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Common property (E.A) and existence of fixed points in Menger spaces

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Abstract
The aim of this paper is to prove some common fixed-point theorems for weakly compatible mappings in Menger spaces satisfying common property (E.A). Some examples are also given which demonstrate the validity of our results. As an application of our main result, we present a common fixed-point theorem for four finite families of self-mappings in Menger spaces. Our result is an improved probabilistic version of the result of Sedghi *et al.* [Gen. Math. 18:3-12, 2010].
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1 Introduction

In 1922, Banach proved the principal contraction result [1]. As we know, there have been published many works about fixed-point theory for different kinds of contractions on some spaces such as quasi-metric spaces [2], cone metric spaces [3], convex metric spaces [4], partially ordered metric spaces [5–9], G-metric spaces [10–14], partial metric spaces [15, 16], quasi-partial metric spaces [17], fuzzy metric spaces [18], and Menger spaces [19]. Also, studies either on approximate fixed point or on qualitative aspects of numerical procedures for approximating fixed points are available in the literature; see [4, 20, 21].

Jungck and Rhoades [22] weakened the notion of compatibility by introducing the notion of weakly compatible mappings (extended by Singh and Jain [23] to probabilistic metric space) and proved common fixed-point theorems without assuming continuity of the involved mappings in metric spaces. In 2002, Aamri and Moutawakil [24] introduced the notion of property (E.A) (extended by Kubiaczyk and Sharma [25] to probabilistic metric space) for self-mappings which contained the class of noncompatible mappings due to Pant [26]. Further, Liu *et al.* [27] defined the notion of common property (E.A) (extended by Ali *et al.* [28] to probabilistic metric space) which contains the property (E.A) and proved several fixed-point theorems under hybrid contractive conditions. Since then, there has been continuous and intense research activity in fixed-point theory and by now there exists an extensive literature (*e.g.* [29–33] and the references therein).

Many mathematicians proved several common fixed-point theorems for contraction mappings in Menger spaces by using different notions *viz.* compatible mappings, weakly compatible mappings, property (E.A), common property (E.A) (see [28, 34–51]).

In the present paper, we prove some common fixed-point theorems for weakly compatible mappings in Menger space using the common property (E.A). Some examples are also derived which demonstrate the validity of our results. As an application of our main result, we extend the related results to four finite families of self-mappings in Menger spaces.

2 Preliminaries

In the sequel, $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{N}$ denote the set of real numbers, the set of nonnegative real numbers, and the set of positive integers, respectively.

Definition 2.1 [52] A triangular norm $*$ (shortly t-norm) is a binary operation on the unit interval $[0,1]$ such that for all $a,b,c,d \in [0,1]$ the following conditions are satisfied:
(1) $a * 1 = a$,
(2) $a * b = b * a$,
(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
(4) $a * (b * c) = (a * b) * c$.

Examples of t-norms are $a * b = \min\{a,b\}$, $a * b = ab$, and $a * b = \max\{a + b - 1, 0\}$.

Definition 2.2 [52] A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is called a distribution function if it is nondecreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$. We shall denote the set of all distribution functions on $(-\infty, \infty)$ by $\Im$, while H will always denotes the specific distribution function defined by

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0; \\ 1, & \text{if } t > 0. \end{cases}$$

If X is a nonempty set, $\mathcal{F} : X \times X \to \Im$ is called a probabilistic distance on X and $F(x,y)$ is usually denoted by $F_{x,y}$.

Definition 2.3 [52] The ordered pair $(X, \mathcal{F})$ is called a probabilistic metric space (shortly, PM-space) if X is a nonempty set and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions:
(1) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,
(2) $F_{x,y}(0) = 0$ for all $x, y \in X$,
(3) $F_{x,y}(t) = F_{y,x}(t)$ for all $x, y \in X$ and for all $t > 0$,
(4) $F_{x,z}(t) = 1$, $F_{z,y}(s) = 1 \Rightarrow F_{x,y}(t + s) = 1$ for $x, y, z \in X$ and $t, s > 0$.

Every metric space (X, d) can always be realized as a probabilistic metric space defined by $F_{x,y}(t) = H(t - d(x,y))$ for all $x, y \in X$ and $t > 0$. So probabilistic metric spaces offer a wider framework (than that of the metric spaces) and are general enough to cover even wider statistical situations.

Definition 2.4 [19] A Menger space $(X, \mathcal{F}, *)$ is a triplet where $(X, \mathcal{F})$ is a probabilistic metric space and $*$ is a t-norm satisfying the following condition:

$$F_{x,y}(t + s) \geq F_{x,z}(t) * F_{z,y}(s),$$

for all $x, y, z \in X$ and $t, s > 0$.

Throughout this paper, $(X,\mathcal{F},*)$ is considered to be a Menger space with condition $\lim_{t\to\infty}\mathcal{F}_{x,y}(t) = 1$ for all $x,y\in X$. Every fuzzy metric space $(X,M,*)$ may be a Menger space by considering $\mathcal{F}: X\times X\to\Im$ defined by $F_{x,y}(t) = M(x,y,t)$ for all $x,y\in X$.

Definition 2.5 [52] Let $(X,\mathcal{F},*)$ be a Menger space and $*$ be a t-norm. Then

(1) a sequence $\{x_n\}$ in X is said to converge to a point x in X if and only if for every $\epsilon > 0$ and $\lambda\in(0,1)$, there exists an integer $N\in\mathbb{N}$ such that $F_{x_n,x}(\epsilon) > 1-\lambda$ for all $n\geq N$;

(2) a sequence $\{x_n\}$ in X is said to be Cauchy if for every $\epsilon > 0$ and $\lambda\in(0,1)$, there exists an integer $N\in\mathbb{N}$ such that $F_{x_n,x_m}(\epsilon) > 1-\lambda$ for all $n,m\geq N$.

A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.6 [53] A pair (A,S) of self-mappings of a Menger space $(X,\mathcal{F},*)$ is said to be compatible if $\lim_{n\to\infty}F_{ASx_n,SAx_n}(t) = 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = z$ for some $z\in X$.

Definition 2.7 [28] A pair (A,S) of self-mappings of a Menger space $(X,\mathcal{F},*)$ is said to be noncompatible if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Ax_n = z = \lim_{n\to\infty}Sx_n$ for some $z\in X$, but, for some $t>0$, either $\lim_{n\to\infty}F_{ASx_n,SAx_n}(t)\neq 1$ or the limit does not exist.

Definition 2.8 [25] A pair (A,S) of self-mappings of a Menger space $(X,\mathcal{F},*)$ is said to satisfy property (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = z,$$

for some $z\in X$.

Remark 2.1 From Definition 2.8, it is easy to see that any two noncompatible self-mappings of $(X,\mathcal{F},*)$ satisfy property (E.A) but the reverse need not be true (see [40, Example 1]).

Definition 2.9 [34] Two pairs (A,S) and (B,T) of self-mappings of a Menger space $(X,\mathcal{F},*)$ are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$, $\{y_n\}$ in X such that

$$\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = \lim_{n\to\infty}By_n = \lim_{n\to\infty}Ty_n = z,$$

for some $z\in X$.

Definition 2.10 [22] A pair (A,S) of self-mappings of a nonempty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, *i.e.* if $Az = Sz$ for some $z\in X$, then $ASz = SAz$.

Remark 2.2 If self-mappings A and S of a Menger space $(X,\mathcal{F},*)$ are compatible then they are weakly compatible but the reverse need not be true (see [23, Example 1]).

Remark 2.3 It is noticed that the notion of weak compatibility and the (E.A) property are independent to each other (see [54, Example 2.2]).

Definition 2.11 [41] Two families of self-mappings $\{A_i\}$ and $\{S_j\}$ are said to be pairwise commuting if:

(1) $A_iA_j = A_jA_i$, $i,j \in \{1,2,\ldots,m\}$,
(2) $S_kS_l = S_lS_k$, $k,l \in \{1,2,\ldots,n\}$,
(3) $A_iS_k = S_kA_i$, $i \in \{1,2,\ldots,m\}$, $k \in \{1,2,\ldots,n\}$.

3 Main results

Let Φ is a set of all increasing and continuous functions $\phi : (0,1] \to (0,1]$, such that $\phi(t) > t$ for every $t \in (0,1)$.

Example 3.1 Let $\phi : (0,1] \to (0,1]$ defined by $\phi(t) = t^{\frac{1}{2}}$. It is easy to see that $\phi \in \Phi$.

Before proving our main theorems, we begin with the following lemma.

Lemma 3.1 *Let A, B, S and T be self-mappings of a Menger space $(X,\mathcal{F},*)$, where $*$ is a continuous t-norm. Suppose that*

(1) *$A(X) \subset T(X)$ or $B(X) \subset S(X)$,*
(2) *the pair (A,S) or (B,T) satisfies property $(E.A)$,*
(3) *$B(y_n)$ converges for every sequence $\{y_n\}$ in X whenever $T(y_n)$ converges or $A(x_n)$ converges for every sequence $\{x_n\}$ in X whenever $S(x_n)$ converges,*
(4) *there exist $\phi \in \Phi$ and $1 \le k < 2$ such that*

$$F_{Ax,By}(t) \geq \phi\left(\min\left\{\begin{array}{c} F_{Sx,Ty}(t), \\ \sup_{t_1+t_2=\frac{2}{k}t}\min\{F_{Sx,Ax}(t_1), F_{Ty,By}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t}\max\{F_{Sx,By}(t_3), F_{Ty,Ax}(t_4)\} \end{array}\right\}\right) \tag{3.1}$$

holds for all $x,y \in X$, $t > 0$. Then the pairs (A,S) and (B,T) share the common property $(E.A)$.

Proof Suppose the pair (A,S) satisfies property (E.A), then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \tag{3.2}$$

for some $z \in X$. Since $A(X) \subset T(X)$, hence for each $\{x_n\} \subset X$ there corresponds a sequence $\{y_n\} \subset X$ such that $Ax_n = Ty_n$. Therefore,

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ax_n = z. \tag{3.3}$$

Thus in all, we have $Ax_n \to z$, $Sx_n \to z$ and $Ty_n \to z$. By (3), the sequence $\{By_n\}$ converges and in all we need to show that $By_n \to z$ as $n \to \infty$. Let $By_n \to l$ for $t > 0$ as $n \to \infty$. Then, it is enough to show that $z = l$. Suppose that $z \neq l$, then there exists $t_0 > 0$ such that

$$F_{z,l}\left(\frac{2}{k}t_0\right) > F_{z,l}(t_0). \tag{3.4}$$

In order to establish the claim embodied in (3.4), let us assume that (3.4) does not hold. Then we have $\mathcal{F}_{z,l}(\frac{2}{k}t) \leq \mathcal{F}_{z,l}(t)$ for all $t > 0$. Repeatedly using this equality, we obtain

$$F_{z,l}(t) \geq F_{z,l}\left(\frac{2}{k}t\right) \geq \cdots \geq F_{z,l}\left(\left(\frac{2}{k}\right)^n t\right) \to 1,$$

as $n \to \infty$. This shows that $F_{z,l}(t) = 1$ for all $t > 0$, which contradicts $z \neq l$, and hence (3.4) is proved. Using inequality (3.1), with $x = x_n$, $y = y_n$, we get

$$\begin{aligned} F_{Ax_n,By_n}(t_0) &\geq \phi\left(\min\left\{\begin{array}{c} F_{Sx_n,Ty_n}(t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{Sx_n,Ax_n}(t_1), F_{Ty_n,By_n}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{Sx_n,By_n}(t_3), F_{Ty_n,Ax_n}(t_4)\} \end{array}\right\}\right) \\ &\geq \phi\left(\min\left\{\begin{array}{c} F_{Sx_n,Ty_n}(t_0), \\ \min\{F_{Sx_n,Ax_n}(\epsilon), F_{Ty_n,By_n}(\frac{2}{k}t_0 - \epsilon)\}, \\ \max\{F_{Sx_n,By_n}(\frac{2}{k}t_0 - \epsilon), F_{Ty_n,Ax_n}(\epsilon)\} \end{array}\right\}\right), \end{aligned}$$

for all $\epsilon \in (0, \frac{2}{k}t_0)$. As $n \to \infty$, it follows that

$$\begin{aligned} F_{z,l}(t_0) &\geq \phi\left(\min\left\{\begin{array}{c} F_{z,z}(t_0), \\ \min\{F_{z,z}(\epsilon), F_{z,l}(\frac{2}{k}t_0 - \epsilon)\}, \\ \max\{F_{z,l}(\frac{2}{k}t_0 - \epsilon), F_{z,z}(\epsilon)\} \end{array}\right\}\right) \\ &= \phi\left(F_{z,l}\left(\frac{2}{k}t_0 - \epsilon\right)\right) \\ &> F_{z,l}\left(\frac{2}{k}t_0 - \epsilon\right), \end{aligned}$$

as $\epsilon \to 0$, we have

$$F_{z,l}(t_0) \geq F_{z,l}\left(\frac{2}{k}t_0\right),$$

which contradicts (3.4). Therefore, $z = l$. Hence the pairs (A, S) and (B, T) share the common property (E.A). □

Remark 3.1 In general, the converse of Lemma 3.1 is not true (see [28, Example 3.1]).

Now we prove a common fixed-point theorem for two pairs of mappings in Menger space which is an extension of the main result of Sedghi *et al.* [55] in a version of Menger space.

Theorem 3.1 *Let A, B, S and T be self-mappings of a Menger space $(X, \mathcal{F}, *)$, where $*$ is a continuous t-norm satisfying inequality* (3.1) *of Lemma* 3.1. *Suppose that*

(1) *the pairs (A, S) and (B, T) share the common property $(E.A)$,*
(2) *$S(X)$ and $T(X)$ are closed subsets of X.*

Then the pairs (A, S) and (B, T) have a coincidence point each. Moreover, A, B, S, and T have a unique common fixed point provided both pairs (A, S) and (B, T) are weakly compatible.

Proof Since the pairs (A,S) and (B,T) share the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z, \tag{3.5}$$

for some $z \in X$. Since $S(X)$ is a closed subset of X, hence $\lim_{n\to\infty} Sx_n = z \in S(X)$. Therefore, there exists a point $u \in X$ such that $Su = z$. Now we assert that $Au = Su$. Suppose that $Au \neq Su$, then there exists $t_0 > 0$ such that

$$F_{Au,Su}\left(\frac{2}{k}t_0\right) > F_{Au,Su}(t_0). \tag{3.6}$$

In order to establish the claim embodied in (3.6), let us assume that (3.6) does not hold. Then we have $F_{Au,Su}(\frac{2}{k}t) \leq F_{Au,Su}(t)$ for all $t > 0$. Repeatedly using this equality, we obtain

$$F_{Au,Su}(t) \geq F_{Au,Su}\left(\frac{2}{k}t\right) \geq \cdots \geq F_{Au,Su}\left(\left(\frac{2}{k}\right)^n t\right) \to 1,$$

as $n \to \infty$. This shows that $F_{Au,Su}(t) = 1$ for all $t > 0$, which contradicts $Au \neq Su$ and hence (3.6) is proved. Using inequality (3.1), with $x = u$, $y = y_n$, we get

$$\begin{aligned} F_{Au,By_n}(t_0) &\geq \phi\left(\min\left\{\begin{array}{c} F_{Su,Ty_n}(t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{Su,Au}(t_1), F_{Ty_n,By_n}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{Su,By_n}(t_3), F_{Ty_n,Au}(t_4)\} \end{array}\right\}\right) \\ &\geq \phi\left(\min\left\{\begin{array}{c} F_{z,Ty_n}(t_0), \\ \min\{F_{z,Au}(\frac{2}{k}t_0 - \epsilon), F_{By_n,Ty_n}(\epsilon)\}, \\ \max\{F_{z,By_n}(\epsilon), F_{Ty_n,z}(\frac{2}{k}t_0 - \epsilon)\} \end{array}\right\}\right), \end{aligned}$$

for all $\epsilon \in (0, \frac{2}{k}t_0)$. As $n \to \infty$, it follows that

$$\begin{aligned} F_{Au,z}(t_0) &\geq \phi\left(\min\left\{\begin{array}{c} F_{z,z}(t_0), \\ \min\{F_{z,Au}(\frac{2}{k}t_0 - \epsilon), F_{z,z}(\epsilon)\}, \\ \max\{F_{z,z}(\epsilon), F_{z,z}(\frac{2}{k}t_0 - \epsilon)\} \end{array}\right\}\right) \\ &= \phi\left(F_{z,Au}\left(\frac{2}{k}t_0 - \epsilon\right)\right) \\ &> F_{Au,z}\left(\frac{2}{k}t_0 - \epsilon\right), \end{aligned}$$

as $\epsilon \to 0$, we have

$$F_{Au,z}(t_0) \geq F_{Au,z}\left(\frac{2}{k}t_0\right),$$

which contradicts (3.6). Therefore $Au = Su = z$ and hence u is a coincidence point of (A,S).

If $T(X)$ is a closed subset of X. Therefore there exists a point $v \in X$ such that $Tv = z$. Now we assert that $Bv = Tv = z$. Let, on the contrary, $Bv \neq Tv$. As earlier, there exists $t_0 > 0$ such that

$$F_{Bv,Tv}\left(\frac{2}{k}t_0\right) > F_{Bv,Tv}(t_0). \tag{3.7}$$

To support the claim, let it be untrue. Then we have $F_{Bv,Tv}(\frac{2}{k}t) \leq F_{Bv,Tv}(t)$ for all $t > 0$. Repeatedly using this equality, we obtain

$$F_{Bv,Tv}(t) \geq F_{Bv,Tv}\left(\frac{2}{k}t\right) \geq \cdots \geq F_{Bv,Tv}\left(\left(\frac{2}{k}\right)^n t\right) \to 1,$$

as $n \to \infty$. This shows that $F_{Bv,Tv}(t) = 1$ for all $t > 0$, which contradicts $Bv \neq Tv$ and hence (3.7) is proved. Using inequality (3.1), with $x = x_n$, $y = v$, we get

$$\begin{aligned} F_{Ax_n,Bv}(t_0) &\geq \phi\left(\min\left\{\begin{array}{c} F_{Sx_n,Tv}(t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0}\min\{F_{Sx_n,Ax_n}(t_1), F_{Tv,Bv}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0}\max\{F_{Sx_n,Bv}(t_3), F_{Tv,Ax_n}(t_4)\} \end{array}\right\}\right) \\ &\geq \phi\left(\min\left\{\begin{array}{c} F_{Sx_n,z}(t_0), \\ \min\{F_{Sx_n,Ax_n}(\epsilon), F_{z,Bv}(\frac{2}{k}t_0-\epsilon)\}, \\ \max\{F_{Sx_n,Bv}(\frac{2}{k}t_0-\epsilon), F_{z,Ax_n}(\epsilon)\} \end{array}\right\}\right), \end{aligned}$$

for all $\epsilon \in (0, \frac{2}{k}t_0)$. As $n \to \infty$, it follows that

$$\begin{aligned} F_{z,Bv}(t_0) &\geq \phi\left(\min\left\{\begin{array}{c} F_{z,z}(t_0), \\ \min\{F_{z,z}(\epsilon), F_{z,Bv}(\frac{2}{k}t_0-\epsilon)\}, \\ \max\{F_{z,Bv}(\frac{2}{k}t_0-\epsilon), F_{z,z}(\epsilon)\} \end{array}\right\}\right) \\ &= \phi\left(F_{z,Bv}\left(\frac{2}{k}t_0-\epsilon\right)\right) \\ &> F_{z,Bv}\left(\frac{2}{k}t_0-\epsilon\right), \end{aligned}$$

as $\epsilon \to 0$, we have

$$F_{z,Bv}(t_0) \geq F_{z,Bv}\left(\frac{2}{k}t_0\right),$$

which contradicts (3.7). Therefore $Bv = Tv = z$, which shows that v is a coincidence point of the pair (B, T).

Since the pair (A, S) is weakly compatible, therefore $Az = ASu = SAu = Sz$. Now we assert that z is a common fixed point of (A, S). If $z \neq Az$, then on using (3.1) with $x = z$, $y = v$, we get, for some $t_0 > 0$,

$$F_{Az,Bv}(t_0) \geq \phi\left(\min\left\{\begin{array}{c} F_{Sz,Tv}(t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0}\min\{F_{Sz,Az}(t_1), F_{Tv,Bv}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0}\max\{F_{Sz,Bv}(t_3), F_{Tv,Az}(t_4)\} \end{array}\right\}\right),$$

$$F_{Az,z}(t_0) \geq \phi\left(\min\left\{\begin{array}{c} F_{Az,z}(t_0), \\ \min\{F_{Az,Az}(\epsilon), F_{z,z}(\frac{2}{k}t_0-\epsilon)\}, \\ \max\{F_{Az,z}(\epsilon), F_{z,Az}(\frac{2}{k}t_0-\epsilon)\} \end{array}\right\}\right),$$

for all $\epsilon \in (0, \frac{2}{k}t_0)$. As $\epsilon \to 0$, we have

$$\begin{aligned} F_{Az,z}(t_0) &\geq \phi\left(\min\left\{F_{Az,z}(t_0), F_{z,Az}\left(\frac{2}{k}t_0\right)\right\}\right) \\ &= \phi\big(F_{Az,z}(t_0)\big) \\ &> F_{Az,z}(t_0), \end{aligned}$$

which is a contradiction. Hence $Az = Sz = z$, *i.e.* z is a common fixed point of (A, S). Also the pair (B, T) is weakly compatible, therefore $Bz = BTv = TBv = Tz$. Now we show that z is also a common fixed point of (B, T). If $z \neq Bz$, then on using (3.1) with $x = u$, $y = z$, we get, for some $t_0 > 0$,

$$F_{Au,Bz}(t_0) \geq \phi\left(\min\left\{\begin{array}{c} F_{Su,Tz}(t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{Su,Au}(t_1), F_{Tz,Bz}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{Su,Bz}(t_3), F_{Tz,Au}(t_4)\} \end{array}\right\}\right),$$

$$F_{z,Bz}(t_0) \geq \phi\left(\min\left\{\begin{array}{c} F_{z,Bz}(t_0), \\ \min\{F_{z,z}(\epsilon), F_{Bz,Bz}(\frac{2}{k}t_0 - \epsilon)\}, \\ \max\{F_{z,Bz}(\epsilon), F_{Bz,z}(\frac{2}{k}t_0 - \epsilon)\} \end{array}\right\}\right),$$

for all $\epsilon \in (0, \frac{2}{k}t_0)$. As $\epsilon \to 0$, we have

$$\begin{aligned} F_{z,Bz}(t_0) &\geq \phi\left(\min\left\{F_{z,Bz}(t_0), F_{Bz,z}\left(\frac{2}{k}t_0\right)\right\}\right) \\ &= \phi\big(F_{z,Bz}(t_0)\big) \\ &> F_{z,Bz}(t_0), \end{aligned}$$

which is a contradiction. Therefore $Bz = z = Tz$, which shows that z is a common fixed point of the pair (B, T). Therefore z is a common fixed point of both pairs (A, S) and (B, T). The uniqueness of common fixed point is an easy consequence of inequality (3.1). □

Remark 3.2 Theorem 3.1 is an improved probabilistic version of the result of Sedghi *et al.* [55, Theorem 1] for two pairs of self-mappings without any requirement on containment of ranges amongst the involved mappings.

The following example illustrates Theorem 3.1.

Example 3.2 Let $(X, \mathcal{F}, *)$ be a Menger space, where $X = [2, 19]$, with continuous t-norm $*$ is defined by $a * b = ab$ for all $a, b \in [0, 1]$ and

$$F_{x,y}(t) = \left(\frac{t}{t+1}\right)^{|x-y|}$$

for all $x, y \in X$. The function ϕ is defined as in Example 3.1. Define the self-mappings A, B, S, and T by

$$A(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3,19]; \\ 3, & \text{if } x \in (2,3], \end{cases} \qquad B(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3,19]; \\ 2.5, & \text{if } x \in (2,3], \end{cases}$$

$$S(x) = \begin{cases} 2, & \text{if } x = 2; \\ 10, & \text{if } x \in (2,3]; \\ \frac{x+77}{40}, & \text{if } x \in (3,19], \end{cases} \qquad T(x) = \begin{cases} 2, & \text{if } x = 2; \\ 13, & \text{if } x \in (2,3); \\ 14, & \text{if } x = 3; \\ \frac{x+77}{40}, & \text{if } x \in (3,19]. \end{cases}$$

We take $\{x_n\} = \{3 + \frac{1}{n}\}$, $\{y_n\} = \{2\}$ or $\{x_n\} = \{2\}$, $\{y_n\} = \{3 + \frac{1}{n}\}$. We have

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 2 \in X.$$

Therefore, both pairs (A, S) and (B, T) satisfy the common property (E.A).

It is noted that $A(X) = \{2, 3\} \nsubseteq [2, 2.4] \cup \{13, 14\} = T(X)$ and $B(X) = \{2, 2.5\} \nsubseteq [2, 2.4] \cup \{10\} = S(X)$. On the other hand, $S(X)$ and $T(X)$ are closed subsets of X. Thus, all the conditions of Theorem 3.1 are satisfied and 2 is a unique common fixed point of the pairs (A, S) and (B, T), which also remains a point of coincidence as well. Also, all the involved mappings are even discontinuous at their unique common fixed point 2.

Remark 3.3 In fact, the mapping $\mathcal{F}$ in Example 3.2 is also a fuzzy metric. However, the result of Sedghi *et al.* [55, Theorem 1] cannot be used for this case since $A(X) \nsubseteq T(X)$ and $B(X) \nsubseteq S(X)$.

Theorem 3.2 *The conclusion of Theorem* 3.1 *remains true if the condition* (2) *of Theorem* 3.1 *is replaced by the following*:

(2)′ $\overline{A(X)} \subset T(X)$ *and* $\overline{B(X)} \subset S(X)$, *where* $\overline{A(X)}$ *is the closure range of* A *and* $\overline{B(X)}$ *is the closure range of* B.

Proof Since the pairs (A, S) and (B, T) satisfy the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$. Then since $z \in \overline{A(X)}$ and $\overline{A(X)} \subset T(X)$ there exists a point $v \in X$ such that $z = Tv$. By the proof of Theorem 3.1, we can show that the pair (B, T) has a coincidence point, call it v, *i.e.* $Bv = Tv$. Since $z \in \overline{B(X)}$ and $\overline{B(X)} \subset S(X)$ there exists a point $u \in X$ such that $z = Su$. Similarly we can also prove that the pair (A, S) has a coincidence point, call it u, *i.e.* $Au = Su$. The rest of the proof is on the lines of the proof of Theorem 3.1, hence it is omitted. □

Corollary 3.1 *The conclusions of Theorems* 3.1-3.2 *remain true if condition* (2) *of Theorem* 3.1 *and condition* (2)′ *of Theorem* 3.2 *are replaced by the following*:

(2)″ $A(X)$ *and* $B(X)$ *are closed subsets of* X *provided* $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$.

Theorem 3.3 *Let $(X,\mathcal{F},*)$ be a Menger space, where $*$ is a continuous t-norm. Let A, B, S and T be mappings from X into itself and satisfying the conditions* (1)-(4) *of Lemma* 3.1. *Suppose that*

(5) *$S(X)$ (or $T(X)$) is a closed subset of X.*

Then the pairs (A,S) and (B,T) have a coincidence point each. Moreover, A, B, S and T have a unique common fixed point provided both pairs (A,S) and (B,T) are weakly compatible.

Proof In view of Lemma 3.1, the pairs (A,S) and (B,T) share the common property (E.A), *i.e.* there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$.

If $S(X)$ is a closed subset of X, then on the lines of Theorem 3.1, we can show that the pair (A,S) has coincidence point, say u, *i.e.* $Au = Su = z$. Since $A(X) \subset T(X)$ and $Au \in A(X)$, there exists $v \in X$ such that $Au = Tv$. The rest of the proof runs along the lines of the proof of Theorem 3.1, therefore details are omitted. □

Remark 3.4 Theorem 3.3 is also a partial improvement of Theorem 3.1 besides relaxing the closedness of one of the subspaces.

Example 3.3 In setting of Example 3.2, replace the self-mappings A, B, S and T by

$$A(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3,19]; \\ 3, & \text{if } x \in (2,3], \end{cases} \qquad B(x) = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3,19]; \\ 4, & \text{if } x \in (2,3], \end{cases}$$

$$S(x) = \begin{cases} 2, & \text{if } x = 2; \\ 14, & \text{if } x \in (2,3]; \\ \frac{x+1}{2}, & \text{if } x \in (3,19], \end{cases} \qquad T(x) = \begin{cases} 2, & \text{if } x = 2; \\ 11 + x, & \text{if } x \in (2,3]; \\ \frac{x+1}{2}, & \text{if } x \in (3,19]. \end{cases}$$

It is noted that $A(X) = \{2,3\} \subset [2,10] \cup (13,14] = T(X)$ and $B(X) = \{2,4\} \subset [2,10] \cup \{14\} = S(X)$. Also the pairs (A,S) and (B,T) commute at 2 which is their common coincidence point. Thus all the conditions of Theorems 3.2-3.3 and Corollary 3.1 are satisfied and 2 is a unique common fixed point of A, B, S and T. Here, it may be pointed out that Theorem 3.1 is not applicable to this example as $S(X)$ is not a closed subset of X. Also, notice that some mappings in this example are even discontinuous at their unique common fixed point 2.

By choosing A, B, S, and T suitably, we can drive a multitude of common fixed-point theorems for a pair or triod of self-mappings. If we take $A = B$ and $S = T$ in Theorem 3.1 then we get the following natural result which is an improved probabilistic version of the result of Sedghi *et al.* [55, Theorem 1].

Corollary 3.2 *Let $(X,\mathcal{F},*)$ be a Menger space, where $*$ is a continuous t-norm. Let A and S be mappings from X into itself and satisfying the following conditions*:

(1) *The pair (A,S) shares property (E.A),*

(2) *$S(X)$ is a closed subset of X,*

(3) *there exist $\phi \in \Phi$ and $1 \leq k < 2$ such that*

$$F_{Ax,Ay}(t) \geq \phi\left(\min\left\{\begin{array}{c} F_{Sx,Sy}(t), \\ \sup_{t_1+t_2=\frac{2}{k}t} \min\{F_{Sx,Ax}(t_1), F_{Sy,Ay}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{F_{Sx,Ay}(t_3), F_{Sy,Ax}(t_4)\} \end{array}\right\}\right) \tag{3.8}$$

holds for all $x, y \in X$ and $t > 0$. Then the pair (A, S) has a coincidence point. Moreover, A and S have a unique common fixed point provided the pair (A, S) is weakly compatible.

Our next theorem is proved for six self-mappings in Menger space, which extends earlier proved Theorem 3.1.

Theorem 3.4 *Let $(X, \mathcal{F}, *)$ be a Menger space, where $*$ is a continuous t-norm. Let A, B, R, S, H and T be mappings from X into itself and satisfying the following conditions:*

(1) *The pairs (A, SR) and (B, TH) share the common property (E.A),*
(2) *$SR(X)$ and $TH(X)$ are closed subsets of X,*
(3) *there exist $\phi \in \Phi$ and $1 \leq k < 2$ such that*

$$F_{Ax,By}(t) \geq \phi\left(\min\left\{\begin{array}{c} F_{SRx,THy}(t), \\ \sup_{t_1+t_2=\frac{2}{k}t} \min\{F_{SRx,Ax}(t_1), F_{THy,By}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{F_{SRx,By}(t_3), F_{THy,Ax}(t_4)\} \end{array}\right\}\right) \tag{3.9}$$

holds for all $x, y \in X$ and $t > 0$. Then the pairs (A, SR) and (B, TH) have a coincidence point each. Moreover, A, B, R, S, H, and T have a unique common fixed point provided the pairs (A, SR) and (B, TH) commute pairwise (i.e. $AS = SA$, $AR = RA$, $SR = RS$, $BT = TB$, $BH = HB$, and $TH = HT$).

Proof By Theorem 3.1, A, B, TH and SR have a unique common fixed point z in X. We show that z is a unique common fixed point of the self-mappings A, R and S. If $z \neq Rz$, then on using (3.9) with $x = Rz$, $y = z$, we get, for some $t_0 > 0$,

$$F_{A(Rz),Bz}(t_0) \geq \phi\left(\min\left\{\begin{array}{c} F_{SR(Rz),THz}(t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{F_{SR(Rz),A(Rz)}(t_1), F_{THz,Bz}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{F_{SR(Rz),Bz}(t_3), F_{THz,A(Rz)}(t_4)\} \end{array}\right\}\right),$$

$$F_{Rz,z}(t_0) \geq \phi\left(\min\left\{\begin{array}{c} F_{Rz,z}(t_0), \\ \min\{F_{Rz,Rz}(\epsilon), F_{z,z}(\frac{2}{k}t_0 - \epsilon)\}, \\ \max\{F_{Rz,z}(\epsilon), F_{z,Rz}(\frac{2}{k}t_0 - \epsilon)\} \end{array}\right\}\right),$$

for all $\epsilon \in (0, \frac{2}{k}t_0)$. As $\epsilon \to 0$, we have

$$\begin{aligned} F_{Rz,z}(t_0) &\geq \phi\left(\min\left\{F_{Rz,z}(t_0), F_{z,Rz}\left(\frac{2}{k}t_0 - \epsilon\right)\right\}\right) \\ &= \phi\big(F_{Rz,z}(t_0)\big) \\ &> F_{Rz,z}(t_0), \end{aligned}$$

which is a contradiction. Therefore, $Rz = z$ and so $S(Rz) = S(z) = z$. Similarly, we get $Tz = Hz = z$. Hence z is a unique common fixed point of self-mappings A, B, R, S, H and T in X. □

Corollary 3.3 *Let $(X, \mathcal{F}, *)$ be a Menger space, where $*$ is a continuous t-norm. Let $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$ and $\{T_g\}_{g=1}^q$ be four finite families from X into itself such that $A = A_1A_2\cdots A_m$, $B = B_1B_2\cdots B_n$, $S = S_1S_2\cdots S_p$ and $T = T_1T_2\cdots T_q$, which satisfy the inequality* (3.1). *If the pairs (A, S) and (B, T) share the common property $(E.A)$ along with closedness of $S(X)$ and $T(X)$, then (A, S) and (B, T) have a point of coincidence each.*

Moreover, $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$ and $\{T_g\}_{g=1}^q$ have a unique common fixed point provided the pairs of families $(\{A_i\}, \{S_k\})$ and $(\{B_r\}, \{T_g\})$ are commute pairwise, where $i \in \{1, 2, \ldots, m\}$, $k \in \{1, 2, \ldots, p\}$, $r \in \{1, 2, \ldots, n\}$ and $g \in \{1, 2, \ldots, q\}$.

Proof The proof of this theorem is similar to that of Theorem 3.1 contained in Imdad *et al.* [41], hence details are omitted. □

Remark 3.5 Corollary 3.3 extends the result of Sedghi *et al.* [55, Theorem 2] to four finite families of self-mappings.

By setting $A_1 = A_2 = \cdots = A_m = A$, $B_1 = B_2 = \cdots = B_n = B$, $S_1 = S_2 = \cdots = S_p = S$, and $T_1 = T_2 = \cdots = T_q = T$ in Corollary 3.3, we deduce the following.

Corollary 3.4 *Let $(X, \mathcal{F}, *)$ be a Menger space, where $*$ is a continuous t-norm. Let A, B, S and T be mappings from X into itself such that the pairs (A^m, S^p) and (B^n, T^q) share the common property $(E.A)$. Then there exist $\phi \in \Phi$, $1 \leq k < 2$ and $m, n, p, q \in \mathbb{N}$ such that*

$$F_{A^m x, B^n y}(t) \geq \phi\left(\min\left\{\begin{array}{c} F_{S^p x, T^q y}(t), \\ \sup_{t_1+t_2=\frac{2}{k}t} \min\{F_{S^p x, A^m x}(t_1), F_{T^q y, B^n y}(t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{F_{S^p x, B^n y}(t_3), F_{T^q y, A^m x}(t_4)\} \end{array}\right\}\right) \tag{3.10}$$

holds for all $x, y \in X$ and $t > 0$. If $S^p(X)$ and $T^q(X)$ are closed subsets of X, then the pairs (A, S) and (B, T) have a point of coincidence each. Further, A, B, S, and T have a unique common fixed point provided both pairs (A^m, S^p) and (B^n, T^q) commute pairwise.

Conclusion

Theorem 3.1 is proved for two pairs of weakly compatible mappings in Menger spaces using common property (E.A). Theorem 3.1 is an improved probabilistic version of the result of Sedghi *et al.* [55, Theorem 1] for two pairs of mappings without any requirement on containment of ranges amongst the involved mappings. Several results (Theorem 3.2, Theorem 3.3 and Corollary 3.1) are also defined for the existence of fixed points in Menger spaces. Example 3.2 and Example 3.3 are furnished in support of our results. As an extension of our main result, Theorem 3.4 is proved for six self-mappings using the notion of pairwise commuting whereas Corollary 3.3 extends Theorem 3.1 to four finite families of self-mappings.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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