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Convergence of the q-Stancu-Szász-Beta type operators

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Abstract

In this paper, we study on q-Stancu-Szász-Beta type operators. We give these operators convergence properties and obtain a weighted approximation theorem in the interval $[0,\infty)$.

MSC: 41A25; 41A36

Keywords: q-Stancu type operators; Szász-Beta type operators; weighted approximation

1 Introduction

In [1], Mahmudov constructed q-Szász operators and obtained rate of global convergence in the frame of weighted spaces and a Voronovskaja type theorem for these operators. In [2], Gupta and Mahmudov studied on the q-analog of the Szász-Beta type operators. In [3], Yüksel and Dinlemez gave a Voronovskaja type theorem for q-analog of a certain family Szász-Beta type operators. In [4], Govil and Gupta introduced the q-analog of certain Beta-Szász-Stancu operators. They estimated the moments and established direct results in terms of modulus of continuity and an asymptotic formula for the q-operators. In [5–14], interesting generalization about q-calculus were given. Our aims are to give approximation properties and a weighted approximation theorem for q-Stancu-Szász-Beta type operators. We use without further explanation the basic notations and formulas, from the theory of q-calculus as set out in [15–19]. Let $A>0$ and f be a real valued continuous function defined on the interval $[0,\infty)$. For $0<q\le 1$, q-Stancu-Szász-Beta type operators are defined as

$$B_{n,q}^{(\alpha,\beta)}(f,x)=\sum_{k=0}^{\infty}s_{n,k}^{q}(x)\int_{0}^{\infty/A}b_{n,k}^{q}(t)f\left(\frac{[n]_q t+\alpha}{[n]_q+\beta}\right)d_q t, \tag{1.1}$$

where

$$s_{n,k}^{q}(x)=\left([n]_q x\right)^k\frac{e^{-[n]_q x}}{[k]_q!}$$

and

$$b_{n,k}^{q}(x)=\frac{q^{k^2}x^k}{B_q(k+1,n)(1+x)_q^{n+k+1}}.$$

If we write $q = 1$ and $\alpha = \beta = 0$ in (1.1), then the operators $B_{n,q}^{(\alpha,\beta)}(f,x)$ are reduced to Szász-Beta type operators studied in [20–23].

2 Auxiliary results

For the sake of brevity, the notation $F_s^q(n) = \prod_{i=1}^{s}[n-i]_q$ and $G_\beta^q(n) = ([n]_q+\beta)$ will be used throughout the article. Now we are ready to give the following lemma for the Korovkin test functions.

Lemma 1 *Let $e_m(t) = t^m$, $m = 0,1,2$, we get*

$$\text{(i)}\quad B_{n,q}^{(\alpha,\beta)}(e_0,x) = 1,$$

$$\text{(ii)}\quad B_{n,q}^{(\alpha,\beta)}(e_1,x) = \frac{[n]_q^2 x}{q^2 G_\beta^q(n)F_1^q(n)} + \frac{[n]_q}{qG_\beta^q(n)F_1^q(n)} + \frac{\alpha}{G_\beta^q(n)},$$

$$\begin{aligned}\text{(iii)}\quad B_{n,q}^{(\alpha,\beta)}(e_2,x) = {}& \frac{[n]_q^4 x^2}{q^6 G_\beta^q(n)^2 F_2^q(n)} + \left\{\frac{[n]_q^3}{q^5 G_\beta^q(n)^2 F_2^q(n)}\right.\\ &\left. + \frac{(1+[2]_q)[n]_q^3}{q^4 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha[n]_q^2}{q^2 G_\beta^q(n)^2 F_1^q(n)}\right\}x\\ &+ \frac{[2]_q[n]_q^2}{q^3 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha[n]_q}{qG_\beta^q(n)^2F_1^q(n)} + \frac{\alpha^2}{G_\beta^q(n)^2}.\end{aligned}$$

Proof Using the q-Gamma and q-Beta functions in [15, 24], we obtain the following equality:

$$\begin{aligned}&q^{k^2}\int_0^{\infty/A}\frac{1}{B(k+1,n)}\frac{t^{k+m}}{(1+t)_q^{n+k+1}}\,d_qt\\ &\quad = \frac{[m+k]_q![n-m-1]_q!q^{\{2k^2-(k+m)(k+m+1)\}/2}}{[k]_q![n-1]_q!}.\end{aligned}\tag{2.1}$$

Then, using (2.1), for $m = 0$, we get

$$\begin{aligned}B_{n,q}^{(\alpha,\beta)}(e_0,x) &= e^{-[n]_qx}\sum_{k=0}^{\infty}\frac{([n]_qx)^k}{[k]_q!}q^{k(k-1)/2}\\ &= e^{-[n]_qx}E_q^{[n]_qx} = 1,\end{aligned}$$

and the proof of (i) is finished. With a direct computation, we obtain (ii) as follows:

$$\begin{aligned}B_{n,q}^{(\alpha,\beta)}(e_1,x) = {}&\frac{[n]_q}{G_\beta^q(n)F_1^q(n)}\sum_{k=1}^{\infty}\frac{([n]_qx)^k}{[k-1]_q!}q^{k(k-3)-2/2}e^{-[n]_qx}\\ &+ \frac{[n]_q}{G_\beta^q(n)F_1^q(n)}\sum_{k=0}^{\infty}\frac{([n]_qx)^k}{[k]_q!}q^{k(k-1)-2/2}e^{-[n]_qx}\\ &+ \frac{\alpha}{G_\beta^q(n)}\sum_{k=0}^{\infty}\frac{([n]_qx)^k}{[k]_q!}q^{k(k-1)/2}e^{-[n]_qx}\end{aligned}$$

$$
\begin{aligned}
&= \frac{[n]_q^2 x}{q^2 G_\beta^q(n) F_1^q(n)} E_q^{[n]_q x} e^{-[n]_q x} + \frac{[n]_q}{q G_\beta^q(n) F_1^q(n)} E_q^{[n]_q x} e^{-[n]_q x} \\
&\quad + \frac{\alpha}{G_\beta^q(n)} E_q^{[n]_q x} e^{-[n]_q x} \\
&= \frac{[n]_q^2 x}{q^2 G_\beta^q(n) F_1^q(n)} + \frac{[n]_q}{q G_\beta^q(n) F_1^q(n)} + \frac{\alpha}{G_\beta^q(n)}.
\end{aligned}
$$

Using the equality

$$
[n]_q = [s]_q + q^s [n-s]_q, \quad 0 \le s \le n, \tag{2.2}
$$

we get

$$
\begin{aligned}
B_{n,q}^{(\alpha,\beta)}(e_2, x) &= \frac{[n]_q^4 x^2}{q^6 G_\beta^q(n)^2 F_2^q(n)} \\
&\quad + \left\{ \frac{[n]_q^3}{q^5 G_\beta^q(n)^2 F_2^q(n)} + \frac{(1+[2]_q)[n]_q^3}{q^4 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha [n]_q^2}{q^2 G_\beta^q(n)^2 F_1^q(n)} \right\} x \\
&\quad + \frac{[2]_q [n]_q^2}{q^3 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha [n]_q}{q G_\beta^q(n)^2 F_1^q(n)} + \frac{\alpha^2}{G_\beta^q(n)^2},
\end{aligned}
$$

and so we have the proof of (iii). □

To obtain our main results we need to compute the second moment.

Lemma 2 *Let $q \in (0,1)$ and $n > 2$. Then we have the following inequality*:

$$
B_{n,q}^{(\alpha,\beta)}\big((t-x)^2, x\big) \le \left(\frac{2(1-q^4)}{q^6} + \frac{164(\alpha+\beta+1)^2 [n]_q}{q^6 F_2^q(n)} \right) x(x+1) + \frac{6(\alpha+1)^2}{q^3 G_\beta^q(n)}.
$$

Proof From the linearity of the $B_{n,q}^{(\alpha,\beta)}$ operators and Lemma 1, we write the second moment as

$$
\begin{aligned}
&B_{n,q}^{(\alpha,\beta)}\big((t-x)^2, x\big) \\
&\quad = \left\{ \frac{[n]_q^4}{q^6 G_\beta^q(n)^2 F_2^q(n)} - \frac{2[n]_q^2}{q^2 G_\beta^q(n) F_1^q(n)} + 1 \right\} x^2 \\
&\qquad + \left\{ \frac{\{1+(1+[2]_q)q\}[n]_q^3}{q^5 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha [n]_q^2}{q^2 G_\beta^q(n)^2 F_1^q(n)} - \frac{2[n]_q}{q G_\beta^q(n) F_1^q(n)} - \frac{2\alpha}{G_\beta^q(n)} \right\} x \\
&\qquad + \frac{[2]_q [n]_q^2}{q^3 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha [n]_q}{q G_\beta^q(n)^2 F_1^q(n)} + \frac{\alpha^2}{G_\beta^q(n)^2} \\
&\quad \le \left\{ \frac{[n]_q^4}{q^6 G_\beta^q(n)^2 F_2^q(n)} - \frac{2[n]_q^2}{q^2 G_\beta^q(n) F_1^q(n)} + 1 + \frac{\{1+(1+[2]_q)q\}[n]_q^3}{q^5 G_\beta^q(n)^2 F_2^q(n)} \right. \\
&\qquad \left. + \frac{2\alpha [n]_q^2}{q^2 G_\beta^q(n)^2 F_1^q(n)} \right\} x(x+1) + \frac{[2]_q [n]_q^2}{q^3 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha [n]_q}{q G_\beta^q(n)^2 F_1^q(n)} + \frac{\alpha^2}{G_\beta^q(n)^2}
\end{aligned}
$$

$$\begin{aligned}
&\leq \left\{ \frac{[n]_q^4(1+q^6) - 2q^4[n-2]_q^4 + 2\beta q^6[n]_q[n-1]_q[n-2]_q}{q^6 G_\beta^q(n)^2 F_2^q(n)} \right. \\
&\quad \left. + \frac{(q+q^2+[2]_q q^2)[n]_q^3}{q^6 G_\beta^q(n)^2 F_2^q(n)} + \frac{q^6\beta^2[n-1]_q[n-2]_q}{q^6 G_\beta^q(n)^2 F_2^q(n)} + \frac{2\alpha q^4[n]_q^2[n-2]_q}{q^6 G_\beta^q(n)^2 F_2^q(n)} \right\} x(x+1) \\
&\quad + \frac{\{[2]_q + 2\alpha q^2 + \alpha^2 q^3\}[n]_q}{q^3 G_\beta^q(n) F_2^q(n)}.
\end{aligned}$$

From (2.2), we have

$$\begin{aligned}
&B_{n,q}^{(\alpha,\beta)}\big((t-x)^2, x\big) \\
&\quad \leq \left\{ \frac{[n-2]_q^4(q^{14}+q^8-2q^4)}{q^6 G_\beta^q(n)^2 F_2^q(n)} \right. \\
&\qquad + \frac{(1+q^6)\{4[2]_q q^6[n-2]_q^3 + 6[2]_q^2 q^4[n-2]_q^2 + 4[2]_q^3 q^2[n-2]_q + [2]_q^4\}}{q^6 G_\beta^q(n)^2 F_2^q(n)} \\
&\qquad \left. + \frac{(q+q^2+[2]_q q^2 + 2\beta q^6 + 2\alpha q^4)[n]_q^3 + \beta^2 q^6[n]_q^2}{q^6 G_\beta^q(n)^2 F_2^q(n)} \right\} x(x+1) \\
&\qquad + \frac{([2]_q + q^2)([2]_q + 2\alpha q^2 + \alpha^2 q^3)}{q^3 G_\beta^q(n) F_1^q(n)} \\
&\quad \leq \left(\frac{2(1-q^4)}{q^6} + \frac{164(\alpha+\beta+1)^2[n]_q}{q^6 F_2^q(n)} \right) x(x+1) + \frac{6(\alpha+1)^2}{q^3 G_\beta^q(n)}.
\end{aligned}$$

And the proof of Lemma 2 is now finished. □

3 Direct estimates

Now in our considerations, $C_B[0,\infty)$ denotes the set of all bounded-continuous functions from $[0,\infty)$ to $\mathbb{R}$. $C_B[0,\infty)$ is a normed space with the norm $\|f\|_B = \sup\{|f(x)| : x \in [0,\infty)\}$. We denote the first modulus of continuity on the finite interval $[0,b]$, $b > 0$,

$$\omega_{[0,b]}(f,\delta) = \sup_{0<h\leq\delta, x\in[0,b]} \big|f(x+h) - f(x)\big|. \tag{3.1}$$

The Peetre K-functional is defined by

$$K_2(f,\delta) = \inf\big\{\|f-g\|_B + \delta\|g''\|_B : g \in W_\infty^2\big\}, \quad \delta > 0,$$

where $W_\infty^2 = \{g \in C_B[0,\infty) : g', g'' \in C_B[0,\infty)\}$. By Theorem 2.4 in [25], p.177, there exists a positive constant C such that

$$K_2(f,\delta) \leq C\omega_2(f,\sqrt{\delta}), \tag{3.2}$$

where

$$\omega_2(f,\sqrt{\delta}) = \sup_{0<h\leq\sqrt{\delta}} \sup_{x\in[0,\infty)} \big|f(x+2h) - 2f(x+h) - f(x)\big|.$$

Gadzhiev proved the weighted Korovkin-type theorems in [26]. We give the Gadzhiev results in weighted spaces. Let $\rho(x) = 1 + x^2$ and the weighted spaces $C_\rho[0,\infty)$ denote the space of all continuous functions f, satisfying $|f(x)| \leq M_f \rho(x)$, where M_f is a constant depending only on f. $C_\rho[0,\infty)$ is a normed space with the norm $\|f\|_\rho = \sup\{\frac{|f(x)|}{\rho(x)} : x \in \mathbb{R}^+ \cup \{0\}\}$ and $C^*_\rho[0,\infty)$ denotes the subspace of all functions $f \in C_\rho[0,\infty)$ for which $\lim_{|x|\to\infty} \frac{|f(x)|}{\rho(x)}$ exists finitely.

Thus we are ready to give direct results. The following lemma is routine and its proof is omitted.

Lemma 3 *Let*

$$\overline{B}^{(\alpha,\beta)}_{n,q}(f,x) = B^{(\alpha,\beta)}_{n,q}(f,x) - f\bigl(D^{(\alpha,\beta)}_{n,q}(x)\bigr) + f(x). \tag{3.3}$$

Then the following assertions hold for the operators (3.3):

(i) $\overline{B}^{(\alpha,\beta)}_{n,q}(1,x) = 1$,

(ii) $\overline{B}^{(\alpha,\beta)}_{n,q}(t,x) = x$,

(iii) $\overline{B}^{(\alpha,\beta)}_{n,q}(t-x,x) = 0$,

where $D^{(\alpha,\beta)}_{n,q}(x) = \frac{[n]_q^2 x}{q^2 G^q_\beta(n) F^q_1(n)} + \frac{[n]_q}{q G^q_\beta(n) F^q_1(n)} + \frac{\alpha}{G^q_\beta(n)}$.

Lemma 4 *Let* $q \in (0,1)$ *and* $n > 2$. *Then for every* $x \in [0,\infty)$ *and* $f'' \in C_B[0,\infty)$, *we have the inequality*

$$\bigl|\overline{B}^{(\alpha,\beta)}_{n,q}(f,x) - f(x)\bigr| \leq \delta^{(\alpha,\beta)}_{n,q}(x)\bigl\|f''\bigr\|_B,$$

where $\delta^{(\alpha,\beta)}_{n,q}(x) = (\frac{2(1-q^4)}{q^6} + \frac{263(\alpha+\beta+1)^2}{q^6 F^q_1(n)})x(x+1) + \frac{5(\alpha+1)^2}{q^3 G^q_\beta(n)}$.

Proof Using Taylor's expansion

$$f(t) = f(x) + (t-x)f'(x) + \int_x^t (t-u)f''(u)\,du$$

and Lemma 3, we obtain

$$\overline{B}^{(\alpha,\beta)}_{n,q}(f,x) - f(x) = \overline{B}^{(\alpha,\beta)}_{n,q}\left(\int_x^t (t-u)f''(u)\,du, x\right).$$

Then, using Lemma 1 and the inequality

$$\left|\int_x^t (t-u)f''(u)\,du\right| \leq \bigl\|f''\bigr\|_B \frac{(t-x)^2}{2},$$

we get

$$\begin{aligned}
&\bigl|\overline{B}^{(\alpha,\beta)}_{n,q}(f,x) - f(x)\bigr| \\
&\quad\leq \left|B^{(\alpha,\beta)}_{n,q}\left(\int_x^t (t-u)f''(u)\,du, x\right) - \int_x^{D^{(\alpha,\beta)}_{n,q}(x)} \bigl\{D^{(\alpha,\beta)}_{n,q}(x) - u\bigr\}f''(u)\,du\right|
\end{aligned}$$

$$\begin{aligned}
&\leq \frac{\|f''\|_B}{2}\Bigg\{\Bigg(\frac{2(1-q^4)}{q^6}+\frac{164(\alpha+\beta+1)^2[n]_q}{q^6F_2^q(n)}+\bigg(\frac{[n]_q^2}{q^2G_\beta^q(n)F_1^q(n)}-1\bigg)^2 \\
&\quad+\frac{2[n]_q^3}{q^3G_\beta^q(n)^2F_1^q(n)^2}+\frac{2[n]_q^2\alpha}{q^2G_\beta^q(n)^2F_1^q(n)}\Bigg)x(x+1)+\bigg(\frac{[n]_q+\alpha q[n-1]_q}{qG_\beta^q(n)F_1^q(n)}\bigg)^2 \\
&\quad+\frac{6(\alpha+1)^2}{q^3G_\beta^q(n)F_1^q(n)}\Bigg\} \\
&\leq \frac{\|f''\|_B}{2}\Bigg\{\Bigg(\frac{4(1-q^4)}{q^6}+\frac{526(\alpha+\beta+1)^2}{q^6F_1^q(n)}\Bigg)x(x+1)+\frac{10(\alpha+1)^2}{q^3G_\beta^q(n)}\Bigg\}.
\end{aligned}$$

And the proof of the Lemma 4 is now completed. □

Theorem 1 *Let $(q_n)\subset(0,1)$ a sequence such that $q_n\to 1$ as $n\to\infty$. Then for every $n>2$, $x\in[0,\infty)$ and $f\in C_B[0,\infty)$, we have the inequality*

$$\big|B_{n,q_n}^{(\alpha,\beta)}(f,x)-f(x)\big|\leq 2M\omega_2\Big(f,\sqrt{\delta_{n,q_n}^{(\alpha,\beta)}(x)}\Big)+w\big(f,\eta_{n,q_n}^{(\alpha,\beta)}(x)\big),$$

where $\eta_{n,q_n}^{(\alpha,\beta)}(x)=(\frac{[n]_{q_n}^2}{q_n^2G_\beta^{q_n}(n)F_1^{q_n}(n)}-1)x+\frac{[n]_{q_n}}{q_nG_\beta^{q_n}(n)F_1^{q_n}(n)}+\frac{\alpha}{G_\beta^{q_n}(n)}$.

Proof Using (3.3) for any $g\in W_\infty^2$, we obtain the following inequality:

$$\begin{aligned}
\big|B_{n,q_n}^{(\alpha,\beta)}(f,x)-f(x)\big| &\leq \big|\overline{B}_{n,q_n}^{(\alpha,\beta)}(f-g,x)-(f-g)(x)+\overline{B}_{n,q_n}^{(\alpha,\beta)}(g,x)-g(x)\big| \\
&\quad+\bigg|f\bigg(\frac{[n]_{q_n}^2}{q_n^2G_\beta^{q_n}(n)F_1^{q_n}(n)}x+\frac{[n]_{q_n}}{q_nG_\beta^{q_n}(n)F_1^{q_n}(n)}+\frac{\alpha}{G_\beta^{q_n}(n)}\bigg)-f(x)\bigg|.
\end{aligned}$$

From Lemma 4, we get

$$\begin{aligned}
\big|B_{n,q_n}^{(\alpha,\beta)}(f,x)-f(x)\big| &\leq 2\|f-g\|_B+\delta_{n,q_n}^{(\alpha,\beta)}(x)\big\|g''\big\| \\
&\quad+\bigg|f\bigg(\frac{[n]_{q_n}^2}{q_n^2G_\beta^{q_n}(n)F_1^{q_n}(n)}x+\frac{[n]_{q_n}}{q_nG_\beta^{q_n}(n)F_1^{q_n}(n)}+\frac{\alpha}{G_\beta^{q_n}(n)}\bigg)-f(x)\bigg|.
\end{aligned}$$

By using equality (3.1) we have

$$\big|B_{n,q_n}^{(\alpha,\beta)}(f,x)-f(x)\big|\leq 2\|f-g\|_B+\delta_{n,q_n}^{(\alpha,\beta)}(x)\big\|g''\big\|_B+w\big(f,\eta_{n,q_n}^{(\alpha,\beta)}(x)\big).$$

Taking the infimum over $g\in W_\infty^2$ on the right-hand side of the above inequality and using the inequality (3.2), we get the desired result. □

Theorem 2 *Let $(q_n)\subset(0,1)$ a sequence such that $q_n\to 1$ as $n\to\infty$. Then $f\in C_\rho^*[0,\infty)$, and we have*

$$\lim_{n\to\infty}\big\|B_{n,q_n}^{(\alpha,\beta)}(f)-f\big\|_\rho=0.$$

Proof From Lemma 1, it is obvious that $\|B_{n,q_n}^{(\alpha,\beta)}(e_0)-e_0\|_\rho=0$. Since $|\frac{[n]_{q_n}^2}{q_n^2G_\beta^{q_n}(n)F_1^{q_n}(n)}x+\frac{[n]_{q_n}}{q_nG_\beta^{q_n}(n)F_1^{q_n}(n)}+\frac{\alpha}{G_\beta^{q_n}(n)}-x|\leq(x+1)o(1)$ and $\frac{x+1}{1+x^2}$ is positive and bounded from above for each $x \geq 0$, we obtain

$$\left\| B_{n,q_n}^{(\alpha,\beta)}(e_1) - e_1 \right\|_\rho \leq \frac{x+1}{1+x^2} o(1).$$

And then $\lim_{n\to\infty} \| B_{n,q_n}^{(\alpha,\beta)}(e_1) - e_1 \|_\rho = 0$.

Similarly for every $n > 2$, we write

$$\begin{aligned}
\left\| B_{n,q_n}^{(\alpha,\beta)}(e_2) - e_2 \right\|_\rho &= \sup_{x\in[0,\infty)} \left\{ \frac{|(\frac{[n]_{q_n}^4}{q_n^6 G_\beta^{q_n}(n)^2 F_2^{q_n}(n)} - 1)x^2}{1+x^2} \right. \\
&\quad + \frac{\{\frac{(1+(1+[2]_{q_n})q_n)[n]_{q_n}^3 + 2\alpha q^2 [n]_{q_n}^2 [n-1]_{q_n}}{q_n^5 G_\beta^{q_n}(n)^2 F_2^{q_n}(n)}\}x + \frac{[2]_{q_n}[n]_{q_n}^2}{q_n^3 G_\beta^{q_n}(n)^2 F_2^{q_n}(n)}}{1+x^2} \\
&\quad \left. + \frac{\frac{+2\alpha q_n^2 [n]_{q_n}[n-2]_{q_n}}{q_n^3 G_\beta^{q_n}(n)^2 F_2^{q_n}(n)} + \frac{\alpha^2}{G_\beta^{q_n}(n)^2}|}{1+x^2} \right\} \\
&\leq \sup_{x\in[0,\infty)} \frac{1+x+x^2}{1+x^2} o(1),
\end{aligned}$$

we get $\lim_{n\to\infty} \| B_{n,q_n}^{(\alpha,\beta)}(e_2) - e_2 \|_\rho = 0$. Thus, from AD Gadzhiev's theorem in [26], we obtain the desired result of Theorem 2. □

Competing interests
The author declares to have no competing interests.

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References
1. Mahmudov, NI: q-Szász operators which preserve x^2. Math. Slovaca **63**, 1059 (2013)
2. Gupta, V, Mahmudov, NI: Approximation properties of the q-Szasz-Mirakjan-Beta operators. Indian J. Ind. Appl. Math. **3**, 41 (2012)
3. Govil, NK, Gupta, V: q-Beta-Szász-Stancu operators. Adv. Stud. Contemp. Math. **22**, 117 (2012)
4. Yüksel, İ, Dinlemez, Ü: On the approximation by the q-Szász-Beta type operators. Appl. Math. Comput. **235**, 555 (2014)
5. Dinlemez, Ü, Yüksel, İ, Altın, B: A note on the approximation by the q-hybrid summation integral type operators. Taiwan. J. Math. **18**, 781 (2014)
6. Doğru, O, Gupta, V: Monotonicity and the asymptotic estimate of Bleimann Butzer and Hahn operators based on q-integers. Georgian Math. J. **12**, 415 (2005)
7. Doğru, O, Gupta, V: Korovkin-type approximation properties of bivariate q-Meyer-König and Zeller operators. Calcolo **43**, 51 (2006)
8. Gupta, V, Heping, W: The rate of convergence of q-Durrmeyer operators for $0 < q < 1$. Math. Methods Appl. Sci. **31**, 1946 (2008)
9. Gupta, V, Aral, A: Convergence of the q-analogue of Szász-Beta operators. Appl. Math. Comput. **216**, 374 (2010)
10. Gupta, V, Karsli, H: Some approximation properties by q-Szász-Mirakyan-Baskakov-Stancu operators. Lobachevskii J. Math. **33**, 175 (2012)
11. Lupaş, A: A q-analogue of the Bernstein operator. In: Seminar on Numerical and Statistical Calculus, pp. 85-92 (1987)
12. Phillips, GM: Bernstein polynomials based on the q-integers. Ann. Numer. Math. **4**, 511 (1997)
13. Yüksel, İ: Approximation by q-Phillips operators. Hacet. J. Math. Stat. **40**, 191 (2011)
14. Yüksel, I: Approximation by q-Baskakov-Schurer-Szász type operators. AIP Conf. Proc. **1558**, 1136 (2013). doi:10.1063/1.4825708
15. De Sole, A, Kac, VG: On integral representations of q-gamma and q-Beta functions. Atti Accad. Naz. Lincei, Rend. Lincei, Mat. Appl. **16**, 11 (2005)
16. Gupta, V, Agarwal, RP: Convergence Estimates in Approximation Theory. Springer, Cham (2014). ISBN:978-3-319-02764-7
17. Jackson, FH: On q-definite integrals. Q. J. Pure Appl. Math. **41**, 193 (1910)

18. Kac, VG, Cheung, P: Quantum Calculus. Universitext. Springer, New York (2002)
19. Koelink, HT, Koornwinder, TH: q-Special functions, a tutorial. In: Deformation Theory and Quantum Groups with Applications to Mathematical Physics (Amherst, MA, 1990). Contemp. Math., vol. 134, pp. 141-142. Am. Math. Soc., Providence (1992)
20. Deo, N: Direct result on the Durrmeyer variant of Beta operators. Southeast Asian Bull. Math. **32**, 283 (2008)
21. Deo, N: Direct result on exponential-type operators. Appl. Math. Comput. **204**, 109 (2008)
22. Gupta, V, Srivastava, GS, Sahai, A: On simultaneous approximation by Szász-Beta operators. Soochow J. Math. **21**, 1 (1995)
23. Jung, HS, Deo, N, Dhamija, M: Pointwise approximation by Bernstein type operators in mobile interval. Appl. Math. Comput. **214**, 683 (2014)
24. Aral, A, Gupta, V, Agarwal, RP: Applications of q-Calculus in Operator Theory. Springer, New York (2013)
25. De Vore, RA, Lorentz, GG: Constructive Approximation. Springer, Berlin (1993)
26. Gadzhiev, AD: Theorems of the type of P. P. Korovkin type theorems. Mat. Zametki **20**, 781 (1976). English Translation, Math. Notes **20**, 996 (1976)

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