration sequence $\{z^k\}_{k=0}^{+\infty}$ generated by Algorithm 5 converges to a solution $z^* \in R_+^n$ of problem (1).

Proof Suppose that $z^* \in R_+^n$ is a solution of problem (1). Note that $\Gamma = \frac{1}{\gamma}I$, by Theorem 4 and (3), we see that $x^* = \frac{1}{2}(\gamma z^* - \Omega^{-1}(A(z^*) + \Psi(z^*)))$ is a solution of the equation

$$(M_1 + \Omega)x^* = N_1 x^* + (\Omega - M_2)|x^*| + N_2|x^*| - \gamma \Psi(z^*) \tag{5}$$

with $z^* = \frac{1}{\gamma}(|x^*| + x^*)$. Let us consider (4) minus (5); we have

$$(M_1 + \Omega)(x^{k+1} - x^*)$$
$$= N_1(x^k - x^*) + (\Omega - M_2)(|x^k| - |x^*|)$$
$$\quad + N_2(|x^{k+1}| - |x^*|) - \gamma(\Psi(z^k) - \Psi(z^*)).$$

Note that $M_1 - N_1 = M_2 - N_2$, we have

$$(\Omega - M_2)(|x^k| - |x^*|)$$
$$= (\Omega - M_2 + N_2 - N_2)(|x^k| - |x^*|)$$

$$= (\Omega - M_1 + N_1 - N_2)(|x^k| - |x^*|)$$
$$= (\Omega - M_1)(|x^k| - |x^*|) + N_1(|x^k| - |x^*|) - N_2(|x^k| - |x^*|). \tag{6}$$

Since M_1 is a P-matrix and Ω is a positive diagonal matrix, it follows from Lemma 3 that $M_1 + \Omega$ is a nonsingular matrix. Hence, by (5) and (6), we have

$$x^{k+1} - x^*$$
$$= (M_1 + \Omega)^{-1} N_1(x^k - x^*) + (M_1 + \Omega)^{-1}(\Omega - M_1)(|x^k| - |x^*|)$$
$$\quad + (M_1 + \Omega)^{-1} N_1(|x^k| - |x^*|) - (M_1 + \Omega)^{-1} N_2(|x^k| - |x^*|)$$
$$\quad + (M_1 + \Omega)^{-1} N_2(|x^{k+1}| - |x^*|) - (M_1 + \Omega)^{-1} \gamma(\Psi(z^k) - \Psi(z^*)). \tag{7}$$

Taking the facts

$$\|z^k - z^*\| = \left\| \frac{|x^k| + x^k}{\gamma} - \frac{|x^*| + x^*}{\gamma} \right\| \le \frac{2}{\gamma} \|x^k - x^*\|, \tag{8}$$

$\Psi(z)$ is a Lipschitz continuous function, and (8) into account, we have

$$\|\Psi(z^k) - \Psi(z^*)\| \le L \|z^k - z^*\| \le \frac{2L}{\gamma} \|x^k - x^*\|. \tag{9}$$

Thereby, we derive from (7) and (9) that

$$\left(1 - \|(M_1 + \Omega)^{-1} N_2\|\right) \|x^{k+1} - x^*\|$$
$$\le \left(2\left(\|(M_1 + \Omega)^{-1} N_1\| + L\|(M_1 + \Omega)^{-1}\|\right) + \|(M_1 + \Omega)^{-1}(\Omega - M_1)\|\right.$$
$$\quad \left. + \|(M_1 + \Omega)^{-1} N_2\|\right) \|x^k - x^*\|,$$

which is equivalent to

$$\|x^{k+1} - x^*\| \le \frac{2q(\Omega) + f(\Omega) + g(\Omega)}{1 - g(\Omega)} \|x^k - x^*\|$$

with $g(\Omega) < 1$. The condition

$$\frac{2q(\Omega) + f(\Omega) + g(\Omega)}{1 - g(\Omega)} < 1$$

with $g(\Omega) < 1$, which is equivalent to $\rho(\Omega) = 2g(\Omega) + 2q(\Omega) + f(\Omega) < 1$, ensures that the limit $\lim_{k \to +\infty} x^k = x^*$ holds. These results complete the proof. □

Theorem 7 *Let $A \in R^{n \times n}$ be a P-matrix, $M_1 - N_1 = M_2 - N_2 = A$ be two splittings of the matrix A with $M_1 \in R^{n \times n}$ being a symmetric P-matrix. Suppose that $\Omega = \omega I \in R^{n \times n}$ is a positive scalar matrix and ω is a positive constant. $\Psi(z) : R^n \to R^n$ is a Lipschitz continuous function with the Lipschitz constant L. $\lambda_{\max}$ and $\lambda_{\min}$ to denote the largest and smallest eigenvalue of the matrix M_1, respectively. Let $\tau_1 = \|M_1^{-1} N_1\|_2$ and $\tau_2 = \|M_1^{-1} N_2\|_2$ satisfy $\tau_1 + \tau_2 < 1$. If $\lambda_{\min} > L$, the choices of the parameters $\omega, M_1, N_1, M_2, N_2$ satisfy either of the*

following conditions, then the iteration sequence $\{z_k\}_k^{+\infty} \subset R_+^n$ generated by Algorithm 5 converges to the unique solution $z^* \in R_+^n$ of WNCP for any initial vector $x^0 \in R^n$.

(i) *When $0 < \tau_1 + \tau_2 < \frac{\lambda_{\min} - L}{\lambda_{\max}}$,*

$$\omega = \sqrt{\lambda_{\max}\lambda_{\min}}.$$

(ii) *When $\frac{\lambda_{\min} - L}{\lambda_{\max}} < \tau_1 + \tau_2 < \frac{\lambda_{\min} - L}{\sqrt{\lambda_{\max}\lambda_{\min}}}$,*

$$\sqrt{\lambda_{\max}\lambda_{\min}} \le \omega < \frac{[1 - (\tau_1 + \tau_2)]\lambda_{\max}\lambda_{\min} - L\lambda_{\max}}{(\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min}}.$$

(iii) *When $\tau_1 + \tau_2 = \frac{\lambda_{\min} - L}{\lambda_{\max}}$,*

$$\omega \ge \sqrt{\lambda_{\max}\lambda_{\min}}.$$

Proof We first give some formulations, which will be used in the proof. Since M is a symmetric P-matrix and $\tau_1 + \tau_2 < 1$, by the definition of the spectral norm, we have

$$
\begin{aligned}
\left\| (M_1 + \Omega)^{-1} N_1 \right\|_2 &= \left\| (M_1 + \omega I)^{-1} M_1 M_1^{-1} N_1 \right\|_2 \\
&\le \left\| (M_1 + \omega I)^{-1} M_1 \right\|_2 \left\| M_1^{-1} N_1 \right\|_2 \\
&= \max_{\lambda \in \lambda(M_1)} \frac{\lambda \tau_1}{\omega + \lambda} \\
&= \frac{\lambda_{\max}\tau_1}{\omega + \lambda_{\max}}.
\end{aligned}
\tag{10}
$$

Similarly, we have

$$
\begin{aligned}
\left\| (M_1 + \Omega)^{-1} N_2 \right\|_2 &= \left\| (M_1 + \omega I)^{-1} M_1 M_1^{-1} N_1 \right\|_2 \\
&\le \frac{\lambda_{\max}\tau_2}{\omega + \lambda_{\max}}
\end{aligned}
\tag{11}
$$

and

$$
\begin{aligned}
\left\| (M_1 + \Omega)^{-1} \right\|_2 &= \left\| (M_1 + \omega I)^{-1} \right\|_2 \\
&= \max_{\lambda \in \lambda(M_1)} \frac{1}{\omega + \lambda} \\
&= \frac{1}{\omega + \lambda_{\min}}.
\end{aligned}
\tag{12}
$$

In addition, from a simple calculating process, we have

$$
\begin{aligned}
\left\| (M_1 + \Omega)^{-1} (\Omega - M_1) \right\|_2 &= \left\| (M_1 + \omega I)^{-1} (\omega I - M_1) \right\|_2 \\
&= \max_{\lambda \in \lambda(M_1)} \frac{|\omega - \lambda|}{\omega + \lambda} = \max\left\{ \frac{|\omega - \lambda_{\max}|}{\omega + \lambda_{\max}}, \frac{|\omega - \lambda_{\min}|}{\omega + \lambda_{\min}} \right\} \\
&= \begin{cases} \frac{\lambda_{\max} - \omega}{\lambda_{\max} + \omega}, & \omega \le \sqrt{\lambda_{\max}\lambda_{\min}}, \\ \frac{\omega - \lambda_{\min}}{\omega + \lambda_{\min}}, & \omega \ge \sqrt{\lambda_{\max}\lambda_{\min}}. \end{cases}
\end{aligned}
\tag{13}
$$

As follows from (10)-(13), we have

$$
\begin{aligned}
\rho(\Omega) &= 2g(\Omega) + 2q(\Omega) + f(\Omega) \\
&= 2\left\| (M_1 + \Omega)^{-1} N_2 \right\|_2 + 2\left(\left\| (M_1 + \Omega)^{-1} N_1 \right\|_2 \right. \\
&\quad \left. + L \left\| (M_1 + \Omega)^{-1} \right\|_2 \right) + \left\| (M_1 + \Omega)^{-1}(\Omega - M_1) \right\|_2 \\
&= 2\frac{\lambda_{\max}\tau_2}{\omega + \lambda_{\max}} + 2\left(\frac{\lambda_{\max}\tau_1}{\omega + \lambda_{\max}} + \frac{L}{\omega + \lambda_{\min}} \right) + \begin{cases} \frac{\lambda_{\max}-\omega}{\omega+\lambda_{\max}}, & \omega \le \sqrt{\lambda_{\max}\lambda_{\min}}, \\ \frac{\omega-\lambda_{\min}}{\omega+\lambda_{\min}}, & \omega \ge \sqrt{\lambda_{\max}\lambda_{\min}}. \end{cases}
\end{aligned} \tag{14}
$$

We then consider two cases.

(a) When $\omega \le \sqrt{\lambda_{\max}\lambda_{\min}}$, by a simple calculation on (14), we see that ω, τ_1, and τ_2 satisfy $\rho(\Omega) < 1$, which is equivalent to

$$
\omega^2 - \left((\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min} \right)\omega - \left((\tau_1 + \tau_2)\lambda_{\max}\lambda_{\min} + L\lambda_{\max} \right) > 0.
$$

Note that $\omega > 0$ and $\omega \le \sqrt{\lambda_{\max}\lambda_{\min}}$, then the solution of the above inequality is

$$
\theta(\tau_1, \tau_2) < \omega \le \sqrt{\lambda_{\max}\lambda_{\min}},
$$

where

$$
\begin{aligned}
&\theta(\tau_1, \tau_2) \\
&= \frac{\sqrt{((\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min})^2 + 4(\tau_1 + \tau_2)\lambda_{\max}\lambda_{\min} + L\lambda_{\max}}}{2} \\
&\quad + \frac{(\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min}}{2}.
\end{aligned}
$$

Certainly, we have

$$
\theta(\tau_1, \tau_2) < \sqrt{\lambda_{\max}\lambda_{\min}}. \tag{15}
$$

Since $\lambda_{\min} > L$, by the definitions of τ_1, τ_2 and solving (15), we get

$$
0 < \tau_1 + \tau_2 < \frac{\lambda_{\min} - L}{\sqrt{\lambda_{\max}\lambda_{\min}}}.
$$

(b) When $\omega \ge \sqrt{\lambda_{\max}\lambda_{\min}}$, in a same way as (a), ω, τ_1, and τ_2 satisfying $\rho(\Omega) < 1$, which is equivalent to

$$
\left[(\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min} \right]\omega + (\tau_1 + \tau_2 - 1)\lambda_{\max}\lambda_{\min} + L\lambda_{\max} < 0. \tag{16}
$$

If $(\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min} > 0$, that is, $\tau_1 + \tau_2 > \frac{\lambda_{\min}-L}{\lambda_{\max}}$, then

$$
\omega < \frac{[1 - (\tau_1 + \tau_2)]\lambda_{\max}\lambda_{\min} - L\lambda_{\max}}{(\tau_1 + \tau_2)\lambda_{\max} + L - \lambda_{\min}}.
$$

Combined with $\omega \geq \sqrt{\lambda_{max}\lambda_{min}}$, we get

$$\sqrt{\lambda_{max}\lambda_{min}} \leq \omega < \frac{[1-(\tau_1+\tau_2)]\lambda_{max}\lambda_{min} - L\lambda_{max}}{(\tau_1+\tau_2)\lambda_{max} + L - \lambda_{min}}.$$

Naturally, we have

$$\sqrt{\lambda_{max}\lambda_{min}} < \frac{[1-(\tau_1+\tau_2)]\lambda_{max}\lambda_{min} - L\lambda_{max}}{(\tau_1+\tau_2)\lambda_{max} + L - \lambda_{min}}.$$

That is,

$$\tau_1 + \tau_2 < \frac{\lambda_{min} - L}{\sqrt{\lambda_{max}\lambda_{min}}}.$$

This, together with $\tau_1 + \tau_2 > \frac{\lambda_{min}-L}{\lambda_{max}}$, shows that we have

$$\frac{\lambda_{min} - L}{\lambda_{max}} < \tau_1 + \tau_2 < \frac{\lambda_{min} - L}{\sqrt{\lambda_{max}\lambda_{min}}}.$$

If $(\tau_1+\tau_2)\lambda_{max} + L - \lambda_{min} \leq 0$, that is, $\tau_1 + \tau_2 \leq \frac{\lambda_{min}-L}{\lambda_{max}}$, then for any $\omega > 0$ (16) holds. So

$$\omega \geq \sqrt{\lambda_{max}\lambda_{min}}.$$

Hence, from (a) and (b), we see that when $0 < \tau_1 + \tau_2 < \frac{\lambda_{min}-L}{\lambda_{max}}, \omega = \sqrt{\lambda_{max}\lambda_{min}}$; when $\frac{\lambda_{min}-L}{\lambda_{max}} < \tau_1 + \tau_2 < \frac{\lambda_{min}-L}{\sqrt{\lambda_{max}\lambda_{min}}}$, $\sqrt{\lambda_{max}\lambda_{min}} \leq \omega < \frac{[1-(\tau_1+\tau_2)]\lambda_{max}\lambda_{min}-L\lambda_{max}}{(\tau_1+\tau_2)\lambda_{max}+L-\lambda_{min}}$; when $\tau_1 + \tau_2 = \frac{\lambda_{min}-L}{\lambda_{max}}$, $\omega \geq \sqrt{\lambda_{max}\lambda_{min}}$. The proof is completed. □

5 Results and discussion

This study focused on the weakly nonlinear complementarity problems with a large sparse matrix. We proposed an algorithm that is not only computationally more convenient to use but also faster than the modulus-based matrix splitting iteration methods and the convergence conditions are presented when the system matrix is a P-matrix.

Some scholars had already stressed the accelerated modulus-based matrix splitting iteration methods for linear complementarity problems and pointed out that the system matrix is either a positive definite matrix or an H_+-matrix. However, we suggest that the system matrix is a P-matrix, this is more adaptable but also a limitation. Notwithstanding its limitation, this study does suggest that WNCP can be solved faster.

6 Conclusions

In this paper, by reformulating the complementarity problem (1) as an implicit fixed point equation based on splittings of the system matrix A, we establish an accelerated modulus-based matrix splitting iteration algorithm and show the convergence analysis when the involved matrix of the WNCP is a P-matrix.

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