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Boundedness of Toeplitz type operator associated to singular integral operator satisfying a variant of Hörmander's condition on L^p spaces with variable exponent

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Abstract

In this paper, the boundedness for some Toeplitz type operator related to some singular integral operator satisfying a variant of Hörmander's condition on L^p spaces with variable exponent is obtained by using a sharp estimate of the operator.

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Keywords: Toeplitz type operator; singular integral operator; variable L^p space; BMO

1 Introduction

As the development of singular integral operators (see [1, 2]), their commutators have been well studied (see [3, 4]). In [5, 6], some singular integral operators satisfying a variant of Hörmander's condition and the boundedness for the operators and their commutators are obtained (see [6]). In [7–9], some Toeplitz type operators related to singular integral operators and strongly singular integral operators are introduced and the boundedness for the operators generated by BMO and Lipschitz functions are obtained. In the last years, the theory of L^p spaces with variable exponent has been developed because of its connections with some questions in fluid dynamics, calculus of variations, differential equations and elasticity (see [10–13] and their references). Karlovich and Lerner study the boundedness of the commutators of singular integral operators on L^p spaces with variable exponent (see [14]). Motivated by these papers, the main purpose of this paper is to introduce some Toeplitz type operator related to some singular integral operator satisfying a variant of Hörmander's condition and prove the boundedness for the operator on L^p spaces with variable exponent by using a sharp estimate of the operator.

2 Preliminaries and results

First, let us introduce some notations. Throughout this paper, Q will denote a cube of R^n with sides parallel to the axes. For any locally integrable function f and $\delta > 0$, the sharp function of f is defined by

$$f_\delta^\#(x) = \sup_{Q \ni x} \left(\frac{1}{|Q|} \int_Q \left| f(y) - f_Q \right|^\delta dy \right)^{1/\delta},$$

where, and in what follows, $f_Q = |Q|^{-1}\int_Q f(x)\,dx$. It is well known that (see [1, 2])

$$f_\delta^\#(x) \approx \sup_{Q\ni x}\inf_{c\in C}\left(\frac{1}{|Q|}\int_Q \left|f(y)-c\right|^\delta dy\right)^{1/\delta}.$$

We write that $f_\delta^\# = f^\#$ if $\delta = 1$. We say that f belongs to $BMO(R^n)$ if $f^\#$ belongs to $L^\infty(R^n)$ and define $\|f\|_{BMO} = \|f^\#\|_{L^\infty}$. Let M be the Hardy-Littlewood maximal operator defined by

$$M(f)(x) = \sup_{Q\ni x}|Q|^{-1}\int_Q \left|f(y)\right| dy.$$

For $k \in N$, we denote by M^k the operator M iterated k times, *i.e.*, $M^1(f)(x) = M(f)(x)$ and

$$M^k(f)(x) = M\bigl(M^{k-1}(f)\bigr)(x) \quad \text{when } k \geq 2.$$

Let Ψ be a Young function and $\tilde{\Psi}$ be the complementary associated to Ψ. We denote the Ψ-average by, for a function f,

$$\|f\|_{\Psi,Q} = \inf\left\{\lambda > 0 : \frac{1}{|Q|}\int_Q \Psi\left(\frac{|f(y)|}{\lambda}\right) dy \leq 1\right\}$$

and the maximal function associated to Ψ by

$$M_\Psi(f)(x) = \sup_{Q\ni x}\|f\|_{\Psi,Q}.$$

The Young functions to be used in this paper are $\Psi(t) = t(1+\log t)^r$ and $\tilde{\Psi}(t) = \exp(t^{1/r})$, the corresponding average and maximal functions are denoted by $\|\cdot\|_{L(\log L)^r,Q}$, $M_{L(\log L)^r}$ and $\|\cdot\|_{\exp L^{1/r},Q}$, $M_{\exp L^{1/r}}$. Following [4], we know the generalized Hölder inequality,

$$\frac{1}{|Q|}\int_Q \left|f(y)g(y)\right| dy \leq \|f\|_{\Psi,Q}\|g\|_{\tilde{\Psi},Q},$$

and the following inequality: for $r, r_j \geq 1$, $j = 1,\dots,l$ with $1/r = 1/r_1 + \cdots + 1/r_l$ and any $x \in R^n$, $b \in BMO(R^n)$,

$$\|f\|_{L(\log L)^{1/r},Q} \leq M_{L(\log L)^{1/r}}(f) \leq CM_{L(\log L)^l}(f) \leq CM^{l+1}(f),$$
$$\|f - f_Q\|_{\exp L^r,Q} \leq C\|f\|_{BMO},$$
$$|f_{2^{k+1}Q} - f_{2Q}| \leq Ck\|f\|_{BMO}.$$

The non-increasing rearrangement of a measurable function f on R^n is defined by

$$f^*(t) = \inf\bigl\{\lambda > 0 : \bigl|\bigl\{x \in R^n : \left|f(x)\right| > \lambda\bigr\}\bigr| \leq t\bigr\} \quad (0 < t < \infty).$$

For $\lambda \in (0,1)$ and a measurable function f on R^n, the local sharp maximal function of f is defined by

$$M_\lambda^\#(f)(x) = \sup_{Q\ni x}\inf_{c\in C}\bigl((f-c)\chi_Q\bigr)^*\bigl(\lambda|Q|\bigr).$$

Let $p : R^n \to [1, \infty)$ be a measurable function. Denote by $L^{p(\cdot)}(R^n)$ the sets of all Lebesgue measurable functions f on R^n such that $m(\lambda f, p) < \infty$ for some $\lambda = \lambda(f) > 0$, where

$$m(f,p) = \int_{R^n} |f(x)|^{p(x)}\,dx.$$

The sets become Banach spaces with respect to the following norm:

$$\|f\|_{L^{p(\cdot)}} = \inf\{\lambda > 0 : m(f/\lambda, p) \le 1\}.$$

Denote by $M(R^n)$ the sets of all measurable functions $p : R^n \to [1, \infty)$ such that the Hardy-Littlewood maximal operator M is bounded on $L^{p(\cdot)}(R^n)$ and the following holds:

$$1 < p_- = \operatorname*{ess\,inf}_{x\in R^n} p(x), \qquad \operatorname*{ess\,sup}_{x\in R^n} p(x) = p_+ < \infty. \tag{1}$$

In recent years, the boundedness of classical operators on spaces $L^{p(\cdot)}(R^n)$ has attracted a great deal of attention (see [10–14] and their references).

Definition 1 Let $\Phi = \{\phi_1, \dots, \phi_l\}$ be a finite family of bounded functions in R^n. For any locally integrable function f, the Φ sharp maximal function of f is defined by

$$M^{\#}_{\Phi}(f)(x) = \sup_{Q\ni x} \inf_{\{c_1,\dots,c_l\}} \frac{1}{|Q|}\int_Q \left| f(y) - \sum_{i=1}^{l} c_i\phi_i(x_Q - y)\right| dy,$$

where the infimum is taken over all m-tuples $\{c_1, \dots, c_l\}$ of complex numbers and x_Q is the center of Q. For $\eta > 0$, let

$$M^{\#}_{\Phi,\eta}(f)(x) = \sup_{Q\ni x} \inf_{\{c_1,\dots,c_l\}} \left(\frac{1}{|Q|}\int_Q \left| f(y) - \sum_{i=1}^{l} c_j\phi_i(x_Q - y)\right|^{\eta} dy\right)^{1/\eta}.$$

Remark We note that $M^{\#}_{\Phi} \approx M^{\#}_{\Phi}(f)$ if $l = 1$ and $\phi_1 = 1$.

Definition 2 Given a positive and locally integrable function f in R^n, we say that f satisfies the reverse Hölder condition (write this as $f \in RH_\infty(R^n)$) if, for any cube Q centered at the origin, we have

$$0 < \sup_{x\in Q} f(x) \le C\frac{1}{|Q|}\int_Q f(y)\,dy.$$

In this paper, we study some singular integral operator as follows (see [5]).

Definition 3 Let $K \in L^2(R^n)$ and satisfy

$$\|K\|_{L^\infty} \le C,$$

$$|K(x)| \le C|x|^{-n}.$$

There exist functions $B_1, \ldots, B_l \in L^1_{\text{loc}}(R^n - \{0\})$ and $\Phi = \{\phi_1, \ldots, \phi_l\} \subset L^\infty(R^n)$ such that $|\det[\phi_j(y_i)]|^2 \in RH_\infty(R^{nl})$, and for a fixed $\delta > 0$ and any $|x| > 2|y| > 0$,

$$\left| K(x-y) - \sum_{i=1}^{l} B_i(x)\phi_i(y) \right| \le C \frac{|y|^\delta}{|x-y|^{n+\delta}}.$$

For $f \in C_0^\infty$, we define a singular integral operator related to the kernel K by

$$T(f)(x) = \int_{R^n} K(x-y)f(y)\,dy.$$

Let b be a locally integrable function on R^n and T be a singular integral operator with variable Calderón-Zygmund kernels. The Toeplitz type operator associated to T is defined by

$$T_b = \sum_{j=1}^{m} T^{j,1} M_b T^{j,2},$$

where $T^{j,1}$ are the singular integral operators T with variable Calderón-Zygmund kernels or $\pm I$ (the identity operator), $T^{j,2}$ are the linear operators for $j = 1, \ldots, m$ and $M_b(f) = bf$.

Remark Note that the classical Calderón-Zygmund singular integral operator satisfies Definition 3 (see [2, 4]).

We shall prove the following theorems.

Theorem 1 *Let T be a singular integral operator as in Definition* 3, $0 < \delta < 1$ *and* $b \in BMO(R^n)$. *If* $T_1(g) = 0$ *for any* $g \in L^u(R^n)$ $(1 < u < \infty)$, *then there exists a constant* $C > 0$ *such that for any* $f \in L_0^\infty(R^n)$ *and* $\tilde{x} \in R^n$,

$$M^{\#}_{\Phi,\delta}\big(T_b(f)\big)(\tilde{x}) \le C\|b\|_{BMO} \sum_{j=1}^{m} M^2\big(T^{j,2}(f)\big)(\tilde{x}).$$

Theorem 2 *Let T be a singular integral operator as in Definition* 3, $p(\cdot) \in M(R^n)$ *and* $b \in BMO(R^n)$. *If* $T_1(g) = 0$ *for any* $g \in L^u(R^n)$ $(1 < u < \infty)$ *and* $T^{j,2}$ *are the bounded linear operators on* $L^{p(\cdot)}(R^n)$ *for* $k = 1, \ldots, m$, *then* T_b *is bounded on* $L^{p(\cdot)}(R^n)$, *that is,*

$$\big\| T_b(f) \big\|_{L^{p(\cdot)}} \le C\|b\|_{BMO}\|f\|_{L^{p(\cdot)}}.$$

Corollary *Let* $[b, T](f) = bT(f) - T(bf)$ *be a commutator generated by the singular integral operators* T *and* b. *Then Theorems* 1 *and* 2 *hold for* $[b, T]$.

3 Proofs of theorems

To prove the theorems, we need the following lemmas.

Lemma 1 ([1, p.485]) *Let* $0 < p < q < \infty$. *We define that for any function* $f \ge 0$ *and* $1/r = 1/p - 1/q$,

$$\|f\|_{WL^q} = \sup_{\lambda > 0} \lambda \big|\{x \in R^n : f(x) > \lambda\}\big|^{1/q}, \qquad N_{p,q}(f) = \sup_E \|f\chi_E\|_{L^p} / \|\chi_E\|_{L^r},$$

where the sup *is taken for all measurable sets E with* $0 < |E| < \infty$. *Then*

$$\|f\|_{WL^q} \le N_{p,q}(f) \le \big(q/(q-p)\big)^{1/p}\|f\|_{WL^q}.$$

Lemma 2 ([4]) *Let $r_j \ge 1$ for $j = 1, \dots, l$, we denote that* $1/r = 1/r_1 + \cdots + 1/r_l$. *Then*

$$\frac{1}{|Q|}\int_Q \big|f_1(x)\cdots f_l(x)g(x)\big|\,dx \le \|f\|_{\exp L^{r_1},Q}\cdots\|f\|_{\exp L^{r_l},Q}\|g\|_{L(\log L)^{1/r},Q}.$$

Lemma 3 (see [5]) *Let T be a singular integral operator as in Definition* 3. *Then T is weak bounded of* (L^1, L^1).

Lemma 4 ([13]) *Let $p : R^n \to [1,\infty)$ be a measurable function satisfying* (1). *Then $L_0^\infty(R^n)$ is dense in $L^{p(\cdot)}(R^n)$.*

Lemma 5 ([14]) *Let $f \in L^1_{\mathrm{loc}}(R^n)$ and g be a measurable function satisfying*

$$\big|\big\{x \in R^n : |g(x)| > \alpha\big\}\big| < \infty \quad \textit{for all } \alpha > 0.$$

Then

$$\int_{R^n}\big|f(x)g(x)\big|\,dx \le C_n\int_{R^n} M^{\#}_{\lambda_n}(f)(x)M(g)(x)\,dx.$$

Lemma 6 ([5, 14]) *Let* $\delta > 0$, $0 < \lambda < 1$, $f \in L^\delta_{\mathrm{loc}}(R^n)$ *and* $\Phi = \{\phi_1, \dots, \phi_m\} \subset L^\infty(R^n)$ *such that* $|\det[\phi_j(y_i)]|^2 \in RH_\infty(R^{nm})$. *Then*

$$M^{\#}_\lambda(f)(x) \le (1/\lambda)^{1/\delta}M^{\#}_{\Phi,\delta}(f)(x).$$

Lemma 7 ([13, 14]) *Let $p : R^n \to [1,\infty)$ be a measurable function satisfying* (1). *If $f \in L^{p(\cdot)}(R^n)$ and $g \in L^{p'(\cdot)}(R^n)$ with* $p'(x) = p(x)/(p(x)-1)$, *then fg is integrable on R^n and*

$$\int_{R^n}\big|f(x)g(x)\big|\,dx \le C\|f\|_{L^{p(\cdot)}}\|g\|_{L^{p'(\cdot)}}.$$

Lemma 8 ([14]) *Let $p : R^n \to [1,\infty)$ be a measurable function satisfying* (1). *Set*

$$\|f\|'_{L^{p(\cdot)}} = \sup\left\{\int_{R^n}\big|f(x)g(x)\big|\,dx : f \in L^{p(\cdot)}(R^n), g \in L^{p'(\cdot)}(R^n)\right\}.$$

Then $\|f\|_{L^{p(\cdot)}} \le \|f\|'_{L^{p(\cdot)}} \le C\|f\|_{L^{p(\cdot)}}$.

Proof of Theorem 1 It suffices to prove for $f \in L_0^\infty(R^n)$ and some constant C_0 that the following inequality holds:

$$\left(\frac{1}{|Q|}\int_Q\big|T_b(f)(x) - C_0\big|^\delta\,dx\right)^{1/\delta} \le C\|b\|_{BMO}\sum_{j=1}^m M^2\big(T^{j,2}(f)\big)(\tilde{x}),$$

where Q is any cube centered at x_0, $C_0 = \sum_{j=1}^{m}\sum_{i=1}^{l} g_j^i \phi_i(x_0 - x)$ and $g_j^i = \int_{R^n} B_i(x_0 - y) M_{(b-b_{2Q})\chi_{(2Q)^c}} T^{j,2}(f)(y)\,dy$. Without loss of generality, we may assume that $T^{j,1}$ are T $(j = 1,\ldots,m)$. Let $\tilde{x} \in Q$. Fix a cube $Q = Q(x_0, d)$ and $\tilde{x} \in Q$. Write

$$T_b(f)(x) = T_{b-b_{2Q}}(f)(x) = T_{(b-b_{2Q})\chi_{2Q}}(f)(x) + T_{(b-b_{2Q})\chi_{(2Q)^c}}(f)(x) = f_1(x) + f_2(x).$$

Then

$$\begin{aligned}\left(\frac{1}{|Q|}\int_Q \left|T_b(f)(x) - C_0\right|^\delta dx\right)^{1/\delta} &\le C\left(\frac{1}{|Q|}\int_Q \left|f_1(x)\right|^\delta dx\right)^{1/\delta} \\ &\quad + C\left(\frac{1}{|Q|}\int_Q \left|f_2(x) - C_0\right|^\delta dx\right)^{1/\delta} = I + II.\end{aligned}$$

For I, by Lemmas 1, 2 and 3, we obtain

$$\begin{aligned}&\left(\frac{1}{|Q|}\int_Q \left|T^{j,1} M_{(b-b_{2Q})\chi_{2Q}} T^{j,2}(f)(x)\right|^\delta dx\right)^{1/\delta} \\ &\quad\le |Q|^{-1} \frac{\|T^{j,1} M_{(b-b_{2Q})\chi_{2Q}} T^{j,2}(f)\chi_Q\|_{L^\delta}}{|Q|^{1/\delta - 1}} \\ &\quad\le C|Q|^{-1} \left\|T^{j,1} M_{(b-b_{2Q})\chi_{2Q}} T^{j,2}(f)\right\|_{WL^1} \\ &\quad\le C|Q|^{-1} \left\|M_{(b-b_{2Q})\chi_{2Q}} T^{j,2}(f)\right\|_{L^1} \\ &\quad\le C|Q|^{-1} \int_{2Q} \left|b(x) - b_{2Q}\right|\left|T^{j,2}(f)(x)\right| dx \\ &\quad\le C\|b - b_{2Q}\|_{\exp L, 2Q} \left\|T^{j,2}(f)\right\|_{L(\log L), 2Q} \\ &\quad\le C\|b\|_{BMO} M^2\bigl(T^{j,2}(f)\bigr)(\tilde{x}),\end{aligned}$$

thus

$$I \le C\sum_{j=1}^{m}\left(\frac{1}{|Q|}\int_Q \left|T^{j,1} M_{(b-b_{2Q})\chi_{2Q}} T^{j,2}(f)(x)\right|^\delta dx\right)^{1/\delta} \le C\|b\|_{BMO}\sum_{j=1}^{m} M^2\bigl(T^{j,2}(f)\bigr)(\tilde{x}).$$

For II, we get, for $x \in Q$,

$$\begin{aligned}&\left|T^{j,1} M_{(b-b_{2Q})\chi_{(2Q)^c}} T^{j,2}(f)(x) - \sum_{i=1}^{l} g_j^i \phi_i(x_0 - x)\right| \\ &\quad\le \left|\int_{R^n}\left(K(x-y) - \sum_{i=1}^{l} B_i(x_0 - y)\phi_i(x_0 - x)\right)\bigl(b(y) - b_{2Q}\bigr)\chi_{(2Q)^c}(y) T^{j,2}(f)(y)\,dy\right| \\ &\quad\le \sum_{k=1}^{\infty}\int_{2^k d \le |y - x_0| < 2^{k+1} d}\left|K(x-y) - \sum_{i=1}^{l} B_i(x_0 - y)\phi_i(x_0 - x)\right|\left|b(y) - b_{2Q}\right|\left|T^{j,2}(f)(y)\right| dy \\ &\quad\le C\sum_{k=1}^{\infty}\int_{2^k d \le |y - x_0| < 2^{k+1} d}\frac{|x - x_0|^\delta}{|y - x_0|^{n+\delta}}\left|b(y) - b_{2Q}\right|\left|T^{j,2}(f)(y)\right| dy \\ &\quad\le C\sum_{k=1}^{\infty}\frac{d^\delta}{(2^k d)^{n+\delta}}\left(2^k d\right)^n \|b - b_{2Q}\|_{\exp L, 2^{k+1}Q}\left\|T^{j,2}(f)\right\|_{L(\log L), 2^{k+1}Q}\end{aligned}$$

$$\le C\|b\|_{BMO} M^2\big(T^{j,2}(f)\big)(\tilde{x}) \sum_{k=1}^{\infty} k2^{-k\delta}$$
$$\le C\|b\|_{BMO} M^2\big(T^{j,2}(f)\big)(\tilde{x}),$$

thus

$$II \le \frac{C}{|Q|}\int_Q \sum_{j=1}^{m} \left| T^{j,1} M_{(b-b_{2Q})\chi_{(2Q)^c}} T^{j,2}(f)(x) - \sum_{i=1}^{l} g_j^i \phi_i(x_0 - x) \right| dx$$
$$\le C\|b\|_{BMO} \sum_{j=1}^{m} M^2\big(T^{j,2}(f)\big)(\tilde{x}).$$

This completes the proof of Theorem 1. □

Proof of Theorem 2 By Lemmas 4-7, we get, for $f \in L_0^{\infty}(R^n)$ and $g \in L^{p'(\cdot)}(R^n)$,

$$\begin{aligned}
\int_{R^n} \big|T_b(f)(x)g(x)\big|\,dx &\le C\int_{R^n} M^{\#}_{\lambda_n}\big(T_b(f)\big)(x)M(g)(x)\,dx \\
&\le C\int_{R^n} M^{\#}_{\Phi,\delta}\big(T_b(f)\big)(x)M(g)(x)\,dx \\
&\le C\|b\|_{BMO} \sum_{j=1}^{m} \int_{R^n} M^2\big(T^{j,2}(f)\big)(x)M(g)(x)\,dx \\
&\le C\|b\|_{BMO} \sum_{j=1}^{m} \big\|M^2\big(T^{j,2}(f)\big)\big\|_{L^{p(\cdot)}} \big\|M(g)\big\|_{L^{p'(\cdot)}} \\
&\le C\|b\|_{BMO} \sum_{j=1}^{m} \big\|T^{j,2}(f)\big\|_{L^{p(\cdot)}} \big\|M(g)\big\|_{L^{p'(\cdot)}} \\
&\le C\|b\|_{BMO}\|f\|_{L^{p(\cdot)}}\|g\|_{L^{p'(\cdot)}},
\end{aligned}$$

thus, by Lemma 8,

$$\big\|T_b(f)\big\|_{L^{p(\cdot)}} \le \|b\|_{BMO}\|f\|_{L^{p(\cdot)}}.$$

This completes the proof of Theorem 2. □

Competing interests
The author declares that they have no competing interests.

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