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Well-posed symmetric vector quasi-equilibrium problems

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Abstract
In this paper, we consider well-posedness of symmetric vector quasi-equilibrium problems. Based on a nonlinear scalarization technique, we first establish the bounded rationality model *M* for symmetric vector quasi-equilibrium problems, and then introduce a well-posedness concept for symmetric vector quasi-equilibrium problems, which unifies its Hadamard and Tykhonov well-posedness. Finally, sufficient conditions on the well-posedness for symmetric vector quasi-equilibrium problems are given.

MSC: 91A26; 49K40; 90C31

Keywords: well-posedness; bounded rationality model; nonlinear scalarization function; symmetric vector quasi-equilibrium problems; sufficient conditions

1 Introduction

In 2003, Fu [1] introduced the symmetric vector quasi-equilibrium problem (for short, SVQEP) which is a generalization of equilibrium problem proposed by Blum and Oettli [2] and gave an existence theorem for a weak Pareto solution for (SVQEP). It provides a very general model for a wide range of problems, for example, the vector optimization problem, the vector variational inequality problem, the vector complementarity problem and the vector saddle point problem. In 2006, Farajzadeh [3] considered existence theorem of the solution of (SVQEP) in the Hausdorff topological vector space. In 2008, Chen and Gong [4] studied the stability of the set of solutions for (SVQEP), proved a generic stability theorem and gave an existence theorem for essentially connected components of the set of solutions for (SVQEP). In 2012, Zhang [5] introduced the notion of a generalized Levitin-Polyak well-posedness and gave sufficient conditions of the generalized Levitin-Polyak well-posedness for (SVQEP). Recently, by using the same roadmap as Deng and Xiang [6], Zhang *et al.* [7] introduce and study well-posedness in connection with (SVQEP), which unifies its Hadamard and Levitin-Polyak well-posedness.

As is well known, the notion of well-posedness can be divided into two different groups: Hadamard type and Tykhonov type [8, 9]. Roughly speaking, Hadamard type well-posedness is based on the continuous dependence of the optimal solution from the data of the considered optimization problem. Tykhonov types well-posedness such as Tikhonov and Levitin-Polyak well-posedness deal with the behavior of a prescribed class of sequence of approximate solutions. Two kinds of well-posedness have been generalized to various problems related to vector optimization, *e.g.*, vector optimization problems [10–15], vec-

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tor variational inequality problems [10, 16], and vector equilibrium problems [5–7, 17]. Among many approaches for dealing with Tykhonov types well-posedness for vector optimization problems, vector variational inequality problems, and vector equilibrium problems, the nonlinear scalarization technique is of considerable interest. On the other hand, almost all the literature deals with directly specific notions of well-posedness, especially Tykhonov types of well-posedness, while some researchers have investigated a unified approaches to two different types of well-posedness. For one thing, the notion of extended well-posedness for vector optimization problems has been investigated in [12]. In some sense this notion unifies the ideas of Tykhonov and Hadamard well-posedness, allowing perturbations of the objective function and the feasible set. For another, well-posedness under perturbations (called also parametric well-posedness) for vector equilibrium problems has also been investigated in [18]. This kind of well-posedness is a blending of Hadamard and Tikhonov notions, and it gives also links to stability theory and seems well adapted to describe the behaviors of solutions under perturbations.

In this paper, we will introduce a well-posedness concept for (SVQEP), which unifies its Hadamard and Tykhonov well-posedness. The distinguishing feature of our work lies in the use of the scalarization technique and the bounded rationality model M (see [19–22]) to establish well-posedness results of (SVQEP). It is worthy that our research method is different from extended well-posedness and parametric well-posedness. Finally, by using the conditions of the existence theorem of the solutions to (SVQEP) (see [1]), we obtain sufficient conditions of the well-posedness for (SVQEP).

2 Preliminaries

Throughout this paper, unless otherwise specified, let C and D be a compact metric space supplied with distance d_1, d_2, respectively. Let $(Z, \|\cdot\|)$ be a Banach space and P be a nonempty, closed, convex, and pointed cone in Z with apex at the origin and $\operatorname{int} P \neq \emptyset$.

Let $S : C \times D \rightrightarrows C$ and $T : C \times D \rightrightarrows D$ be two set-valued mappings and $F, G : C \times D \to Z$ be two vector-valued mappings. Fu [1] defined a class of symmetric vector quasi-equilibrium problems (for short, SVQEP), which consist in finding $(x, y) \in C \times D$ such that

$$(x, y) \in (S \times T)(x, y), \tag{1}$$

$$F(u, y) - F(x, y) \notin -\operatorname{int} P, \quad \forall u \in S(x, y) \tag{2}$$

and

$$G(x, v) - G(x, y) \notin -\operatorname{int} P, \quad \forall v \in T(x, y). \tag{3}$$

Now we introduce the notion of Tykhonov approximating solution sequence for (SVQEP).

Definition 2.1 A sequence $\{(x_n, y_n)\} \in C \times D$ is called a Tykhonov approximating solution sequence for (SVQEP) if there exists $\{\epsilon_n\} \subset \mathbb{R}^1_+$ with $\epsilon_n \to 0$ such that

$$(x_n, y_n) \in (S \times T)(x_n, y_n), \tag{4}$$

$$F(u, y_n) - F(x_n, y_n) + \epsilon_n e \notin -\operatorname{int} P, \quad \forall u \in S(x_n, y_n) \tag{5}$$

and

$$G(x_n, v) - G(x_n, y_n) + \epsilon_n e \notin -\operatorname{int} P, \quad \forall v \in T(x_n, y_n). \tag{6}$$

Next, we introduce a nonlinear scalarization function and their related properties. For any fixed $e \in \operatorname{int} P$, the nonlinear scalarization function is defined by

$$\xi_e(z) := \inf\{r \in \mathbb{R} : z \in re - P\}, \quad \forall z \in Z.$$

It is well known from [23–25] that ξ_e is continuous, homogeneous, (strictly) monotone (*i.e.*, $\xi_e(z_1) \le \xi_e(z_2)$ if $z_1 - z_2 \in P$ and $\xi_e(z_1) < \xi_e(z_2)$ if $z_2 - z_1 \in \operatorname{int} P$) and convex. For any fixed $e \in \operatorname{int} P$, $z \in Z$, and $r \in \mathbb{R}$, then $\xi_e(z) \ge r$ is equivalent to $z \notin re - \operatorname{int} P$.

Remark 2.1 Note that the nonlinear scalarization function ξ_e is not strongly monotone (see [24]). It is for the reason that the function ξ_e is more useful in dealing with weakly efficient points.

Finally, we recall some useful definitions and lemmas.

Let (X, d) be a metric space. Denote a family of all nonempty compact subsets of X by $K(X)$. For any $A, B \in K(X)$, let

$$h(A, B) = \max\left\{\sup_{a\in A} \inf_{b\in B} d(a, b), \sup_{b\in B} \inf_{a\in A} d(a, b)\right\}$$

denote the Hausdorff metric on $K(X)$. It is well known that $(K(X), h)$ is complete if and only if (X, d) is complete.

Definition 2.2 (see [26]) Let $F : X \rightrightarrows Y$ be a set-valued mapping.

1. F is said to be upper semicontinuous at $x \in X$ if for any open set $U \supset F(x)$, there is an open neighborhood $O(x)$ of x such that $U \supset F(x')$ for each $x' \in O(x)$;
2. F is said to be lower semicontinuous at x if for any open set $U \cap F(x) \neq \emptyset$, there is an open neighborhood $O(x)$ of x such that $U \cap F(x') \neq \emptyset$, for each $x' \in O(x)$;
3. F is said to be an usco mapping if F is upper semicontinuous and $F(x)$ is nonempty compact for each $x \in X$;
4. F is said to be closed if $\operatorname{Graph}(F)$ is closed, where $\operatorname{Graph}(F) = \{(x, y) \in X \times Y : x \in X, y \in F(x)\}$ is the graph of F.

Lemma 2.1 (see [26]) *If $F : X \rightrightarrows Y$ is closed and Y is compact, then F is upper semicontinuous at each $x \in X$.*

3 A unified approach to notions of well-posedness for (SVQEP)

Let Λ be the collection of all problem $\lambda = (S, T, F, G)$ such that

(a) $S : C \times D \rightrightarrows C$ and $T : C \times D \rightrightarrows D$ are continuous with nonempty compact values;
(b) $F, G : C \times D \to Z$ are continuous;
(c) $\sup_{(x,y)\in C\times D} \|F(x, y)\| < +\infty$ and $\sup_{(x,y)\in C\times D} \|G(x, y)\| < +\infty$;
(d) there exists $(x, y) \in C \times D$ meets (1), (2), and (3).

For any $\lambda_1 = (S_1, T_1, F_1, G_1)$, $\lambda_2 = (S_2, T_2, F_2, G_2) \in \Lambda$, we define

$$\rho(\lambda_1, \lambda_2) := \sup_{(x,y)\in C\times D} h_1\bigl(S_1(x,y), S_2(x,y)\bigr) + \sup_{(x,y)\in C\times D} h_2\bigl(T_1(x,y), T_2(x,y)\bigr) + \sup_{(x,y)\in C\times D} \bigl\|F_1(x,y) - F_2(x,y)\bigr\| + \sup_{(x,y)\in C\times D} \bigl\|G_1(x,y) - G_2(x,y)\bigr\|,$$

where h_1, h_2 are Hausdorff metrics on $K(C)$ and $K(D)$, respectively. By Proposition 3.1 in [4], it is easy to prove that (Λ, ρ) is a complete metric space.

Let $X^* = C \times D$, $x^* = (x, y)$, and $d = \max\{d_1, d_2\}$. The bounded rationality model $M = \{\Lambda, X^*, f, \Phi\}$ for (SVQEP) corresponding to $\lambda \in \Lambda$ is defined as follows:

(i) (Λ, ρ) is a metric space and (X^*, d) is a compact metric space;

(ii) the feasible set of λ is defined by

$$f(\lambda) := \bigl\{x^* \in X^* : x^* \in (S \times T)\bigl(x^*\bigr)\bigr\};$$

(iii) the solution set of λ is defined by

$$E(\lambda) := \bigl\{x^* \in X^* : x^* \text{ meets (1), (2) and (3)}\bigr\};$$

(iv) the rationality function of λ is defined by

$$\Phi\bigl(\lambda, x^*\bigr) := \operatorname{Max}\Bigl\{\sup_{u\in S(x,y)} \bigl\{-\xi_e\bigl(F(u,y) - F(x,y)\bigr)\bigr\}, \sup_{v\in T(x,y)} \bigl\{-\xi_e\bigl(G(x,v) - G(x,y)\bigr)\bigr\}\Bigr\}.$$

Remark 3.1 In (iv), a nonlinear scalarization function ξ_e is applied to reduce (SVQEP) to a scalar optimization problem since (SVQEP) does not possess linearity and convexity. Referring to [25], if one needs to solve exactly one representation to catch all the solution of (SQVEP), then the nonlinear scalarization technique is feasible.

Example 3.1 Let $C = D = [0, 2]$, $P = \mathbb{R}_+$, and $e = 1$. For any $(x, y) \in C \times D$, assume that

$$S(x,y) = T(x,y) = [0,1],$$
$$F(x,y) = G(x,y) = -(x+y),$$

and, for any $(u, v) \in C \times D$,

$$F(u,y) = -(u+y), \qquad G(x,v) = -(x+v).$$

Then it is easy to see that

$$f(\lambda) = \bigl\{(x,y) : x \in [0,1], y \in [0,1]\bigr\},$$
$$\Phi\bigl(\lambda, x^*\bigr) = \operatorname{Max}\Bigl\{\sup_{u\in[0,1]} \bigl\{-\xi_e(x-u)\bigr\}, \sup_{v\in[0,1]} \bigl\{-\xi_e(y-v)\bigr\}\Bigr\}.$$

1. If $x^* \in f(\lambda)$, then $x \in [0,1]$ and $y \in [0,1]$. Obviously, $\Phi(\lambda, x^*) \geq 0$.

2. For any $\lambda \in \Lambda$, one has

$$E(\lambda) = \{(x,y) : x = 1, y = 1\} \neq \emptyset.$$

3. It is easy to check that $(x,y) \in E(\lambda)$ if and only if $\Phi(\lambda, x^*) = 0$. Moreover, taking $x = 0$, $y = \frac{1}{2}$, then $(x,y) = (0, \frac{1}{2}) \notin E(\lambda)$ and $\Phi(\lambda, x^*) = 1 \neq 0$.

Lemma 3.1

1. $\forall \lambda \in \Lambda$, $E(\lambda) \neq \emptyset$ *and* $\forall x^* \in f(\lambda)$, $\Phi(\lambda, x^*) \geq 0$.
2. $\forall \lambda \in \Lambda$, $\Phi(\lambda, x^*) \leq \epsilon$ *if and only if* x^* *meets* (5) *and* (6).
3. $x^* \in E(\lambda)$ *if and only if* $\Phi(\lambda, x^*) = 0$.

Proof 1. By the definition of Λ, $\forall \lambda \in \Lambda$, $E(\lambda) \neq \emptyset$. If $x^* = (x,y) \in f(\lambda)$, then $x \in S(x,y)$, $y \in T(x,y)$ and

$$\Phi(\lambda, x^*) \geq \operatorname{Max}\{\{-\xi_e(F(x,y) - F(x,y))\}, \{-\xi_e(G(x,y) - G(x,y))\}\} = 0.$$

2. If x^* meets (5) and (6), then

$$\xi_e(F(u,y) - F(x,y)) \geq -\epsilon, \quad \forall u \in S(x,y)$$

and

$$\xi_e(G(x,v) - G(x,y)) \geq -\epsilon, \quad \forall v \in T(x,y).$$

Thus, we have

$$\Phi(\lambda, x^*) = \operatorname{Max}\Big\{\sup_{u \in S(x,y)}\{-\xi_e(F(u,y) - F(x,y))\}, \sup_{v \in T(x,y)}\{-\xi_e(G(x,v) - G(x,y))\}\Big\} \leq \epsilon.$$

Conversely, if $\Phi(\lambda, x^*) \leq \epsilon$, then we get

$$-\xi_e(F(u,y) - F(x,y)) \geq \epsilon, \quad \forall u \in S(x,y)$$

and

$$-\xi_e(G(x,v) - G(x,y)) \geq \epsilon, \quad \forall v \in T(x,y).$$

It follows that

$$F(u,y) - F(x,y) + \epsilon e \notin -\operatorname{int} P, \quad \forall u \in S(x,y)$$

and

$$G(x,v) - G(x,y) + \epsilon e \notin -\operatorname{int} P, \quad \forall v \in T(x,y).$$

Hence, x^* meets (5) and (6).

3. Using the above results, this result can be obtained. □

Let $x_n^* = (x_n, y_n)$ and $x_{n_k}^* = (x_{n_k}, y_{n_k})$. By Lemma 3.1, we get some new representations on (1)-(6) as follows:

$$(1) \iff x^* \in f(\lambda); \qquad (2) \text{ and } (3) \iff \Phi(\lambda, x^*) = 0;$$
$$(4) \iff x_n^* \in f(\lambda); \qquad (5) \text{ and } (6) \iff \Phi(\lambda, x_n^*) \le \epsilon_n.$$

Therefore, the set of solutions for the problem $\lambda \in \Lambda$ and $\lambda_n \in \Lambda$ ($n = 1, 2, 3, \ldots$) is defined as

$$E(\lambda) := \{x^* \in X^* : x^* \in f(\lambda), \Phi(\lambda, x^*) = 0\},$$
$$E(\lambda_n) := \{x^* \in X^* : x^* \in f(\lambda_n), \Phi(\lambda_n, x^*) = 0\}.$$

The Tykhonov approximating solution set for the problem $\lambda \in \Lambda$ and $\lambda_n \in \Lambda$ ($n = 1, 2, 3, \ldots$) is defined as

$$E(\lambda, \epsilon_n) := \{x^* \in X^* : x^* \in f(\lambda), \Phi(\lambda, x^*) \le \epsilon_n\},$$
$$E(\lambda_n, \epsilon_n) := \{x^* \in X^* : x^* \in f(\lambda_n), \Phi(\lambda_n, x^*) \le \epsilon_n\}.$$

Tykhonov well-posedness for (SVQEP) corresponding to the problem λ is given as follows.

Definition 3.1

1. If $\forall x_n^* \in E(\lambda, \epsilon_n)$, $\epsilon_n > 0$ with $\epsilon_n \to 0$, there must exist a subsequence $\{x_{n_k}^*\} \subset \{x_n^*\}$ such that $x_{n_k}^* \to x^* \in E(\lambda)$, then the problem λ is said to be generalized Tykhonov well-posed (for short GT-wp).
2. If $E(\lambda) = \{x^*\}$ (a singleton), $\forall x_n^* \in E(\lambda, \epsilon_n)$, $\epsilon_n > 0$ with $\epsilon_n \to 0$, we must have $x_n^* \to x^*$, then the problem λ is said to be Tykhonov well-posed (for short T-wp).

Referring to [9], Hadamard well-posedness for (SVQEP) corresponding to the problem λ is defined as follows.

Definition 3.2

1. If $\forall \lambda_n \in \Lambda$, $\lambda_n \to \lambda$, $\forall x_n^* \in E(\lambda_n)$, there must exist a subsequence $\{x_{n_k}^*\} \subset \{x_n^*\}$ such that $x_{n_k}^* \to x^* \in E(\lambda)$, then the problem λ is said to be generalized Hadamard well-posed (for short GH-wp).
2. If $E(\lambda) = \{x^*\}$ (a singleton), $\forall \lambda_n \in \Lambda$, $\lambda_n \to \lambda$, $\forall x_n^* \in E(\lambda_n)$, we must have $x_n^* \to x^*$, then the problem λ is said to be Hadamard well-posed (for short H-wp).

Finally, we establish a well-posedness concept for (SVQEP) corresponding to the problem λ, which unifies its Hadamard and Tykhonov well-posedness.

Definition 3.3

1. If $\forall \lambda_n \in \Lambda$, $\lambda_n \to \lambda$, $\forall x_n^* \in E(\lambda_n, \epsilon_n)$, $\epsilon_n > 0$ with $\epsilon_n \to 0$, there must exist a subsequence $\{x_{n_k}^*\} \subset \{x_n^*\}$ such that $x_{n_k}^* \to x^* \in E(\lambda)$, then the problem λ is said to be generalized well-posed (for short G-wp).
2. If $E(\lambda) = \{x^*\}$ (a singleton), $\forall \lambda_n \in \Lambda$, $\lambda_n \to \lambda$, $\forall x_n^* \in E(\lambda_n, \epsilon_n)$, $\epsilon_n > 0$ with $\epsilon_n \to 0$, we must have $x_n^* \to x^*$, then the problem λ is said to be well-posed (for short wp).

4 Sufficient conditions for well-posedness of (SVQEP)

Assume that the bounded rationality model $M = \{\Lambda, X^*, f, \Phi\}$ for (SVQEP) is given. In order to show sufficient conditions for well-posedness of (SVQEP), we first give the following lemmas.

Lemma 4.1 *$f : \Lambda \rightrightarrows X^*$ is an usco mapping.*

Proof Since X^* is a compact metric space, by Lemma 2.1, it suffices to show that Graph(f) is closed. That is to say, $\forall \lambda_n \in \Lambda$, $\lambda_n \to \lambda \in \Lambda$, $\forall x_n^* \in f(\lambda_n)$, $x_n^* \to x^*$, we need to show that $x^* \in f(\lambda)$.

Let $h_1(S_n(x_n, y_n), S(x_n, y_n)) \leq \epsilon_n$ and $h_2(T_n(x_n, y_n), T(x_n, y_n)) \leq \epsilon_n$. For each $n = 1, 2, 3, \ldots$, since $(x_n, y_n) \in f(\lambda_n)$, then there exists $(x_n, y_n) \in X^*$ such that $(x_n, y_n) \in (S_n \times T_n)(x_n, y_n)$. So there exists $x_n' \in S(x_n, y_n)$ such that $d_1(x_n, x_n') \leq \epsilon_n$. By

$$d_1(x_n', x) \leq d_1(x_n', x_n) + d_1(x_n, x) \to 0,$$

we get $x_n' \to x$. Note that set-value mapping S is continuous on X^*, then we get

$$d_1\big(x, S(x,y)\big) \leq d_1\big(x, x_n'\big) + d_1\big(x_n', S(x_n, y_n)\big) + h_1\big(S(x_n, y_n), S(x,y)\big) \to 0.$$

By compactness of $S(x,y)$, we have $x \in S(x,y)$. Similarly, we can prove that $y \in T(x,y)$. Hence, $(x,y) \in (S \times T)(x,y)$. It shows that $x^* \in f(\lambda)$. □

Lemma 4.2 (see [9, 22]) *Suppose that $f : \Lambda \rightrightarrows X^*$ is a usco mapping. Then, for any $\lambda_n \to \lambda$ and any $x_n^* \in f(\lambda_n)$, there is a subsequence $\{x_{n_k}^*\} \subset \{x_n^*\}$ such that $x_{n_k}^* \to x^* \in f(\lambda)$.*

Lemma 4.3 *Φ is lower semicontinuous at (λ, x^*).*

Proof We only need to show that $\forall \epsilon > 0$, $\forall \lambda_n = (S_n, T_n, F_n, G_n) \in \Lambda$, $\lambda_n \to \lambda = (S, T, F, G) \in \Lambda$, $\forall x_n^* \in X^*$, $x_n^* \to x^* \in X^*$, there exists a positive integer N such that, $\forall n \geq N$,

$$\Phi\big(\lambda_n, x_n^*\big) > \Phi\big(\lambda, x^*\big) - \epsilon. \tag{7}$$

Let

$$\Phi_1\big(\lambda, x^*\big) := \sup_{u \in S(x,y)} \big\{-\xi_e\big(F(u,y) - F(x,y)\big)\big\}$$

and

$$\Phi_2\big(\lambda, x^*\big) := \sup_{v \in T(x,y)} \big\{-\xi_e\big(G(x,v) - G(x,y)\big)\big\}.$$

By the definition of the least upper bound, there exists $u_0 \in S(x,y)$ such that

$$-\xi_e\big(F(u_0, y) - F(x,y)\big) > \Phi_1\big(\lambda, x^*\big) - \frac{\epsilon}{2}. \tag{8}$$

Note that $\sup_{(x,y) \in C \times D} h_1(S_n(x,y), S(x,y)) \to 0$ and $h_1(S(x_n, y_n), S(x,y)) \to 0$, we have

$$h_1\big(S_n(x_n, y_n), S(x,y)\big) \leq h_1\big(S_n(x_n, y_n), S(x_n, y_n)\big) + h_1\big(S(x_n, y_n), S(x,y)\big) \to 0. \tag{9}$$

By (9), there exists $u_n \in S_n(x_n, y_n)$ such that $d_1(u_n, u_0) \to 0$. Since F is continuous on $C \times D$ and $\sup_{(x,y)\in C\times D} \|F_n(x,y) - F(x,y)\| \to 0$, letting $n \to \infty$, we have

$$\left\|F_n(u_n, y_n) - F_n(x_n, y_n) - \big(F(u_0, y) - F(x, y)\big)\right\| \to 0. \tag{10}$$

Using continuity of ξ_e and (10), we have

$$-\xi_e\big(F_n(u_n, y_n) - F_n(x_n, y_n)\big) \to -\xi_e\big(F(u_0, y) - F(x, y)\big). \tag{11}$$

By (11), there exists a positive integer N_1 such that, for any $n \geq N_1$,

$$-\xi_e\big(F_n(u_n, y_n) - F_n(x_n, y_n)\big) > -\xi_e\big(F(u_0, y) - F(x, y)\big) - \frac{\epsilon}{2}. \tag{12}$$

From (8) and (12), for any $n \geq N_1$, we get

$$\begin{aligned}
\Phi_1\big(\lambda_n, x_n^*\big) &= \sup_{u\in S_n(x_n, y_n)} \big\{-\xi_e\big(F_n(u, y_n) - F_n(x_n, y_n)\big)\big\} \\
&\geq -\xi_e\big(F_n(u_n, y_n) - F_n(x_n, y_n)\big) \\
&> -\xi_e\big(F(u_0, y) - F(x, y)\big) - \frac{\epsilon}{2} \\
&> \Phi_1\big(\lambda, x^*\big) - \epsilon.
\end{aligned} \tag{13}$$

Similarly, we can prove that there exists a positive integer N_2 such that, for any $n \geq N_2$,

$$\Phi_2\big(\lambda_n, x_n^*\big) > \Phi_2\big(\lambda, x^*\big) - \epsilon. \tag{14}$$

Let $N = \max\{N_1, N_2\}$, $\forall n \geq N$, by (13) and (14), we get (7), that is,

$$\Phi\big(\lambda_n, x_n^*\big) \geq \operatorname{Max}\big\{\Phi_1\big(\lambda, x^*\big), \Phi_2\big(\lambda, x^*\big)\big\} - \epsilon = \Phi\big(\lambda, x^*\big) - \epsilon. \qquad \square$$

Finally, we give sufficient conditions for G-wp and wp of (SVQEP) corresponding to $\lambda \in \Lambda$.

Theorem 4.1

1. *Every* $\lambda \in \Lambda$ *is G-wp.*
2. *Let* $\lambda \in \Lambda$ *and suppose furthermore* $E(\lambda) = \{x^*\}$ (*a singleton*), *then* λ *is wp.*

Proof 1. $\forall \lambda_n \in \Lambda$, $\lambda_n \to \lambda$, $\forall x_n^* \in E(\lambda_n, \epsilon_n)$, $\epsilon_n > 0$ with $\epsilon_n \to 0$, then we have $x_n^* \in f(\lambda_n)$ and $\Phi(\lambda_n, x_n^*) \leq \epsilon_n$. First, by Lemma 4.1 and Lemma 4.2, if $\lambda_n \to \lambda$, then there exists $\{x_{n_k}^*\} \subset \{x_n^*\}$ such that $x_{n_k}^* \to x^* \in f(\lambda)$. Secondly, by $\Phi(\lambda_n, x_n^*) \leq \epsilon_n$ and Lemma 4.3, we have

$$0 \leq \Phi\big(\lambda, x^*\big) \leq \liminf_{n_k\to\infty} \Phi\big(\lambda_{n_k}, x_{n_k}^*\big) \leq \liminf_{n_k\to\infty} \epsilon_{n_k} = 0,$$

which implies that $\Phi(\lambda, x^*) = 0$. It shows that the problem λ is G-wp.

2. By way of contradiction. If the sequence $\{x_n^*\}$ does not converge x^*, then there exist an open neighborhood O at x^* and a subsequence $\{x_{n_k}^*\}$ of $\{x_n^*\}$ such that $x_{n_k}^* \notin O$. Since

$E(\lambda) = \{x^*\}$ (a singleton), using the above proof, we get $x^*_{n_k} \to x^*$. This is a contradiction to $x^*_{n_k} \notin O$. □

Example 4.1 Let $C = D = [0,2] \times [0,2]$, $Z = \mathbb{R}^2$, $P = \mathbb{R}^2_+$, and $e = (1,1)$. For any $(x,y) \in X \times Y$, assume that

$$S(x,y) = T(x,y) = [0,1] \times [0,1], \quad \forall x,y \in C \times D,$$
$$F(x,y) = G(x,y) = -(x+y),$$

and for any $(u,v) \in C \times D$,

$$F(u,y) = -(u+y), \qquad G(x,y) = -(x+v).$$

Then it is easy to see that, for $\lambda = (S,T,F,G) \in \Lambda$,

$$f(\lambda) = \bigl\{(x,y) | x \in [0,1] \times [0,1], y \in [0,1] \times [0,1]\bigr\}$$

and

$$E(\lambda) = \bigl\{(x,y) \in f(\lambda) | x = (x_1,x_2), y = (y_1,y_2), \max\{x_1,x_2\} = 1, \max\{y_1,y_2\} = 1\bigr\}.$$

Moreover, by Theorem 4.1, the problem λ must be G-wp.

Finally, by Definition 3.1, Definition 3.2, Definition 3.3, and Theorem 4.1, it is easy to check the following.

Corollary 4.1

1. *Every $\lambda \in \Lambda$ must be GT-wp and GH-wp.*
2. *Let $\lambda \in \Lambda$, if $E(\lambda) = \{x^*\}$ (a singleton), then λ must be T-wp and H-wp.*

Remark 4.1 In Theorem 4.1 and Corollary 4.1, $\lambda \in \Lambda$ means that the problem $\lambda = (S,T,F,G)$ holds for all conditions (a), (b), (c), and (d).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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