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Some results in quasi- b -metric-like spaces

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Abstract

In this paper, we introduce a new concept of quasi- b -metric-like spaces as a generalization of b -metric-like spaces and quasi-metric-like spaces. Some fixed point theorems are investigated in quasi- b -metric-like spaces. Moreover, an example is given to support one of our results.

MSC: 47H10; 54H25

Keywords: quasi- b -metric-like spaces; common fixed point; fixed point

1 Introduction and preliminaries

It is well known that the theoretical framework of metric fixed point theory has been an active research field and the contraction mapping principle is one of the most important theorems in functional analysis. Many authors have devoted their attention to generalizing metric spaces and the contraction mapping principle. In [1, 2], Matthews introduced the notion of partial metric space as a part of the study of denotational semantics of dataflow networks. The partial metric space is a generalization of the metric space. Many other generalized metric spaces, such as b -metric spaces [3], partial b -metric spaces [4], quasi-partial metric spaces [5], dislocated metric spaces [6] and b -dislocated metric spaces [7], were introduced. Fixed point theorems were studied in the above generalized metric spaces (see, e.g., [8–18] and the references therein).

The notion of metric-like spaces was introduced by Amini-Harandi in [19].

Definition 1.1 [19] A mapping $\sigma : X \times X \rightarrow [0, +\infty)$, where X is a nonempty set, is said to be metric-like on X if for any $x, y, z \in X$, the following three conditions hold true:

$$(\sigma_1) \quad \sigma(x, y) = 0 \Rightarrow x = y;$$

$$(\sigma_2) \quad \sigma(x, y) = \sigma(y, x);$$

$$(\sigma_3) \quad \sigma(x, z) \leq \sigma(x, y) + \sigma(y, z).$$

The pair (x, σ) is then called a metric-like space.

Recently, the concept of b -metric-like spaces, which is a new generalization of metric-like spaces and partial metric spaces, was introduced by Alghamdi *et al.* [20].

Definition 1.2 [20] A b -metric-like on a nonempty set X is a function $D : X \times X \rightarrow [0, +\infty)$ such that for all $x, y, z \in X$ and a constant $s \geq 1$, the following three conditions hold true:

$$(D_1) \quad \text{if } D(x, y) = 0 \text{ then } x = y;$$

- (D₂) $D(x, y) = D(y, x)$;
 (D₃) $D(x, z) \leq s[D(x, y) + D(y, z)]$.

The pair (x, D) is then called a *b*-metric-like space.

In [20], some concepts in *b*-metric-like spaces were introduced as follows.

Each *b*-metric-like D on X generalizes a topology τ_D on X whose base is the family of open D -balls $B_D(x, \varepsilon) = \{y \in X : |D(x, y) - D(x, x)| < \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

A sequence $\{x_n\}$ in the *b*-metric-like space (X, D) converges to a point $x \in X$ if and only if $D(x, x) = \lim_{n \rightarrow +\infty} D(x, x_n)$.

A sequence $\{x_n\}$ in the *b*-metric-like space (X, D) is called a Cauchy sequence if there exists (and is finite) $\lim_{n, m \rightarrow +\infty} D(x_m, x_n)$.

A *b*-metric-like space is called to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_D to a point $x \in X$ such that $\lim_{n \rightarrow +\infty} D(x, x_n) = D(x, x) = \lim_{n, m \rightarrow +\infty} D(x_m, x_n)$.

In [21], Zhu *et al.* introduced the concept of quasi-metric-like spaces and investigated some fixed point theorems in quasi-metric-like spaces.

Definition 1.3 [21] Let X be a nonempty set. A mapping $\rho : X \times X \rightarrow [0, +\infty)$ is said to be a quasi-metric-like on X if for any $x, y, z \in X$ the following conditions hold:

- (qρ1) $\rho(x, y) = 0 \Rightarrow x = y$;
 (qρ2) $\rho(x, z) \leq \rho(x, y) + \rho(y, z)$.

The pair (x, ρ) is then called a quasi-metric-like space.

In this paper, inspired by Definitions 1.2 and 1.3, we define a quasi-*b*-metric-like which generalizes the quasi-metric-like and *b*-metric-like. Furthermore, we investigate some fixed point theorems in quasi-*b*-metric-like spaces. Also, we give an example to illustrate the usability of one of the obtained results.

2 Main results

In this section, we begin with introducing the concept of a quasi-*b*-metric-like space.

Definition 2.1 A quasi-*b*-metric-like on a nonempty set X is a function $b : X \times X \rightarrow [0, +\infty)$ such that for all $x, y, z \in X$ and a constant $s \geq 1$, the following conditions hold true:

- (qb1) $b(x, y) = 0 \Rightarrow x = y$;
 (qb2) $b(x, z) \leq s[b(x, y) + b(y, z)]$.

The pair (X, b) is then called a quasi-*b*-metric-like space. The number s is called to be the coefficient of (X, b) .

Example 2.1 Let $X = \{0, 1, 2\}$, and let

$$b(x, y) = \begin{cases} 2, & x = y = 0, \\ \frac{1}{2}, & x = 0, y = 1, \\ 2, & x = 1, y = 0, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

Then (X, b) is a quasi- b -metric-like space with the coefficient $s = 2$, but since $b(0, 1) \neq b(1, 0)$, then (X, b) is not a b -metric-like space. It is obvious that (X, b) is not a quasi-metric-like space.

Definition 2.2 Let (X, b) be a quasi- b -metric-like space. Then

- (1) A sequence $\{x_n\}$ in (X, b) converges to a point $x \in X$ if and only if

$$\lim_{n \rightarrow +\infty} b(x_n, x) = \lim_{n \rightarrow +\infty} b(x, x_n) = b(x, x).$$

- (2) A sequence $\{x_n\}$ in (X, b) is called a Cauchy sequence if $\lim_{n, m \rightarrow +\infty} b(x_n, x_m)$ and $\lim_{n, m \rightarrow +\infty} b(x_m, x_n)$ exist and are finite.
 (3) A quasi- b -metric-like space (X, b) is called to be complete if for every Cauchy sequence $\{x_n\}$ in (X, b) , there exists some $x \in X$ such that

$$\lim_{n \rightarrow +\infty} b(x_n, x) = \lim_{n \rightarrow +\infty} b(x, x_n) = b(x, x) = \lim_{n, m \rightarrow +\infty} b(x_n, x_m) = \lim_{n, m \rightarrow +\infty} b(x_m, x_n).$$

- (4) A sequence $\{x_n\}$ in (X, b) is called a 0-Cauchy sequence if

$$\lim_{n, m \rightarrow +\infty} b(x_n, x_m) = \lim_{n, m \rightarrow +\infty} b(x_m, x_n) = 0.$$

- (5) A quasi- b -metric-like space (X, b) is called to be 0-complete if for every 0-Cauchy sequence $\{x_n\}$ in X , there exists some $x \in X$ such that

$$\lim_{n \rightarrow +\infty} b(x_n, x) = \lim_{n \rightarrow +\infty} b(x, x_n) = b(x, x) = 0 = \lim_{n, m \rightarrow +\infty} b(x_n, x_m) = \lim_{n, m \rightarrow +\infty} b(x_m, x_n).$$

It is obvious that every 0-Cauchy sequence is a Cauchy sequence in the quasi- b -metric-like space (X, b) , and every complete quasi- b -metric-like space is a 0-complete quasi- b -metric-like space, but the converse assertions of these facts may not be true.

Remark 2.1 In Example 2.1, let $x_n = 2$ for $n = 1, 2, \dots$, then it is clear that $\lim_{n \rightarrow +\infty} b(x_n, 2) = \lim_{n \rightarrow +\infty} b(2, x_n) = b(2, 2)$ and $\lim_{n \rightarrow +\infty} b(x_n, 1) = \lim_{n \rightarrow +\infty} b(1, x_n) = b(1, 1)$. Therefore, in quasi- b -metric-like spaces, the limit of a convergent sequence is not necessarily unique.

Now we prove our main results.

Theorem 2.1 Let (X, b) be a 0-complete quasi- b -metric-like space with the coefficient $s \geq 1$, and let $f : X \times X \rightarrow X$ be a mapping such that

$$b(f(x), f(y)) \leq \varphi(b(x, y)) \tag{2.1}$$

for all $x, y \in X$, where $\varphi : [0, +\infty) \rightarrow [0, +\infty)$ is a continuous mapping such that $\varphi(t) = 0$ if and only if $t = 0$ and $\varphi(t) < t$ for all $t > 0$. If $\sum_{n=1}^{\infty} s^n \varphi^n(t)$ converges for all $t > 0$, where φ^n is the n th iterate of φ , then f has a unique fixed point. Moreover, for any $x_0 \in X$, the iterative sequence $\{f^n(x_0)\}$ converges to the fixed point.

Proof Let x_0 be an arbitrary point in X . From (2.1), we have

$$b(f^n(x_0), f^{n+1}(x_0)) \leq \varphi(b(f^{n-1}(x_0), f^n(x_0))) \leq \dots \leq \varphi^n(b(x_0, f(x_0))), \quad n > 1 \tag{2.2}$$

and

$$b(f^{n+1}(x_0), f^n(x_0)) \leq \varphi(b(f^n(x_0), f^{n-1}(x_0))) \leq \dots \leq \varphi^n(b(f(x_0), x_0)), \quad n > 1. \quad (2.3)$$

If $b(x_0, f(x_0)) = 0$ or $b(f(x_0), x_0) = 0$, then $x_0 = f(x_0)$, which means that x_0 is a fixed point of f . Suppose that $b(x_0, f(x_0)) > 0$ and $b(f(x_0), x_0) > 0$. Now we show that $\{f^n(x_0)\}$ is a 0-Cauchy sequence. For any integer $r \in \mathbb{Z}^+$ (the set of positive integers), the property (qb2) implies that

$$\begin{aligned} & b(f^n(x_0), f^{n+r}(x_0)) \\ & \leq s[b(f^n(x_0), f^{n+1}(x_0)) + b(f^{n+1}(x_0), f^{n+r}(x_0))] \\ & \leq sb(f^n(x_0), f^{n+1}(x_0)) + s^2[b(f^{n+1}(x_0), f^{n+2}(x_0)) + b(f^{n+2}(x_0), f^{n+r}(x_0))] \\ & \leq sb(f^n(x_0), f^{n+1}(x_0)) + s^2b(f^{n+1}(x_0), f^{n+2}(x_0)) \\ & \quad + s^3[b(f^{n+2}(x_0), f^{n+3}(x_0)) + b(f^{n+3}(x_0), f^{n+r}(x_0))] \\ & \quad \vdots \\ & \leq sb(f^n(x_0), f^{n+1}(x_0)) + s^2b(f^{n+1}(x_0), f^{n+2}(x_0)) + s^3b(f^{n+2}(x_0), f^{n+3}(x_0)) + \dots \\ & \quad + s^{r-1}b(f^{n+r-2}(x_0), f^{n+r-1}(x_0)) + s^{r-1}b(f^{n+r-1}(x_0), f^{n+r}(x_0)) \\ & \leq sb(f^n(x_0), f^{n+1}(x_0)) + s^2b(f^{n+1}(x_0), f^{n+2}(x_0)) + s^3b(f^{n+2}(x_0), f^{n+3}(x_0)) + \dots \\ & \quad + s^{r-1}b(f^{n+r-2}(x_0), f^{n+r-1}(x_0)) + s^r b(f^{n+r-1}(x_0), f^{n+r}(x_0)). \end{aligned} \quad (2.4)$$

Equations (2.2) and (2.4) yield that

$$\begin{aligned} & b(f^n(x_0), f^{n+r}(x_0)) \\ & \leq s\varphi^n(b(x_0, f(x_0))) + s^2\varphi^{n+1}(b(x_0, f(x_0))) + s^3\varphi^{n+2}(b(x_0, f(x_0))) + \dots \\ & \quad + s^{r-1}\varphi^{n+r-2}(b(x_0, f(x_0))) + s^r\varphi^{n+r-1}(b(x_0, f(x_0))) \\ & \leq s^n\varphi^n(b(x_0, f(x_0))) + s^{n+1}\varphi^{n+1}(b(x_0, f(x_0))) + s^{n+2}\varphi^{n+2}(b(x_0, f(x_0))) + \dots \\ & \quad + s^{n+r-2}\varphi^{n+r-2}(b(x_0, f(x_0))) + s^{n+r-1}\varphi^{n+r-1}(b(x_0, f(x_0))) \\ & = \sum_{k=n}^{n+r-1} s^k \varphi^k(b(x_0, f(x_0))). \end{aligned} \quad (2.5)$$

Since $\sum_{n=1}^{\infty} s^n \varphi^n(t)$ converges for all $t > 0$, then $\lim_{n \rightarrow +\infty} b(f^n(x_0), f^{n+r}(x_0)) = 0$, which means that for $m > n$,

$$\lim_{n, m \rightarrow +\infty} b(f^n(x_0), f^m(x_0)) = 0. \quad (2.6)$$

Also, applying (2.3), we proceed similarly as above and obtain $\lim_{n \rightarrow +\infty} b(f^{n+r}(x_0), f^n(x_0)) = 0$, which means that for $m > n$,

$$\lim_{n, m \rightarrow +\infty} b(f^m(x_0), f^n(x_0)) = 0. \quad (2.7)$$

From (2.6) and (2.7), we get that $\{f^n(x_0)\}$ is a 0-Cauchy sequence. Since (X, b) is 0-complete, then the sequence $\{f^n(x_0)\}$ converges to some point $z \in X$, that is,

$$\begin{aligned} \lim_{n \rightarrow +\infty} b(f^n(x_0), z) &= \lim_{n \rightarrow +\infty} b(z, f^n(x_0)) = b(z, z) = 0 = \lim_{n, m \rightarrow +\infty} b(f^n(x_0), f^m(x_0)) \\ &= \lim_{n, m \rightarrow +\infty} b(f^m(x_0), f^n(x_0)). \end{aligned} \tag{2.8}$$

We now show that z is a fixed point of f . By the triangle inequality, we have

$$\begin{aligned} b(z, fz) &\leq s[b(z, f^{n+1}(x_0)) + b(f^{n+1}(x_0), fz)] \\ &= s[b(z, f^{n+1}(x_0)) + b(f(f^n(x_0)), fz)] \\ &\leq sb(z, f^{n+1}(x_0)) + s\varphi(b(f^n(x_0), z)). \end{aligned}$$

Using (2.8) in the above inequalities, we obtain $b(z, fz) = 0$, that is, $fz = z$, hence z is a fixed point of f . Next, we show that z is the unique fixed point of f . Suppose that u is also a fixed point of f , then we claim $b(z, u) = 0$. Suppose that this is not the case, then

$$b(z, u) = b(fz, fu) \leq \varphi(b(z, u)) < b(z, u).$$

It is a contradiction, hence $b(z, u) = 0$, which implies $z = u$, therefore f has a unique fixed point. \square

In Theorem 2.1, taking $\varphi(t) = \lambda t$ with $0 \leq \lambda < \frac{1}{s}$, we can get the following corollary.

Corollary 2.1 *Let (X, b) be a 0-complete quasi- b -metric-like space with the coefficient $s \geq 1$, and let $f : X \rightarrow X$ be a mapping such that*

$$b(fx, fy) \leq \lambda b(x, y) \tag{2.9}$$

for all $x, y \in X$, where $0 \leq \lambda < \frac{1}{s}$. Then f has a unique fixed point in X . Moreover, for any $x_0 \in X$, the iterative sequence $\{f^n(x_0)\}$ converges to the fixed point.

Theorem 2.2 *Let (X, b) be a 0-complete quasi- b -metric-like space with the coefficient $s \geq 1$, and let $F : X \times X \rightarrow X$ be a mapping. If there exists $k \in [0, \frac{1}{s})$ such that*

$$b(F(x, y), F(u, v)) \leq \frac{k}{2}[b(x, u) + b(y, v)] \tag{2.10}$$

for each $(x, y), (u, v) \in X \times X$, then F has a coupled fixed point, that is, there exists (\bar{x}, \bar{y}) such that $\bar{x} = F(\bar{x}, \bar{y})$ and $\bar{y} = F(\bar{y}, \bar{x})$.

Proof Let $M = X \times X$ and define

$$\rho((x_1, y_1), (x_2, y_2)) = b(x_1, x_2) + b(y_1, y_2)$$

for $(x_1, y_1), (x_2, y_2) \in M$. It is straightforward to show that (M, ρ) is a 0-complete quasi- b -metric-like space with the coefficient s . Define $T : M \rightarrow M$ by $T(x, y) = (F(x, y), F(y, x))$. Let

$u = (u_1, u_2), v = (v_1, v_2)$. From (2.10), we have $\rho(Tu, Tv) \leq k\rho(u, v)$. Applying Corollary 2.1, we obtain that T has a unique fixed point in $X \times X$, hence there exists a unique $(\bar{x}, \bar{y}) \in X \times X$ such that $T(\bar{x}, \bar{y}) = (\bar{x}, \bar{y})$, that is, $(F(\bar{x}, \bar{y}), F(\bar{y}, \bar{x})) = (\bar{x}, \bar{y})$. Therefore, $F(\bar{x}, \bar{y}) = \bar{x}$ and $F(\bar{y}, \bar{x}) = \bar{y}$, which implies that F has a unique coupled fixed point. \square

Lemma 2.1 [22] *Let X be a nonempty set and $T : X \rightarrow X$ be a mapping. Then there exists a subset $E \subseteq X$ such that $T(E) = T(X)$ and $T : E \rightarrow X$ is one-to-one.*

The following definitions can be seen in [23–26].

Definition 2.3 Let f and g be two self-mappings on a set X . If $\omega = fx = gx$ for some x in X , then x is called a coincidence point of f and g , where ω is called a point of coincidence of f and g .

Definition 2.4 Let f and g be two self-mappings defined on a set X . Then f and g are said to be weakly compatible if they commute at every coincidence point, i.e., if $fx = gx$ for some $x \in X$, then $fgx = gfx$.

Theorem 2.3 *Let (X, b) be a quasi- b -metric-like space with the coefficient $s \geq 1$, and let f, g be self-mappings on X which satisfy the following condition:*

$$b(fx, fy) \leq \lambda b(gx, gy) \tag{2.11}$$

for all $x, y \in X$, where $0 \leq \lambda < \frac{1}{s}$. If $f(X) \subseteq g(X)$ and $g(X)$ is a 0-complete subset of X , then f and g have a unique point of coincidence in X . Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Proof By Lemma 2.1, there exists $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \rightarrow X$ is one-to-one. Now, define a mapping $h : g(E) \rightarrow g(E)$ by $h(gx) = fx$. Since g is one-to-one on E , h is well defined. Note that $b(h(gx), h(gy)) \leq \lambda b(gx, gy)$ for all $g(x), g(y) \in g(E)$, where $0 \leq \lambda < \frac{1}{s}$. Since $g(E) = g(X)$ is 0-complete, by using Corollary 2.1, there exists a unique $x_0 \in X$ such that $h(gx_0) = gx_0$, hence $fx_0 = gx_0$, which means that f and g have a unique point of coincidence in X . Let $fx_0 = gx_0 = z$, since f and g are weakly compatible, then $fz = gz$, which together with the uniqueness of the point of coincidence implies that $z = fz = gz$. Therefore, z is the unique common fixed point of f and g . \square

Now, we give an example to illustrate the validity of one of our main results.

Example 2.2 Let $X = \{0, 1, 2\}$. Define $b : X \times X \rightarrow [0, +\infty)$ as follows:

$$\begin{aligned} b(0, 0) &= 4, & b(0, 1) &= 4, & b(0, 2) &= \frac{3}{2}; \\ b(1, 0) &= 2, & b(1, 1) &= 5, & b(1, 2) &= 4; \\ b(2, 0) &= 3, & b(2, 1) &= 4, & b(2, 2) &= 0. \end{aligned}$$

Then (X, b) is a complete quasi- b -metric-like space with the coefficient $s = \frac{8}{7}$. Define the mapping $f : X \rightarrow X$ by

$$f0 = 2, \quad f1 = 0, \quad f2 = 2.$$

It is easy to prove that f satisfies all the conditions of Corollary 2.1 with $\lambda \in [\frac{4}{5}, \frac{7}{8})$. Now, by Corollary 2.1, f has a unique fixed point. In fact, 2 is the unique fixed point of f .

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the work. All authors read and approved the final manuscript.

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