Kim and Buong *Journal of Inequalities and Applications* 2014, 2014:440
http://www.journalofinequalitiesandapplications.com/content/2014/1/440

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Convergence rates in regularization for a system of nonlinear ill-posed equations with m-accretive operators

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Abstract
In this paper, we study an operator version of the modified Browder-Tikhonov regularization method for finding a common solution for a system of ill-posed operator equations involving m-accretive operators A_i, $i = 0, \dots, N$, in a reflexive Banach space. The convergence rates of the regularized solutions are estimated not only in the infinite-dimensional space, but also in connection with its finite-dimensional approximations without the weakly sequential continuity of the dual mapping.
MSC: 47H17; 47H20

Keywords: accretive operators; demicontinuous; convex Banach space; modified Browder-Tikhonov regularization; Fréchet differentiability

1 Introduction

Let X be a real reflexive Banach space with the property of approximations and its dual space X^* be strictly convex. The norms of X and X^* are denoted by the symbol $\|\cdot\|$. We write $\langle x, x^*\rangle$ instead of $x^*(x)$ for $x^* \in X^*$ and $x \in X$.

Definition 1.1 A Banach space X is said to be strictly convex if for $x, y \in S_X$ with $x \neq y$, then

$$\bigl\|(1-\lambda)x + \lambda y\bigr\| < 1 \quad \forall \lambda \in (0,1),$$

where S_X is the unit sphere $S_X = \{x \in X : \|x\| = 1\}$.

Definition 1.2 A mapping j from X onto X^* is called the normalized dual mapping of X, if it satisfies the condition

$$\bigl\langle x, j(x)\bigr\rangle = \|x\|^2, \qquad \bigl\|j(x)\bigr\| = \|x\| \quad \forall x \in X.$$

It is well known that if X^* is strictly convex then j is single-valued.

Definition 1.3 An operator A from X to X is said to be accretive, if

$$\bigl\langle A(x) - A(y), j(x-y)\bigr\rangle \geq 0 \quad \forall x, y \in D(A),$$

where $D(A)$ denotes the domain of A. An accretive operator A is said to be an m-accretive, if $\mathcal{R}(A+\lambda I) = X$ for $\lambda > 0$ where $\mathcal{R}(A)$ and I denote the range of A and the identity mapping of X, respectively.

Definition 1.4 An operator A from X to X is said to be

(i) demicontinuous if $x_n \to x$ in X implies $A(x_n) \rightharpoonup A(x)$,

(ii) weakly continuous if $x_n \rightharpoonup x$ implies $A(x_n) \rightharpoonup A(x)$.

It is well known that if A is accretive, and is continuous, demicontinuous, or weakly continuous, then it is m-accretive [1–3].

Definition 1.5 A mapping A from X to X is called Fréchet differentiable at a point $x \in D(A)$, if

$$A(x+h) - A(x) = B(x)h + o\bigl(\|h\|\bigr) \quad \forall x+h \in D(A),$$

where $B(x)$ is a bounded linear mapping from X to X. And the Fréchet derivative of A at $x \in D(A)$ is denoted by $A'(x)$.

Let $\{A_i\}_{i=0}^N$ be a family of $N+1$ accretive operators in X and satisfy one of the above mentioned three continuities.

Our problem is to find a common solution of the following operator equations:

$$A_i(x) = f_i, \quad f_i \in \mathcal{R}(A_i), i = 0, \dots, N. \tag{1.1}$$

Set

$$S = \bigcap_{i=0}^N S_i,$$

where S_i is the solution set of (1.1), that is, $S_i = \{x : A_i(x) = f_i\}$.

Suppose that $S \neq \emptyset$.

For m-accretive operators, some results of the approximating solution for each equation in (1.1) under suitable different conditions are investigated in [4–10], and [11].

The system of equations (1.1) is ill-posed, because each one of the system is ill-posed. By ill-posedness, we mean that its solutions do not depend continuously on the data (A_i, f_i). Therefore, we have to use the stable methods in order to solve the problem. Some stable methods of approximating solution for each equation in (1.1) with m-accretive operator are investigated in [12, 13], and [14] having the weakly sequentially continuous duality mapping j. In [15–19], the authors considered the modified Browder-Tikhonov regularization method with the regularization parameter choice without the property for j, for the case of demicontinuous or weakly continuous accretive operators A_i satisfying the condition

$$\bigl\|A_i(y) - A_i\bigl(x_0^i\bigr) - j^* A_i'\bigl(x_0^i\bigr) j\bigl(y - x_0^i\bigr)\bigr\| \leq \tilde{\tau} \bigl\|y - x_0^i\bigr\| \bigl\|A_i'(x_*) j\bigl(y - x_0^i\bigr)\bigr\| \tag{1.2}$$

for y in some neighborhood of S_i, where $A_i'(x_0^i)$ is the Fréchet derivative of A_i at $x_0^i \in S_i$, $\tilde{\tau}$ is some positive constant, and j^* is the normalized duality mapping of X^*.

In many papers, for each i, the regularized solution of (1.1) is constructed by the following operator equation:

$$A_i^h(x) + \alpha x = f_i^\delta,$$

where (A_i^h, f_i^δ) is the approximation of (A_i, f_i) satisfying the conditions:

$$\left\|A_i^h(x) - A_i(x)\right\| \le hg\big(\|x\|\big), \qquad \left\|f_i - f_i^\delta\right\| \le \delta, \quad h, \delta \to 0, \tag{1.3}$$

$g(t)$ is a nonnegative bounded (image of bounded set is bounded) real function, and A_i^h is also accretive and the same continuity as A_i.

The system of equations (1.1) can be written in the form

$$\mathcal{A}(x) = f, \tag{1.4}$$

where $\mathcal{A} : \mathcal{X} \to \mathcal{X} := X \times \cdots \times X$ is defined by $\mathcal{A}(x) := (A_0(x), \ldots, A_N(x))$, and $f := (f_0, \ldots, f_N)$.

Note that (1.4) can be seen as a special case of (1.1) with $N = 0$. However, one potential advantage of (1.1) over (1.4) can be that it might better reflect the structure of the underlying information $(f_0, \ldots, f_N)$ leading to the couplet system, than a plain concatenation into one single data element f could. In particular, the second advantage is that in estimating convergence rates of regularization solution, which is showed later, we need only the smooth property for one among A_i, while for (1.4) we need the property for A_i, $i = 0, \ldots, N$.

When for each i, A_i is the nonlinear Fréchet differentiable operator from the Hilbert space X to the Hilbert space Y_i with derivative being uniformly bounded in a neighborhood of a common solution, a stable method for problem (1.1) is considered in [20].

In this paper, we show that a common solution of (1.1) involving m-accretive operators A_i, without the weakly sequentially continuous property of j, can be approximated by the modified Browder-Tikhonov regularization method which is described by the following operator equation:

$$A_0^h(x) + \alpha^{1+\mu_0} \sum_{i=1}^{N} \big(A_i^h(x) - f_i^\delta\big) + \alpha x = f_0^\delta, \quad \mu_0 \ge 0, \tag{1.5}$$

where $\alpha > 0$ is a small regularization parameter. Since the operator

$$A_0^h + \alpha^{1+\mu_0} \sum_{i=1}^{N} \big(A_i^h - f_i^\delta\big)$$

has the same properties as each A_i^h, it is also m-accretive. Therefore, (1.5) has a unique solution denoted by x_α^τ, $\tau = (\delta, h)$, for every value $\alpha > 0$.

In the following section, the convergence rates of the regularized solution x_α^τ and its finite-dimensional approximations $x_{\alpha,n}^\tau$ are established under an assumption similar to (1.2).

The symbols '$\to$' and '$\rightharpoonup$' denote strong and weak convergence, respectively, and the notation $a \sim b$ means that $a = o(b)$ and $b = o(a)$.

2 Main results

Assumption A There exists a constant $\tau_0 > 0$ such that

$$\|A_0(x) - A_0(x_0) - j^* A_0'(x_0)^* j(x - x_0)\| \le \tau_0 \|A_0(x) - A_0(x_0)\|, \quad \forall x \in X.$$

Now, we are in a position to introduce the main theorem.

Theorem 2.1 *Let X be a real reflexive Banach space with the property of approximations and its dual space X^* be strictly convex. Let $\{A_i\}_{i=0}^N$ be a family of $N+1$ accretive operators in X and satisfy one of the above mentioned continuities. Assume that the following conditions hold*:

(i) *A_0 is Fréchet differentiable at x_0 with Assumption* A.

(ii) *There exists an element $z \in X$ such that*

$$A_0'(x_0)z = -x_0.$$

(iii) *The parameter α is chosen such that $\alpha \sim (\delta + h)^\mu$, $0 < \mu < 1$.*

Then, for $0 < \delta + h < 1$, we have

$$\|x_\alpha^\tau - x_0\| = o\big((\delta + h)^\theta\big), \quad \theta = \min\{1 - \mu, \mu/2\}.$$

Proof From the property of j, A_i^h, (1.1), (1.3), (1.5), and condition (ii), it follows that

$$\begin{aligned}
\|x_\alpha^\tau - x_0\|^2 &= \langle x_\alpha^\tau - x_0, j(x_\alpha^\tau - x_0)\rangle \\
&= \frac{1}{\alpha}\left\langle f_0^\delta - A_0^h(x_\alpha^\tau) + \alpha^{1+\mu_0}\sum_{i=1}^N \big(f_i^\delta - A_i^h(x_\alpha^\tau)\big) - \alpha x_0, j(x_\alpha^\tau - x_0)\right\rangle \\
&\le \frac{1 + N\alpha^{1+\mu_0}}{\alpha}\big[\delta + hg(\|x_0\|)\big]\|x_\alpha^\tau - x_0\| \\
&\quad + \big\langle z, A_0'(x_0)^* j(x_\alpha^\tau - x_0)\big\rangle.
\end{aligned} \tag{2.1}$$

Therefore, $\{x_\alpha^\tau\}$ is a bounded set. Since

$$\big\langle z, A_0'(x_0)^* j(x_\alpha^\tau - x_0)\big\rangle \le \|z\| \|A_0'(x_0)^* j(x_\alpha^\tau - x_0)\|,$$

by virtue of Assumption A, we have

$$\begin{aligned}
&\|A_0'(x_0)^* j(x_\alpha^\tau - x_0)\| \\
&\quad = \|j^* A_0'(x_0)^* j(x_\alpha^\tau - x_0)\| \\
&\quad \le (\tau_0 + 1)\|A_0(x_\alpha^\tau) - f_0\| \\
&\quad \le (\tau_0 + 1)\big[\|A_0^h(x_\alpha^\tau) - f_0^\delta\| + \delta + hg(\|x_\alpha^\delta\|)\big] \\
&\quad \le (\tau_0 + 1)\left[\alpha^{1+\mu_0}\sum_{i=1}^N \|A_i^h(x_\alpha^\tau) - f_i^\delta\| + \alpha\|x_\alpha^\delta\| + \delta + hg(\|x_\alpha^\delta\|)\right].
\end{aligned}$$

Since $\alpha \sim (\delta + h)^{\mu}$, $0 < \mu < 1$, and $g(t)$ is a bounded function, from (2.1) and the last inequality, we obtain

$$\|x_{\alpha}^{\tau} - x_0\|^2 \leq C_1(\delta + h)^{1-\mu}\|x_{\alpha}^{\tau} - x_0\| + C_2(\delta + h)^{\mu}, \quad 0 < \delta + h < 1,$$

where C_1 and C_2 are positive constants. Now, by using the implication

$$a, b, c \geq 0, p > q, \quad a^p \leq ba^q + c \quad \Longrightarrow \quad a^p = o\bigl(b^{p/(p-q)} + c\bigr),$$

we obtain

$$\|x_{\alpha}^{\tau} - x_0\| = o\bigl((\delta + h)^{\theta}\bigr), \quad \theta = \min\{1 - \mu, \mu/2\}.$$

This completes the proof. □

Now, we consider the problem of approximating (1.5) by the sequence of finite-dimensional problems

$$A_{0,n}^h(x) + \alpha^{1+\mu_0}\sum_{i=1}^{N}\bigl(A_{i,n}^h(x) - f_{i,n}^{\delta}\bigr) + \alpha x = f_{0,n}^{\delta}, \quad x \in X_n, \tag{2.2}$$

where $f_{i,n}^{\delta} = P_n f_i^{\delta}$, $A_{i,n}^h = P_n A_i^h P_n$, P_n is the linear projection from X onto X_n, $P_n x \to x$ for all $x \in X$, $\|P_n\| \leq C_0$, C_0 is some positive constant, and $\{X_n\}$ is the sequence of finite-dimensional subspaces of X such that

$$X_1 \subset X_2 \subset \cdots \subset X.$$

It is easy to see that $A_{i,n}^h$ are also m-accretive. The aspects of existence and convergence of the solution $x_{\alpha,n}^{\tau}$ of problem (2.2), as $n \to \infty$, to the solution x_{α}^{τ} of the operator equation (1.5) for each $\alpha > 0$ has been studied in [21]. The question under which conditions the sequence $\{x_{\alpha,n}^{\tau}\}$ converges to a solution x_0, as $\alpha, \delta, h \to 0$ and $n \to \infty$, and the convergence rates of $\{x_{\alpha,n}^{\tau}\}$ are subject of our further investigations.

In addition, suppose that j satisfies the following inequality:

$$\|j(x) - j(y)\| \leq C(R)\|x - y\|^{\nu}, \quad 0 < \nu < 1, \tag{2.3}$$

where $C(R)$, $R > 0$ is positive increasing function on $R = \max\{\|x\|, \|y\|\}$ (see [11]).

Set

$$\gamma_n = \|(I - P_n)x_0\|.$$

Theorem 2.2 *Let X be a real reflexive Banach space with the property of approximations and its dual space X^* be strictly convex. Let $\{A_i\}_{i=0}^N$ be a family of $N+1$ accretive operators in X and satisfy one of the above mentioned continuities. Suppose that the following conditions hold*:

(i) *A_0 is Fréchet differentiable with Assumption* A *and the derivative A_0' being uniformly bounded at x_0.*

(ii) *There exists an element* $z \in X$ *such that*

$$A_0'(x_0)z = -x_0.$$

(iii) *The parameter* α *is chosen such that* $\alpha \sim (\delta + h + \gamma_n)^\mu$, $0 < \mu < 1$.
Then, for $0 < \delta + h < 1$, *we have*

$$\|x_{\alpha,n}^\tau - x_0\| = o\big((\delta + h + \gamma_n)^\theta + \gamma_n^{\nu/2}\big), \quad \theta = \min\{1 - \mu, \mu/2\}.$$

Proof Set $x_0^n = P_n x_0$. From (2.2) and the property $j^n(x) = j(x)$ for all $x \in X_n$, where $j^n = P_n^* j P_n$ is the dual mapping of X_n (see [13]), it follows that

$$\begin{aligned}
\|x_{\alpha,n}^\tau - x_0^n\|^2 &= \langle x_{\alpha,n}^\tau - x_0^n, j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&= \frac{1}{\alpha}\langle f_{0,n}^\delta - A_{0,n}^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\quad + \langle -x_0^n, j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\quad + \alpha^{\mu_0}\sum_{i=1}^N \langle f_{i,n}^\delta - A_{i,n}^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle.
\end{aligned} \tag{2.4}$$

Clearly,

$$\begin{aligned}
&\langle f_{0,n}^\delta - A_{0,n}^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\quad = \langle f_0^\delta - f_0 + A_0(x_0) - A_0(x_0^n) + A_0(x_0^n) - A_0^h(x_0^n), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\qquad + \langle A_0^h(x_0^n) - A_0^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\quad \le \big[\delta + \|A_0(x_0) - A_0(x_0^n)\| + hg(\|x_0^n\|)\big]\|x_{\alpha,n}^\tau - x_0^n\|.
\end{aligned}$$

Due to condition (i) and $x_0^n \to x_0$ as $n \to \infty$, we have

$$\|A_0(x_0^n) - A_0(x_0)\| \le C_0'\gamma_n,$$

where C_0' is a positive constant such that

$$\|A_0'(x_0)\| \le C_0'$$

for x in a neighborhood of x_0. Thus, we have

$$\langle f_{0,n}^\delta - A_{0,n}^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \le \big[\delta + hg(\|x_0^n\|) + C_0'\gamma_n\big]\|x_{\alpha,n}^\tau - x_0^n\|. \tag{2.5}$$

Each term of the sum in (2.4) is estimated as follows:

$$\begin{aligned}
&\langle f_{i,n}^\delta - A_{i,n}^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\quad = \langle f_i^\delta - A_i^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle \\
&\quad = \langle f_i^\delta - A_i^h(x_0^n) + A_i^h(x_0^n) - A_i^h(x_{\alpha,n}^\tau), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle
\end{aligned}$$

$$\begin{aligned}&\leq \langle f_i^\delta - A_i^h(x_0^n), j^n(x_{\alpha,n}^\tau - x_0^n)\rangle\\&\leq [\delta + hg(\|x_0^n\|) + \|A_i(x_0) - A_i(x_0^n)\|]\|x_{\alpha,n}^\tau - x_0^n\|.\end{aligned}\tag{2.6}$$

By virtue of the continuity of A_i, there exists a positive constant C' such that

$$\|A_i(x_0) - A_i(x_0^n)\| \leq C', \quad i = 1, \dots, N.$$

From (2.4)-(2.6), we see that

$$\begin{aligned}\|x_{\alpha,n}^\tau - x_0^n\|^2 \leq{}& \left[\frac{1}{\alpha}(\delta + h + C_0'\gamma_n) + \alpha^{\mu_0}(\delta + hg(\|x_0^n\|) + C')\right]\|x_{\alpha,n}^\tau - x_0^n\|\\&+ \langle -x_0, j(x_{\alpha,n}^\tau - x_0^n)\rangle.\end{aligned}\tag{2.7}$$

Consequently, $\{x_{\alpha,n}^\tau\}$ is bounded as $\delta, h, \alpha \to 0$ and $n \to \infty$. Obviously, from (2.3), Assumption A, and condition (ii), it follows that

$$\begin{aligned}\langle -x_0, j(x_{\alpha,n}^\tau - x_0^n)\rangle &= \langle z, A_0'(x_0)^*[j(x_{\alpha,n}^\tau - x_0^n) - j(x_{\alpha,n}^\tau - x_0)]\rangle + \langle z, A_0'(x_0)^* j(x_{\alpha,n}^\tau - x_0)\rangle\\&\leq C(R_1)\|A_0'(x_0)^*\|\|z\|\gamma_n^\nu + \|z\|\|A_0'(x_0)^* j(x_{\alpha,n}^\tau - x_0)\|,\end{aligned}$$

where R_1 is a positive constant with $R_1 \geq \max\{\|x_0\|, \|x_{\alpha,n}^\tau\|\}$.

On the other hand,

$$\begin{aligned}\|A_0'(x_0)^* j(x_{\alpha,n}^\tau - x_0)\| &\leq (\tau_0 + 1)\|A_0(x_{\alpha,n}^\tau) - f_0\|\\&\leq (\tau_0 + 1)[\|A_0^h(x_{\alpha,n}^\tau) - f_0^h\| + \delta + hg(\|x_{\alpha,n}^\tau\|)].\end{aligned}$$

By virtue of the Hahn-Banach theorem, there exists an element $y^* \in X^*$ with $\|y^*\| = 1$ such that

$$\|A_0^h(x_{\alpha,n}^\tau) - f_0^h\| = \langle A_0^h(x_{\alpha,n}^\tau) - f_0^h, y^*\rangle.$$

Since

$$\langle A_0^h(x_{\alpha,n}^\tau) - f_0^h, y^*\rangle = \langle A_0^h(x_{\alpha,n}^\tau) - f_0^h, (I^* - P_n^*)y^*\rangle + \langle A_0^h(x_{\alpha,n}^\tau) - f_0^h, P_n^* y^*\rangle$$

and

$$\|(I^* - P_n^*)y^*\| \leq 1/2,$$

for sufficiently large n, where I^* is the identity operator in X^*, we have

$$\|A_0^h(x_{\alpha,n}^\tau) - f_0^h\| \leq 2\|A_{0,n}^h(x_{\alpha,n}^\tau) - f_{0,n}^h\|.$$

Therefore,

$$\|A_0'(x_0)^*j(x_{\alpha,n}^\tau - x_0)\| \le (\tau_0+1)C_0\Bigg[\alpha^{1+\mu_0}\sum_{i=1}^N \|A_i^h(x_{\alpha,n}^\tau) - f_i^\delta\| + \alpha\|x_{\alpha,n}^\tau\| + \delta + hg(\|x_{\alpha,n}^\tau\|)\Bigg].$$

Thus, (2.7) has the form

$$\|x_{\alpha,n}^\tau - x_0^n\|^2 \le \tilde{C}_1(\delta+h+\gamma_n)^{1-\mu}\|x_{\alpha,n}^\tau - x_0^n\| + \tilde{C}_2\big[(\delta+h+\gamma_n)^\mu + \gamma_n^\nu\big],$$

where $\tilde{C}_i > 0$ $(i = 1,2)$. Consequently, we have

$$\|x_{\alpha,n}^\tau - x_0\| = O\big((\delta+h+\gamma_n)^\theta + \gamma_n^{\nu/2}\big), \quad \theta = \min\{1-\mu, \mu/2\}.$$

This completes the proof. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The main idea of this paper was proposed by JKK. JKK and NB prepared the manuscript initially and performed all the steps of proof in this research. All authors read and approved the final manuscript.

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Acknowledgements
This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012R1A1A2042138).

Received: 18 April 2014 Accepted: 21 October 2014 Published: 03 Nov 2014

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10.1186/1029-242X-2014-440
Cite this article as: Kim and Buong: Convergence rates in regularization for a system of nonlinear ill-posed equations with *m*-accretive operators. *Journal of Inequalities and Applications* 2014, 2014:440