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A generalization on weak contractions in partially ordered *b*-metric spaces and its application to quadratic integral equations

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Abstract
We introduce the notion of almost generalized (ψ,φ,L)-contractive mappings, and establish the coincidence and common fixed point results for this class of mappings in partially ordered complete *b*-metric spaces. Our results extend and improve several known results from the context of ordered metric spaces to the setting of ordered *b*-metric spaces. As an application, we prove the existence of a unique solution to a class of nonlinear quadratic integral equations.

Keywords: fixed point; common fixed point; coincidence point; integral equations; *b*-metric space; partially ordered set

1 Introduction

Fixed points theorems in partially ordered metric spaces were firstly obtained in 2004 by Ran and Reurings [1], and then by Nieto and Rodríguez-López [2]. In this direction several authors obtained further results under weak contractive conditions (see, *e.g.*, [3–8]). Berinde initiated in [9] the concept of almost contractions and obtained several interesting fixed point theorems. This has been a subject of intense study since then; see, *e.g.*, [10–20]. Some authors used related notions as 'condition (B)' (Babu *et al.* [21]) and 'almost generalized contractive condition' for two maps (Ćirić *et al.* [22]), and for four maps (Aghajani *et al.* [23]). See also a note by Pacurar [15]. On the other hand, the concept of b-metric space was introduced by Czerwik in [24]. After that, several interesting results of the existence of fixed point for single-valued and multivalued operators in b-metric spaces have been obtained (see [25–40]). Pacurar [41] proved some results on sequences of almost contractions and fixed points in b-metric spaces. Recently, Hussain and Shah [42] obtained results on KKM mappings in cone b-metric spaces. Using the concepts of partially ordered metric spaces, almost generalized contractive condition, and b-metric spaces, we define a new concept of almost generalized (ψ,φ,L)-contractive condition. In this paper, some coincidence and common fixed point theorems for mappings satisfying almost generalized (ψ,φ,L)-contractive condition in the setup of partially ordered complete b-metric spaces are proved. Consistent with [43] and [40, p.264], the following definitions and results will be needed in the sequel.

Definition 1.1 [43] Let X be a (nonempty) set and $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{R}^+$ is said to be a b-metric space iff for all $x, y, z \in X$, the following conditions are satisfied:

(i) $d(x,y) = 0$ iff $x = y$,
(ii) $d(x,y) = d(y,x)$,
(iii) $d(x,y) \leq s[d(x,z) + d(z,y)]$.

The pair (X,d) is called a b-metric space with the parameter s.

It should be noted that the class of b-metric spaces is effectively larger than that of metric spaces, since a b-metric is a metric, when $s = 1$.

The following example shows that in general a b-metric does not necessarily need to be a metric (see, also, [40]).

Example 1.1 [44] Let (X,d) be a metric space and $\rho(x,y) = (d(x,y))^p$, where $p > 1$ is a real number. Then ρ is a b-metric with $s = 2^{p-1}$. However, if (X,d) is a metric space, then (X,ρ) is not necessarily a metric space. For example, if $X = \mathbb{R}$ is the set of real numbers and $d(x,y) = |x - y|$ is the usual Euclidean metric, then $\rho(x,y) = (x - y)^s$ is a b-metric on $\mathbb{R}$ with $s = 2$, but it is not a metric on $\mathbb{R}$.

Also, the following example of a b-metric space is given in [45].

Example 1.2 [45] Let X be the set of Lebesgue measurable functions on $[0,1]$ such that $\int_0^1 |f(x)|^2\,dx < \infty$. Define $D : X \times X \to [0,\infty)$ by $D(f,g) = \int_0^1 |f(x) - g(x)|^2\,dx$. As $(\int_0^1 |f(x) - g(x)|^2\,dx)^{\frac{1}{2}}$ is a metric on X, then, from the previous example, D is a b-metric on X, with $s = 2$, where the b-metric D is defined with $D(x,y) = \|d(x,y)\|$, d is a cone metric (also see [46–49]).

Khamsi [50] also showed that each cone metric space over a normal cone has a b-metric structure.

Definition 1.2 [6] We shall say that the mapping T is g-nondecreasing if

$$gx \leq gy \quad \Longrightarrow \quad Tx \leq Ty.$$

2 Main results

Throughout the paper, let Ψ be the family of all functions $\psi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

(a) ψ is continuous,
(b) ψ is nondecreasing,
(c) $\psi(0) = 0 < \psi(t)$ for every $t > 0$.

We denote by Φ the set of all functions $\varphi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

(i) φ is right continuous,
(ii) φ is nondecreasing,
(iii) $\varphi(t) < t$ for every $t > 0$.

Let $(X, d, \leq)$ be a partially ordered b-metric space and $T : X \to X$ and $g : X \to X$ be two mappings. Set

$$M(x,y) = \max\left\{d(gx,gy), d(gx,Tx), d(gy,Ty), \frac{d(gx,Ty) + d(gy,Tx)}{2s}\right\}$$

and

$$N(x,y) = \min\bigl\{d(gx,Tx), d(gy,Ty), d(gx,Ty), d(gy,Tx)\bigr\}.$$

Now, we introduce the following definition.

Definition 2.1 Let $(X,d,\preceq)$ be a partially ordered b-metric space. We say that $T : X \to X$ is an almost generalized (ψ,φ,L)-contractive mapping with respect to $g : X \to X$ for some $\psi \in \Psi$, $\varphi \in \Phi$, and $L \geq 0$ if

$$\psi\bigl(s^3 d(Tx,Ty)\bigr) \leq \varphi\bigl(\psi\bigl(M(x,y)\bigr)\bigr) + L\psi\bigl(N(x,y)\bigr) \tag{2.1}$$

for all $x, y \in X$ with $gx \preceq gy$.

Now, we establish some results for the existence of coincidence point and common fixed point of mappings satisfying almost generalized (ψ,φ,L)-contractive condition in the setup of partially ordered b-metric spaces. The first result in this paper is the following coincidence point theorem.

Theorem 2.1 *Suppose that $(X,d,\preceq)$ is a partially ordered complete b-metric space. Let $T : X \to X$ be an almost generalized (ψ,φ,L)-contractive mapping with respect to $g : X \to X$, and T and g are continuous such that T is a monotone g-nondecreasing mapping, commutative with g and $T(X) \subseteq g(X)$. If there exists $x_0 \in X$ such that $gx_0 \preceq Tx_0$, then T and g have a coincidence point in X.*

Proof By the given assumptions, there exists $x_0 \in X$ such that $gx_0 \preceq Tx_0$. Since $T(X) \subseteq g(X)$, we can define $x_1 \in X$ such that $gx_1 = Tx_0$, then $gx_0 \preceq Tx_0 = gx_1$. Also there exists $x_2 \in X$ such that $gx_2 = Tx_1$. Since T is a monotone g-nondecreasing mapping, we have

$$gx_1 = Tx_0 \preceq Tx_1 = gx_2.$$

Continuing in this way, we construct a sequence $\{x_n\}$ in X such that for all $n = 0, 1, 2, \ldots,$

$$gx_{n+1} = Tx_n \tag{2.2}$$

for which

$$gx_0 \preceq gx_1 \preceq gx_2 \preceq \cdots \preceq gx_n \preceq gx_{n+1} \preceq \cdots. \tag{2.3}$$

If there exists $k_0 \in \mathbb{N}$ such that $gx_{k_0+1} = gx_{k_0}$, then $gx_{k_0} = Tx_{k_0}$. This means that x_{k_0} is a coincidence point of T, g, and the proof is finished. Thus, $gx_{n+1} \neq gx_n$ for all $n \in \mathbb{N}$. From (2.2) and (2.3) and the inequality (2.1) with $(x,y) = (x_n, x_{n+1})$, we have

$$\begin{aligned} \psi\bigl(d(gx_{n+1},gx_{n+2})\bigr) &\leq \psi\bigl(s^3 d(gx_{n+1},gx_{n+2})\bigr) = \psi\bigl(s^3 d(Tx_n,Tx_{n+1})\bigr) \\ &\leq \varphi\bigl(\psi\bigl(M(x_n,x_{n+1})\bigr)\bigr) + L\psi\bigl(N(x_n,x_{n+1})\bigr), \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} M(x_n, x_{n+1}) &= \max\biggl\{ d(gx_n, gx_{n+1}), d(gx_n, Tx_n), d(gx_{n+1}, Tx_{n+1}), \\ &\qquad \frac{d(gx_n, Tx_{n+1}) + d(gx_{n+1}, Tx_n)}{2s} \biggr\} \\ &= \max\biggl\{ d(gx_n, gx_{n+1}), d(gx_n, gx_{n+1}), d(gx_{n+1}, gx_{n+2}), \frac{d(gx_n, gx_{n+2})}{2s} \biggr\} \end{aligned}$$

and

$$N(x_n, x_{n+1}) = \min\bigl\{ d(gx_n, Tx_n), d(gx_{n+1}, Tx_{n+1}), d(gx_n, Tx_{n+1}), d(gx_{n+1}, Tx_n) \bigr\} = 0.$$

Since

$$\frac{d(gx_n, gx_{n+2})}{2s} \le \frac{d(gx_n, gx_{n+1}) + d(gx_{n+1}, gx_{n+2})}{2} \le \max\bigl\{ d(gx_n, gx_{n+1}), d(gx_{n+1}, gx_{n+2}) \bigr\},$$

then we get

$$\begin{aligned} &M(x_n, x_{n+1}) = \max\bigl\{ d(gx_n, gx_{n+1}), d(gx_{n+1}, gx_{n+2}) \bigr\}, \\ &N(x_n, x_{n+1}) = 0. \end{aligned} \tag{2.5}$$

By (2.4) and (2.5), we have

$$\psi\bigl(d(gx_{n+1}, gx_{n+2})\bigr) \le \varphi\bigl(\psi\bigl(\max\bigl\{ d(gx_n, gx_{n+1}), d(gx_{n+1}, gx_{n+2}) \bigr\}\bigr)\bigr). \tag{2.6}$$

Suppose that $\max\{d(gx_n, gx_{n+1}), d(gx_{n+1}, gx_{n+2})\} = d(gx_{n+1}, gx_{n+2}) > 0$ for some $n \in \mathbb{N}$, then by (2.6)

$$\psi\bigl(d(gx_{n+1}, gx_{n+2})\bigr) \le \varphi\bigl(\psi\bigl(d(gx_{n+1}, gx_{n+2})\bigr)\bigr) < \psi\bigl(d(gx_{n+1}, gx_{n+2})\bigr);$$

a contradiction. Hence,

$$\max\bigl\{ d(gx_n, gx_{n+1}), d(gx_{n+1}, gx_{n+2}) \bigr\} = d(gx_n, gx_{n+1})$$

and thus

$$\psi\bigl(d(gx_{n+1}, gx_{n+2})\bigr) \le \varphi\bigl(\psi\bigl(d(gx_n, gx_{n+1})\bigr)\bigr) < \psi\bigl(d(gx_n, gx_{n+1})\bigr).$$

Thus, we get

$$\psi\bigl(d(gx_{n+1}, gx_{n+2})\bigr) < \psi\bigl(d(gx_n, gx_{n+1})\bigr)$$

for all $n \in \mathbb{N}$. Now, from

$$\begin{aligned} \psi\bigl(d(gx_n, gx_{n+1})\bigr) &\le \varphi\bigl(\psi\bigl(d(gx_{n-1}, gx_n)\bigr)\bigr) \le \varphi^2\bigl(\psi\bigl(d(gx_{n-2}, gx_{n-1})\bigr)\bigr) \\ &\le \cdots \le \varphi^n\bigl(\psi\bigl(d(gx_0, gx_1)\bigr)\bigr) \end{aligned}$$

and the property of φ, we obtain $\lim_{n\to\infty}\psi(d(gx_n,gx_{n+1}))=0$, and consequently

$$\lim_{n\to\infty} d(gx_n,gx_{n+1})=0. \tag{2.7}$$

Now, we shall prove that $\{gx_n\}$ is a Cauchy sequence in (X,d). Suppose, on the contrary, that $\{gx_n\}$ is not a Cauchy sequence. Then there exist $\epsilon>0$ and subsequences $\{gx_{m(k)}\}$, $\{gx_{n(k)}\}$ of $\{gx_n\}$ with $m(k)>n(k)\geq k$ such that

$$d(gx_{n(k)},gx_{m(k)})\geq\epsilon. \tag{2.8}$$

Additionally, corresponding to $n(k)$, we may choose $m(k)$ such that it is the smallest integer satisfying (2.8) and $m(k)>n(k)\geq k$. Thus,

$$d(gx_{n(k)},gx_{m(k)-1})<\epsilon. \tag{2.9}$$

Using the triangle inequality in b-metric space and (2.8) and (2.9) we obtain

$$\begin{aligned}\epsilon&\leq d(gx_{m(k)},gx_{n(k)})\leq sd(gx_{m(k)},gx_{m(k)-1})+sd(gx_{m(k)-1},gx_{n(k)})\\&<sd(gx_{m(k)},gx_{m(k)-1})+s\epsilon.\end{aligned}$$

Taking the upper limit as $k\to\infty$ and using (2.7) we obtain

$$\epsilon\leq\limsup_{k\to\infty} d(gx_{n(k)},gx_{m(k)})\leq s\epsilon. \tag{2.10}$$

Also

$$\begin{aligned}\epsilon&\leq d(gx_{n(k)},gx_{m(k)})\leq sd(gx_{n(k)},gx_{m(k)+1})+sd(gx_{m(k)+1},gx_{m(k)})\\&\leq s^2d(gx_{n(k)},gx_{m(k)})+s^2d(gx_{m(k)},gx_{m(k)+1})+sd(gx_{m(k)+1},gx_{m(k)})\\&\leq s^2d(gx_{n(k)},gx_{m(k)})+\left(s^2+s\right)d(gx_{m(k)},gx_{m(k)+1}).\end{aligned}$$

So from (2.7) and (2.10), we have

$$\frac{\epsilon}{s}\leq\limsup_{k\to\infty} d(gx_{n(k)},gx_{m(k)+1})\leq s^2\epsilon. \tag{2.11}$$

Also

$$\begin{aligned}\epsilon&\leq d(gx_{m(k)},gx_{n(k)})\leq sd(gx_{m(k)},gx_{n(k)+1})+sd(gx_{n(k)+1},gx_{n(k)})\\&\leq s^2d(gx_{m(k)},gx_{n(k)})+s^2d(gx_{n(k)},gx_{n(k)+1})+sd(gx_{n(k)+1},gx_{n(k)})\\&\leq s^2d(gx_{m(k)},gx_{n(k)})+\left(s^2+s\right)d(gx_{n(k)},gx_{n(k)+1}).\end{aligned}$$

So from (2.7) and (2.10), we have

$$\frac{\epsilon}{s}\leq\limsup_{k\to\infty} d(gx_{m(k)},gx_{n(k)+1})\leq s^2\epsilon. \tag{2.12}$$

Also

$$d(gx_{n(k)+1}, gx_{m(k)}) \leq sd(gx_{n(k)+1}, gx_{m(k)+1}) + sd(gx_{m(k)+1}, gx_{m(k)}),$$

so from (2.7) and (2.12), we have

$$\frac{\epsilon}{s^2} \leq \limsup_{k\to\infty} d(gx_{n(k)+1}, gx_{m(k)+1}). \tag{2.13}$$

Linking (2.7), (2.10), (2.11) together with (2.12) we get

$$\begin{aligned}
&\limsup_{k\to\infty} M(x_{n(k)}, x_{m(k)}) \\
&\quad = \max\biggl\{\limsup_{k\to\infty} d(gx_{n(k)}, gx_{m(k)}), \limsup_{k\to\infty} d(gx_{n(k)}, gx_{n(k)+1}), \limsup_{k\to\infty} d(gx_{m(k)}, gx_{m(k)+1}), \\
&\qquad \frac{\limsup_{k\to\infty} d(gx_{n(k)}, gx_{m(k)+1}) + \limsup_{k\to\infty} d(gx_{m(k)}, gx_{n(k)+1})}{2s}\biggr\} \\
&\quad \leq \max\left\{s\epsilon, 0, 0, \frac{s^2\epsilon + s^2\epsilon}{2s}\right\} = s\epsilon.
\end{aligned}$$

So,

$$\limsup_{k\to\infty} M(x_{n(k)}, x_{m(k)}) \leq \epsilon s. \tag{2.14}$$

Similarly, we have

$$\limsup_{k\to\infty} N(x_{n(k)}, x_{m(k)}) = 0. \tag{2.15}$$

Since $m(k) > n(k)$ from (2.2), we have

$$gx_{n(k)} \preceq gx_{m(k)}.$$

Thus,

$$\begin{aligned}
\psi\left(s^3 d(gx_{n(k)+1}, gx_{m(k)+1})\right) &= \psi\left(s^3 d(Tx_{n(k)}, Tx_{m(k)})\right) \\
&\leq \varphi\left(\psi\left(M(x_{n(k)}, x_{m(k)})\right)\right) + L\psi\left(N(x_{n(k)}, x_{m(k)})\right).
\end{aligned}$$

Passing to the upper limit as $k \to \infty$, and using (2.13), (2.14), and (2.15), we get

$$\begin{aligned}
\psi(s\epsilon) &\leq \psi\left(s^3 \limsup_{k\to\infty} d(gx_{n(k)+1}, gx_{m(k)+1})\right) = \limsup_{k\to\infty} \psi\left(s^3 d(gx_{n(k)+1}, gx_{m(k)+1})\right) \\
&= \limsup_{k\to\infty} \psi\left(s^3 d(Tx_{n(k)}, Tx_{m(k)})\right) \\
&\leq \limsup_{k\to\infty} \varphi\left(\psi\left(M(x_{n(k)}, x_{m(k)})\right)\right) + \limsup_{k\to\infty} L\psi\left(N(x_{n(k)}, x_{m(k)})\right) \\
&= \varphi\left(\psi\left(\limsup_{k\to\infty} M(x_{n(k)}, x_{m(k)})\right)\right) + L\psi\left(\limsup_{k\to\infty} N(x_{n(k)}, x_{m(k)})\right) \\
&\leq \varphi\left(\psi(\epsilon s)\right) < \psi(s\epsilon),
\end{aligned}$$

which is a contradiction. Thus, we proved that $\{gx_n\}$ is a Cauchy sequence in (X,d). Since X is a complete b-metric space, there exists $x \in X$ such that

$$\lim_{n\to\infty} gx_{n+1} = x. \tag{2.16}$$

From the commutativity of T and g, we have

$$g(gx_{n+1}) = g\bigl(T(x_n)\bigr) = T(gx_n). \tag{2.17}$$

Letting $n \to \infty$ in (2.17) and from the continuity of T and g, we get

$$gx = \lim_{n\to\infty} g(gx_{n+1}) = \lim_{n\to\infty} T(gx_n) = T\Bigl(\lim_{n\to\infty} gx_n\Bigr) = T(x).$$

This implies that x is a coincidence point of T and g. This completes the proof. □

Now, we will prove the following result.

Theorem 2.2 *Suppose that $(X,d,\leq)$ is a partially ordered complete b-metric space. Let $T: X \to X$ be an almost generalized (ψ,φ,L)-contractive mapping with respect to $g: X \to X$, T is a g-nondecreasing mapping and $T(X) \subseteq g(X)$. Also suppose*

$$\begin{aligned} &\textit{if } \{gx_n\} \subset X \textit{ is a nondecreasing sequence with } gx_n \to gz \textit{ in } gX,\\ &\textit{then } gx_n \leq gz, gz \leq g(gz)\ \forall n \textit{ hold.} \end{aligned} \tag{2.18}$$

Also suppose gX is closed. If there exists $x_0 \in X$ such that $gx_0 \leq Tx_0$, then T and g have a coincidence. Further, if T and g commute at their coincidence points, then T and g have a common fixed point.

Proof As in the proof of Theorem 2.1, we can show that $\{gx_n\}$ is a Cauchy sequence. Since gX is a closed, there exists $x \in X$ such that

$$\lim_{n\to\infty} gx_{n+1} = gx. \tag{2.19}$$

Now we show that x is a coincidence point of T and g. Since from (2.18) and (2.19) we have $gx_n \leq gx$ for all n, then by the triangle inequality in a b-metric space and (2.1), we get

$$\begin{aligned} d(gx,Tx) &\leq sd(gx,gx_{n+1}) + sd(gx_{n+1},Tx) = sd(gx,gx_{n+1}) + sd(Tx_n,Tx),\\ \psi\bigl(d(gx,Tx)\bigr) &\leq \lim_{n\to\infty} \psi\bigl(sd(Tx_n,Tx)\bigr) \leq \lim_{n\to\infty} \psi\bigl(s^3 d(Tx_n,Tx)\bigr)\\ &\leq \lim_{n\to\infty} \bigl[\varphi\bigl(\psi\bigl(M(x_n,x)\bigr)\bigr) + L\psi\bigl(N(x_n,x)\bigr)\bigr]\\ &\leq \varphi\bigl(\psi\bigl(d(gx,Tx)\bigr)\bigr) < \psi\bigl(d(gx,Tx)\bigr). \end{aligned}$$

Indeed,

$$\begin{aligned} \lim_{n\to\infty} M(x_n,x) &= \lim_{n\to\infty} \max\left\{d(gx_n,gx), d(gx_n,Tx_n), d(gx,Tx), \frac{d(gx_n,Tx) + d(gx,Tx_n)}{2s}\right\}\\ &= d(gx,Tx) \end{aligned}$$

and

$$\lim_{n\to\infty} N(x_n,x) = \lim_{n\to\infty} \min\{d(gx_n,Tx_n),d(gx,Tx),d(gx_n,Tx),d(gx,Tx_n)\} = 0.$$

Hence $d(gx,Tx) = 0$, that is, $Tx = gx$. Thus we proved that T and g have a coincidence. Suppose now that T and g commute at x. Set $y = Tx = gx$. Then

$$Ty = T(gx) = g(Tx) = gy.$$

Since from (2.18) we have $gx \leq g(gx) = gy$ and as $gx = Tx$ and $gy = Ty$, from (2.1) we obtain

$$\begin{aligned}\psi\bigl(d(Tx,Ty)\bigr) &\leq \psi\bigl(s^3 d(Tx,Ty)\bigr) \leq \varphi\bigl(\psi\bigl(M(x,y)\bigr)\bigr) + L\psi\bigl(N(x,y)\bigr)\\ &= \varphi\left(\psi\left(\max\left\{d(gx,gy),d(gx,Tx),d(gy,Ty),\frac{d(gx,Ty)+d(gy,Tx)}{2s}\right\}\right)\right)\\ &\quad + L\psi\bigl(\min\{d(gx,Tx),d(gy,Ty),d(gx,Ty),d(gy,Tx)\}\bigr)\\ &= \varphi\bigl(\psi\bigl(d(Tx,Ty)\bigr)\bigr) < \psi\bigl(d(Tx,Ty)\bigr).\end{aligned}$$

Hence $d(Tx,Ty) = 0$, that is, $y = Tx = Ty$. Therefore, $Ty = gy = y$. Thus we proved that T and g have a common fixed point. □

In the following, we deduce some fixed point theorems from our main results given by Theorems 2.1 and 2.2.

Corollary 2.3 *Let $(X,d,\leq)$ be a partially ordered complete b-metric space and $T : X \to X$ is a nondecreasing mapping. Suppose there exist $\psi \in \Psi$, $\varphi \in \Phi$, and $L \geq 0$ such that*

$$\psi\bigl(s^3 d(Tx,Ty)\bigr) \leq \varphi\bigl(\psi\bigl(M(x,y)\bigr)\bigr) + L\psi\bigl(N(x,y)\bigr),$$

where

$$M(x,y) = \max\left\{d(x,y),d(x,Tx),d(y,Ty),\frac{d(x,Ty)+d(y,Tx)}{2s}\right\}$$

and

$$N(x,y) = \min\{d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)\}$$

for all $x,y \in X$ with $x \leq y$. Also suppose either

(a) *if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \leq z$, for all n, holds, or*

(b) *T is continuous.*

If there exists $x_0 \in X$ such that $x_0 \leq Tx_0$, then T has a fixed point in X.

Example 2.1 Let X be the set of Lebesgue measurable functions on $[0,1]$ such that $\int_0^1 |x(t)|\,dt < \infty$. Define $D : X \times X \to [0,\infty)$ by

$$D(x,y) = \left(\int_0^1 \bigl|x(t)-y(t)\bigr|\,dt\right)^2.$$

Then D is a b-metric on X, with $s = 2$. Also, this space can also be equipped with a partial order given by

$$x, y \in X, \quad x \preceq y \quad \Longleftrightarrow \quad x(t) \leq y(t) \quad \text{for any } t \in [a, b].$$

The operator $T : X \to X$ defined by

$$Tx(t) = t^n + e^t + \frac{\sqrt{2}}{4} \ln\left(\left|x(t)\right| + 1\right). \tag{2.20}$$

Now, we prove that T has a fixed point. For all $x, y \in X$ with $x \preceq y$, we have

$$\begin{aligned}
\sqrt{2^3 D(Tx, Ty)} &= \sqrt{2^3 \left(\int_0^1 \left|Tx(t) - Ty(t)\right| dt\right)^2} \\
&\leq 2\sqrt{2} \int_0^1 \left| \frac{\sqrt{2}}{4} \ln\left(\left|x(t)\right| + 1\right) - \frac{\sqrt{2}}{4} \ln\left(\left|y(t)\right| + 1\right) \right| dt \\
&\leq \int_0^1 \left|\left(\ln\left(\left|x(t)\right| + 1\right) - \ln\left(\left|y(t)\right| + 1\right)\right)\right| dt \\
&\leq \int_0^1 \ln\left(\frac{|x(t)| + 1}{|y(t)| + 1}\right) dt \\
&\leq \int_0^1 \ln\left(1 + \frac{|x(t) - y(t)|}{|y(t)| + 1}\right) dt \\
&\leq \ln\left(1 + \int_0^1 \left|x(t) - y(t)\right| dt\right) \\
&\leq \ln\left(1 + \sqrt{\left(\int_0^1 \left|x(t) - y(t)\right| dt\right)^2}\right) \\
&\leq \ln\left(1 + \sqrt{D(x, y)}\right).
\end{aligned}$$

Now, if we define $\varphi(t) = \ln(1 + t)$, $\psi(t) = \sqrt{t}$, and $x_0 = 0$. Thus, by Corollary 2.3 we see that T has a fixed point.

Remark 2.1 Corollary 2.3 extends and generalizes many existing fixed point theorems in the literature [2, 3, 51, 52].

The following result is the immediate consequence of Corollary 2.3.

Corollary 2.4 *Let $(X, d, \preceq)$ be a partially ordered complete b-metric space and $T : X \to X$ is a nondecreasing mapping. Suppose there exists $\varphi \in \Phi$ such that*

$$s^3 d(Tx, Ty) \leq \varphi\left(\max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2s}\right\}\right) \tag{2.21}$$

for all $x, y \in X$ with $x \preceq y$. Also suppose either

(a) *if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \preceq z$, for all n, holds, or*

(b) *T is continuous.*

If there exists $x_0 \in X$ such that $x_0 \le Tx_0$, then T has a fixed point in X.

Remark 2.2 Corollary 2.4 is a generalization to [3, Theorem 1.3].

Taking $\varphi(t) = \lambda t$, $0 < \lambda < 1$, in Corollary 2.4 we obtain the following generalization of the results in [1, 53].

Corollary 2.5 *Let $(X, d, \le)$ be a partially ordered complete b-metric space and $T : X \to X$ is a nondecreasing mapping. Suppose there exists $\varphi \in \Phi$ such that*

$$s^3 d(Tx, Ty) \le \lambda \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2s}\right\}$$

for all $x, y \in X$ with $x \le y$. Also suppose either

(a) *if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \le z$, for all n, holds, or*

(b) *T is continuous.*

If there exists $x_0 \in X$ such that $x_0 \le Tx_0$, then T has a fixed point in X.

Corollary 2.6 *Let $(X, d, \le)$ be a partially ordered complete b-metric space and $T : X \to X$ is a nondecreasing mapping. Suppose there exist $\psi \in \Psi$ and $0 \le \lambda < 1$ such that*

$$\psi\left(s^3 d(Tx, Ty)\right) \le \lambda \psi\left(d(x,y)\right)$$

for all $x, y \in X$ with $x \le y$. Also suppose either

(a) *if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \le z$, for all n, holds, or*

(b) *T is continuous.*

If there exists $x_0 \in X$ such that $x_0 \le Tx_0$, then T has a fixed point in X.

3 Application to integral equations

Here, in this section, we wish to study the existence of a unique solution to a nonlinear quadratic integral equation, as an application to the our fixed point theorem. Consider the integral equation

$$x(t) = h(t) + \lambda \int_0^1 k(t,s) f\left(s, x(s)\right) ds, \quad t \in I = [0,1], \lambda \ge 0. \tag{3.1}$$

Let Γ denote the class of those functions $\gamma : [0, +\infty) \to [0, +\infty)$ for which $\gamma \in \Phi$ and $(\gamma(t))^p \le \gamma(t^p)$, for all $p \ge 1$.

For example, $\gamma_1(t) = kt$, where $0 \le k < 1$ and $\gamma_2(t) = \frac{t}{t+1}$ are in Γ.

We will analyze (3.1) under the following assumptions:

(a_1) $f : I \times \mathbb{R} \to \mathbb{R}$ is continuous monotone nondecreasing in x, $f(t,x) \ge 0$ and there exist constant $0 \le L < 1$ and $\gamma \in \Gamma$ such that for all $x, y \in \mathbb{R}$ and $x \ge y$

$$\left|f(t,x) - f(t,y)\right| \le L\gamma(x - y).$$

(a_2) $h : I \to \mathbb{R}$ is a continuous function.

(a_3) $k : I \times I \to \mathbb{R}$ is continuous in $t \in I$ for every $s \in I$ and measurable in $s \in I$ for all $t \in I$ such that

$$\int_0^1 k(t,s)\,ds \le K$$

and $k(t,s) \ge 0$.

(a_4) There exists $\alpha \in C(I)$ such that

$$\alpha(t) \le h(t) + \lambda \int_0^1 k(t,s) f\bigl(s, \alpha(s)\bigr)\,ds.$$

(a_5) $L^p \lambda^p K^p \le \frac{1}{2^{3p-3}}$.

We consider the space $X = C(I)$ of continuous functions defined on $I = [0,1]$ with the standard metric given by

$$\rho(x,y) = \sup_{t \in I} \bigl|x(t) - y(t)\bigr| \quad \text{for } x, y \in C(I).$$

This space can also be equipped with a partial order given by

$$x, y \in C(I), \quad x \le y \quad \Longleftrightarrow \quad x(t) \le y(t) \quad \text{for any } t \in I.$$

Now for $p \ge 1$, we define

$$d(x,y) = \bigl(\rho(x,y)\bigr)^p = \Bigl(\sup_{t \in I} \bigl|x(t) - y(t)\bigr|\Bigr)^p = \sup_{t \in I} \bigl|x(t) - y(t)\bigr|^p \quad \text{for } x, y \in C(I).$$

It is easy to see that (X, d) is a complete b-metric space with $s = 2^{p-1}$ [44].

For any $x, y \in X$ and each $t \in I$, $\max\{x(t), y(t)\}$ and $\min\{x(t), y(t)\}$ belong to X and are upper and lower bounds of x, y, respectively. Therefore, for every $x, y \in X$, one can take $\max\{x,y\}, \min\{x,y\} \in X$ which are comparable to x, y. Now, we formulate the main result of this section.

Theorem 3.1 *Under assumptions* (a_1)-(a_5), (3.1) *has a unique solution in* $C(I)$.

Proof We consider the operator $T : X \to X$ defined by

$$T(x)(t) = h(t) + \lambda \int_0^1 k(t,s) f\bigl(s, x(s)\bigr)\,ds \quad \text{for } t \in I.$$

By virtue of our assumptions, T is well defined (this means that if $x \in X$ then $T(x) \in X$). For $x \le y$, and $t \in I$ we have

$$\begin{aligned} T(x)(t) - T(y)(t) &= h(t) + \lambda \int_0^1 k(t,s) f\bigl(s, x(s)\bigr)\,ds - h(t) - \lambda \int_0^1 k(t,s) f\bigl(s, y(s)\bigr)\,ds \\ &= \lambda \int_0^1 k(t,s)\bigl[f\bigl(s, x(s)\bigr) - f\bigl(s, y(s)\bigr)\bigr]\,ds \le 0. \end{aligned}$$

Therefore, T has the monotone nondecreasing property. Also, for $x \leq y$, we have

$$\begin{aligned}\left|T(x)(t) - T(y)(t)\right| &= \left|h(t) + \lambda \int_0^1 k(t,s)f\bigl(s,x(s)\bigr)\,ds - h(t) - \lambda \int_0^1 k(t,s)f\bigl(s,y(s)\bigr)\,ds\right| \\ &\leq \lambda \int_0^1 k(t,s)\bigl|f\bigl(s,x(s)\bigr) - f\bigl(s,y(s)\bigr)\bigr|\,ds \\ &\leq \lambda \int_0^1 k(t,s)L\gamma\bigl(y(s) - x(s)\bigr)\,ds.\end{aligned}$$

Since the function γ is nondecreasing and $x \leq y$, we have

$$\gamma\bigl(y(s) - x(s)\bigr) \leq \gamma\Bigl(\sup_{t\in I}\bigl|x(s) - y(s)\bigr|\Bigr) = \gamma\bigl(\rho(x,y)\bigr),$$

hence

$$\left|T(x)(t) - T(y)(t)\right| \leq \lambda \int_0^1 k(t,s)L\gamma\bigl(\rho(x,y)\bigr)\,ds \leq \lambda KL\gamma\bigl(\rho(x,y)\bigr).$$

Then we obtain

$$\begin{aligned}d\bigl(T(x), T(y)\bigr) &= \sup_{t\in I}\bigl|T(x)(t) - T(y)(t)\bigr|^p \\ &\leq \bigl\{\lambda KL\gamma\bigl(\rho(x,y)\bigr)\bigr\}^p = \lambda^p K^p L^p \gamma\bigl(\rho(x,y)\bigr)^p \\ &\leq \lambda^p K^p L^p \gamma\bigl(\rho(x,y)^p\bigr) = \lambda^p K^p L^p \gamma\bigl(d(x,y)\bigr) \\ &\leq \lambda^p K^p L^p \varphi\left(\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2s}\right\}\right) \\ &\leq \frac{1}{2^{3p-3}}\varphi\left(\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2s}\right\}\right).\end{aligned}$$

This proves that the operator T satisfies the contractive condition (2.21) appearing in Corollary 2.4. Also, let α, β be the functions appearing in assumption (a_4); then, by (a_4), we get $\alpha \leq T(\alpha)$. So, (3.1) has a solution and the proof is complete. □

Example 3.1 Consider the following functional integral equation:

$$x(t) = \frac{t^2}{1+t^4} + \frac{1}{27}\int_0^1 \frac{e^{-s}\sin t}{2(1+t)}\frac{|x(s)|}{1+|x(s)|}\,ds \tag{3.2}$$

for $t \in [0,1]$. Observe that this equation is a special case of (3.1) with

$$\begin{aligned}h(t) &= \frac{t^2}{1+t^4}, \\ k(t,s) &= \frac{e^{-s}}{1+t}, \\ f(t,x) &= \frac{\sin t}{2}\frac{|x|}{1+|x|}.\end{aligned}$$

Indeed, by using $\gamma(t) = \frac{1}{3}t$ we see that $\gamma \in \Phi$ and $(\gamma(t))^p = (\frac{1}{3}t)^p = \frac{1}{3^p}t^p \leq \frac{1}{3}t^p = \gamma(t^p)$, for all $p \geq 1$. Further, for arbitrarily fixed $x, y \in \mathbb{R}$ such that $x \geq y$ and for $t \in [0,1]$ we obtain

$$\begin{aligned}\bigl|f(t,x) - f(t,y)\bigr| &= \left|\frac{\sin t}{2}\frac{|x|}{1+|x|} - \frac{\sin t}{2}\frac{|y|}{1+|y|}\right| \\ &\leq \frac{1}{2}|x-y| = \frac{1}{6}\gamma(x-y).\end{aligned}$$

Thus, the function f satisfies assumption (a_1) with $L = \frac{1}{6}$. It is also easily seen that h is a continuous function. Further, notice that the function k is continuous in $t \in I$ for every $s \in I$ and measurable in $s \in I$ for all $t \in I$ and $k(t,s) \geq 0$. Moreover, we have

$$\begin{aligned}\int_0^1 k(t,s)\,ds &= \int_0^1 \frac{e^{-s}}{1+t}\,ds = \frac{1-e^{-1}}{1+t} \\ &\leq 1 - e^{-1} \leq \frac{2}{3} = K.\end{aligned}$$

If we put $\alpha(t) = \frac{3t^2}{4(1+t^4)}$, we have

$$\begin{aligned}\alpha(t) &= \frac{3t^2}{4(1+t^4)} \leq \frac{t^2}{1+t^4} \\ &\leq \frac{t^2}{1+t^4} + \frac{1}{27}\int_0^1 \frac{e^{-s}\sin t}{2(1+t)}\frac{|\alpha(s)|}{1+|\alpha(s)|}\,ds \\ &= h(t) + \lambda\int_0^1 k(t,s)f\bigl(s,\alpha(s)\bigr)\,ds.\end{aligned}$$

This shows that assumption (a_4) holds. Taking $L = \frac{1}{6}$, $K = \frac{2}{3}$ and $\lambda = \frac{1}{27}$, then inequality $L^p\lambda^pK^p \leq \frac{1}{2^{3p-3}}$ appearing in assumption (a_5) has the following form:

$$\frac{1}{6^p} \times \frac{1}{27^p} \times \frac{2^p}{3^p} \leq \frac{1}{2^{3p-3}}.$$

It is easily seen that each number $p \geq 1$ satisfies the above inequality. Consequently, all the conditions of Theorem 3.1 are satisfied. Hence the integral equation (3.2) has a unique solution in $C(I)$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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