Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Some inequalities for nonnegative tensors

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Abstract

Let $\mathcal{A}$ be a nonnegative tensor and $x = (x_i) > 0$ its Perron vector. We give lower bounds for $x_t^{m-1}/\sum x_{i_2}\cdots x_{i_m}$ and upper bounds for $x_s^{m-1}/\sum x_{i_2}\cdots x_{i_m}$, where $x_s = \max_{1\le i\le n} x_i$ and $x_t = \min_{1\le i\le n} x_i$.

MSC: 15A18; 15A69; 65F15; 65F10

Keywords: Perron vector; nonnegative tensor; spectral radius; eigenvalues

1 Introduction

Eigenvalue problems of higher order tensors have become an important topic of study in a new applied mathematics branch, numerical multilinear algebra, and they have a wide range of practical applications [1–7]. The main difficulty in tensor problems is that they are generally nonlinear. Therefore, large amounts of results for matrices are never in force for higher order tensors. However, there are still some results preserved in the case of higher order tensors.

Throughout this paper we consider an mth order n-dimensional tensor $\mathcal{A}$ consisting of n^m entries in $\mathbb{R}$:

$$\mathcal{A} = (a_{i_1,i_2,\ldots,i_m}), \quad a_{i_1,i_2,\ldots,i_m} \in \mathbb{R}, 1 \le i_1, i_2, \ldots, i_m \le n.$$

The tensor $\mathcal{A}$ is called nonnegative (or positive) if all the entries $a_{i_1,i_2,\ldots,i_m} \ge 0$ (or $a_{i_1,i_2,\ldots,i_m} > 0$). We also denote by $\mathbb{C}$ the field of complex numbers.

Definition 1.1 A pair $(\lambda, x) \in \mathbb{C} \times \mathbb{C}^n/\{0\}$ is called an eigenvalue-eigenvector pair of $\mathcal{A}$, if they satisfy

$$\mathcal{A}x^{m-1} = \lambda x^{[m-1]}, \tag{1}$$

where n-dimensional column vectors $\mathcal{A}x^{m-1}$ and $x^{[m-1]}$ are defined as

$$\mathcal{A}x^{m-1} := (a_{ii_2\cdots i_n} x_{i_2}\cdots x_{i_n})_{1\le i\le n} \quad \text{and} \quad x^{[m-1]} := \left(x_i^{m-1}\right)_{1\le i\le n}.$$

This definition was introduced by Qi [8] when m is even and $\mathcal{A}$ is symmetric. Independently, Lim [9] gave such a definition but restricted x to be a real vector and λ to be a real number. Let

$$\rho(\mathcal{A}) = \max\{|\lambda| : \lambda \text{ is an eigenvalue of } \mathcal{A}\},$$

where $|\lambda|$ denotes the modulus of λ. We call $\rho(\mathcal{A})$ the spectral radius of tensor $\mathcal{A}$.

In [4], Chang *et al.* generalized the Perron-Frobenius theorem from nonnegative matrices to irreducible nonnegative tensors. Some further results based on this theorem are discussed by Yang [10, 11].

Definition 1.2 The tensor $\mathcal{A}$ is called reducible if there exists a nonempty proper index subset $\mathbb{J} \subset \{1, 2, \dots, n\}$ such that

$$a_{i_1,i_2,\dots,i_m} = 0, \quad \forall i_1 \in \mathbb{J}, \forall i_2, \dots, i_m \notin \mathbb{J}.$$

If $\mathcal{A}$ is not reducible, then we call $\mathcal{A}$ irreducible.

Theorem 1.3 [4] *If $\mathcal{A}$ is irreducible and nonnegative, then there exist a number $\rho(\mathcal{A}) > 0$ and a vector $x_0 > 0$, such that*

$$\mathcal{A}x_0^{m-1} = \rho(\mathcal{A})x_0^{[m-1]}.$$

Moreover, if λ is an eigenvalue with a nonnegative eigenvector, then $\lambda = \rho(\mathcal{A})$. If λ is an eigenvalue of $\mathcal{A}$, then $|\lambda| \leq \rho(\mathcal{A})$.

We call x_0 a Perron vector of $\mathcal{A}$ corresponding to its largest nonnegative eigenvalue $\rho(\mathcal{A})$.

In this paper, we are interested in studying some bounds for the Perron vector of $\mathcal{A}$. For this purpose, we define

$$x_s = \max_{1\leq i\leq n} x_i, \qquad x_t = \min_{1\leq i\leq n} x_i.$$

In the following we first give a new and simple bound for x_s/x_t. Then we give some lower bounds for $x_t^{m-1}/\sum x_{i_2}\cdots x_{i_m}$ and upper bounds for $x_s^{m-1}/\sum x_{i_2}\cdots x_{i_m}$, which can be used to get another bound for x_s/x_t.

The paper is organized as follows. In Section 2, some efforts of establishing the bounds of the nonnegative tensor are made. An application of these bounds is studied in Section 3.

2 Bounds

In [12], the authors have studied the perturbation bound for the spectral radius of $\mathcal{A}$, and they show that a Perron vector x must be known in advance so that the perturbation bound can be computed. Here we cite a lemma for use below.

Lemma 2.1 [12] *Suppose $\mathcal{A}$ is nonnegative such that $m \geq 3$ and x is its Perron vector x. Then*

$$1 \leq \frac{x_s}{x_t} \leq \min_{2\leq k\leq m} \left\{ \frac{\max_{1\leq i_1,i_k\leq n} \sum^{n}_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\dots,i_m}}{\min_{1\leq i_1,i_k\leq n} \sum^{n}_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\dots,i_m}} \right\}. \tag{2}$$

Define the ith row sum of $\mathcal{A}$ as

$$R_i(\mathcal{A}) = \sum_{i_2,\ldots,i_m=1}^{n} a_{ii_2\cdots i_m},$$

and denote the largest, the smallest, and the average row sums of $\mathcal{A}$ by

$$R_{\max}(\mathcal{A}) = \max_{i=1,\ldots,n} R_i(\mathcal{A}), \qquad R_{\min}(\mathcal{A}) = \min_{i=1,\ldots,n} R_i(\mathcal{A}).$$

Let

$$l = \min_{i_1,\ldots,i_m} a_{i_1\cdots i_m}, \qquad L = \max_{i_1,\ldots,i_m} a_{i_1\cdots i_m}.$$

Theorem 2.2 *Suppose $\mathcal{A}$ is nonnegative with Perron vector x. Then*

$$\frac{x_s}{x_t} \geq \max\left\{\left\{\frac{R_{\max}(\mathcal{A})}{R_{\min}(\mathcal{A})}\right\}^{\frac{1}{2(m-1)}}, \max_{2\leq k\leq m}\left\{\frac{\max_{1\leq i_1,i_k\leq n}\sum^{n}_{\underbrace{i_2,\ldots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\ldots,i_m}}{\min_{1\leq i_1,i_k\leq n}\sum^{n}_{\underbrace{i_2,\ldots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\ldots,i_m}}\right\}^{\frac{1}{2m-3}}\right\}. \tag{3}$$

Proof Since, for any $i = 1, 2, \ldots, n$,

$$\rho(\mathcal{A})x_t^{m-1} \leq \rho(\mathcal{A})x_i^{m-1} = \sum_{i_2,\ldots,i_m=1}^{n} a_{ii_2\cdots i_m}x_{i_2}\cdots x_{i_m} \leq \sum_{i_2,\ldots,i_m=1}^{n} a_{ii_2\cdots i_m}x_s^{m-1},$$

we have

$$\rho(\mathcal{A})x_t^{m-1} \leq R_{\min}(\mathcal{A})x_s^{m-1}. \tag{4}$$

Similarly,

$$\rho(\mathcal{A})x_s^{m-1} \geq R_{\max}(\mathcal{A})x_t^{m-1}. \tag{5}$$

It follows from (4) and (5) that

$$\frac{x_s^{m-1}}{x_t^{m-1}} \geq \frac{x_t^{m-1}}{x_s^{m-1}}\frac{R_{\max}(\mathcal{A})}{R_{\min}(\mathcal{A})}$$

and therefore

$$\frac{x_s}{x_t} \geq \left\{\frac{R_{\max}(\mathcal{A})}{R_{\min}(\mathcal{A})}\right\}^{\frac{1}{2(m-1)}}.$$

If we assume $\|x\|_1 = 1$, then, for any $i = 1, 2, \dots, n$,

$$\begin{aligned}\rho(\mathcal{A})x_t^{m-1} \le \rho(\mathcal{A})x_i^{m-1} &= \sum_{i_2,\dots,i_m=1}^{n} a_{ii_2\cdots i_m} x_{i_2}\cdots x_{i_m} \\ &\le \left(\sum_{i_2,\dots,i_m=1}^{n} a_{ii_2\cdots i_m} x_{i_k}\right) x_s^{m-2} \le \left(\min_{1\le i_1,i_k\le n} \sum_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}}^{n} a_{i_1,i_2,\dots,i_m}\right) x_s^{m-2}. \end{aligned} \tag{6}$$

Similarly,

$$\rho(\mathcal{A})x_s^{m-1} \ge \left(\max_{1\le i_1,i_k\le n} \sum_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}}^{n} a_{i_1,i_2,\dots,i_m}\right) x_t^{m-2},$$

and therefore

$$\frac{x_s}{x_t} \ge \left\{ \frac{\max_{1\le i_1,i_k\le n} \sum^{n}_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\dots,i_m}}{\min_{1\le i_1,i_k\le n} \sum^{n}_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\dots,i_m}} \right\}^{\frac{1}{2m-3}}.$$

The result follows. □

Remark By the obvious inequality

$$1 \le \max\left\{ \left\{\frac{R_{\max}(\mathcal{A})}{R_{\min}(\mathcal{A})}\right\}^{\frac{1}{2(m-1)}}, \max_{2\le k\le m} \left\{ \frac{\max_{1\le i_1,i_k\le n} \sum^{n}_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\dots,i_m}}{\min_{1\le i_1,i_k\le n} \sum^{n}_{\underbrace{i_2,\dots,i_m}_{\text{except } i_k}} a_{i_1,i_2,\dots,i_m}} \right\}^{\frac{1}{2m-3}} \right\},$$

we see that our new lower bound in (3) is sharper than that in (2).

If the tensor $\mathcal{A}$ is positive, we derive a new upper bound for x_s/x_t in terms of the entries of $\mathcal{A}$ and the spectral radius $\rho(\mathcal{A})$.

Theorem 2.3 *If $\mathcal{A}$ is positive with Perron vector x, then*

$$\begin{aligned}\frac{l}{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l(m-1)n} &\le \frac{x_t^{m-1}}{\sum_{i_2,\dots,i_m=1}^{n} x_{i_2}\cdots x_{i_m}} \\ &\le \frac{L}{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + L(m-1)n},\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + [(m-1)n-1]L}{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + L(m-1)n} \cdot \frac{1}{(m-1)n-1} &\le \frac{x_s^{m-1}}{\sum_{i_2,\dots,i_m=1}^{n} x_{i_2}\cdots x_{i_m}} \\ &\le \frac{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l}{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l(m-1)n},\end{aligned} \tag{8}$$

$$\begin{aligned}&\left\{\frac{\rho(\mathcal{A})-R_{\max}(\mathcal{A})+[(m-1)n-1]L}{L}\cdot\frac{1}{(m-1)n-1}\right\}^{\frac{1}{m-1}}\\&\quad\le\frac{x_s}{x_t}\le\left\{\frac{\rho(\mathcal{A})-R_{\min}(\mathcal{A})+l}{l}\right\}^{\frac{1}{m-1}}.\end{aligned}\tag{9}$$

Proof First, we prove the right side of (9). Now we consider

$$\begin{aligned}\rho(\mathcal{A})x_t^{m-1}-l\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}&=\sum_{i_2,\ldots,i_m=1}^{n}a_{ti_2\cdots i_m}x_{i_2}\cdots x_{i_m}-l\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}\\&\ge x_t^{m-1}\bigl[R_{\min}(\mathcal{A})-l(m-1)n\bigr]\end{aligned}\tag{10}$$

and

$$\begin{aligned}\frac{x_s^{m-1}}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}&=1-\frac{\sum_{\substack{i_2,\ldots,i_m=1\\ \text{except } i_2=\cdots=i_m=s}}^{n}x_{i_2}\cdots x_{i_m}}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}\\&\le 1-\frac{[(m-1)n-1]}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}x_t^{m-1},\end{aligned}\tag{11}$$

so we can get

$$\begin{aligned}\frac{x_t^{m-1}}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}&\ge\frac{l}{\rho(\mathcal{A})-R_{\min}(\mathcal{A})+l(m-1)n},\\ \frac{x_s^{m-1}}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}&\le\frac{\rho(\mathcal{A})-R_{\min}(\mathcal{A})+l}{\rho(\mathcal{A})-R_{\min}(\mathcal{A})+l(m-1)n},\end{aligned}$$

then

$$\frac{x_s}{x_t}\le\left\{\frac{\rho(\mathcal{A})-R_{\min}(\mathcal{A})+l}{l}\right\}^{\frac{1}{m-1}}.$$

On the other hand,

$$\begin{aligned}L\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}-\rho(\mathcal{A})x_t^{m-1}&=L\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}-\sum_{i_2,\ldots,i_m=1}^{n}a_{ti_2\cdots i_m}x_{i_2}\cdots x_{i_m}\\&\ge x_t^{m-1}\bigl[L(m-1)n-R_{\max}(\mathcal{A})\bigr]\end{aligned}\tag{12}$$

and

$$\begin{aligned}\frac{x_t^{m-1}}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}&=1-\frac{\sum_{\substack{i_2,\ldots,i_m=1\\ \text{except } i_2=\cdots=i_m=t}}^{n}x_{i_2}\cdots x_{i_m}}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}\\&\le 1-\frac{[(m-1)n-1]}{\sum_{i_2,\ldots,i_m=1}^{n}x_{i_2}\cdots x_{i_m}}x_s^{m-1},\end{aligned}\tag{13}$$

so we can get

$$\frac{x_t^{m-1}}{\sum_{i_2,\ldots,i_m=1}^n x_{i_2}\cdots x_{i_m}} \le \frac{L}{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + L(m-1)n},$$
$$\frac{x_s^{m-1}}{\sum_{i_2,\ldots,i_m=1}^n x_{i_2}\cdots x_{i_m}} \ge \frac{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + [(m-1)n-1]L}{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + L(m-1)n} \cdot \frac{1}{(m-1)n-1},$$

then

$$\frac{x_s}{x_t} \ge \left\{ \frac{\rho(\mathcal{A}) - R_{\max}(\mathcal{A}) + [(m-1)n-1]L}{L} \cdot \frac{1}{(m-1)n-1} \right\}^{\frac{1}{m-1}}.$$

This completes the proof. □

In the following corollary, we get some bounds for $x_t / \sum_{j=1}^n x_j$ and $x_s / \sum_{j=1}^n x_j$ in terms of the entries of $\mathcal{A}$ and the spectral radius $\rho(\mathcal{A})$.

Corollary 2.4 *If $\mathcal{A}$ is positive with Perron vector x. Then*

$$\frac{x_t}{\sum_{i_k=1}^n x_{i_k}} \ge \frac{l}{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + nl}, \tag{14}$$
$$\frac{x_s}{\sum_{i_k=1}^n x_{i_k}} \le \frac{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l}{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + nl}, \tag{15}$$
$$\frac{x_s}{x_t} \le \frac{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l}{l}. \tag{16}$$

Proof From

$$\rho(\mathcal{A})x_t^{m-1} = \sum_{i_2,\ldots,i_m=1}^n a_{ti_2\cdots i_m} x_{i_2}\cdots x_{i_m} \ge \left(\sum_{i_2,\ldots,i_m=1}^n a_{ti_2\cdots i_m} x_{i_k} \right) x_t^{m-2},$$

we have

$$\begin{aligned} \rho(\mathcal{A})x_t - l\sum_{i_k=1}^n x_{i_k} &\ge \sum_{i_2,\ldots,i_m=1}^n a_{ti_2\cdots i_m} x_{i_k} - l\sum_{i_k=1}^n x_{i_k} \\ &\ge x_t\big[R_{\min}(\mathcal{A}) - nl\big]. \end{aligned} \tag{17}$$

Similar to the proof of Theorem 2.3, we can get the results. □

Remark Similar to the proof of Theorem 2.3, we can get the lower bound of $\frac{x_s}{x_t}$.

3 Application to the perturbation bound

Suppose $\mathcal{A} \ge 0$ and $\mathcal{B}$ is another tensor satisfying $\mathcal{A} \le \mathcal{B}$. In this section we are interested in the bound of $\rho(\mathcal{B}) - \rho(\mathcal{A})$.

Lemma 3.1 [10] *$0 \le \mathcal{A} \le \mathcal{B}$, then $\rho(\mathcal{A}) \le \rho(\mathcal{B})$.*

We let $\mathbb{P} = \{(y_1, y_2, \ldots, y_n) | y_i \geq 0\}$ be the positive cone, and let the interior of $\mathbb{P}$ be denoted by $\mathrm{int}(\mathbb{P}) = \{(y_1, y_2, \ldots, y_n) | y_i > 0\}$.

Theorem 3.2 *Let $\mathcal{A}$ be weakly irreducible. For a nonzero $x \in \mathbb{P}$, we define*

$$\mu(x) = \min_{x_i>0} \frac{(\mathcal{A}x^{m-1})_i}{x_i^{m-1}}, \qquad \nu(x) = \max_{x_i>0} \frac{(\mathcal{A}x^{m-1})_i}{x_i^{m-1}}.$$

Then

$$\mu(x) \leq \rho(\mathcal{A}) \leq \nu(x). \tag{18}$$

Proof Since

$$\min_{x_i>0} \frac{(\mathcal{A}x^{m-1})_i}{x_i^{m-1}} \leq \frac{(\mathcal{A}x^{m-1})_i}{x_i^{m-1}} \leq \max_{x_i>0} \frac{(\mathcal{A}x^{m-1})_i}{x_i^{m-1}}, \quad x_i \neq 0,$$
$$\mu(x)x^{m-1} \leq \mathcal{A}x^{m-1} \leq \nu(x)x^{m-1}.$$

By the result of Lemma 5.4 in [10], we have

$$\mu(x) \leq \rho(\mathcal{A}) \leq \nu(x). \qquad \square$$

Theorem 3.3 *Let $\mathcal{A}$, $\mathcal{B}$ be irreducible and positive such that $\omega = \min_{i_1,\ldots,i_m}(b_{i_1,\ldots,i_m} - a_{i_1,\ldots,i_m}) \geq 0$ and $\mu = \max_{i_1,\ldots,i_m}(b_{i_1,\ldots,i_m} - a_{i_1,\ldots,i_m})$. Then*

$$\frac{\omega[\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l(m-1)n]}{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l} \leq \rho(\mathcal{B}) - \rho(\mathcal{A}) \leq \frac{\mu[\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l(m-1)n]}{l}.$$

Proof Let x be the Perron vector of $\mathcal{A}$. Define i as follows:

$$\frac{(\mathcal{B}x^{m-1})_i}{x_i^{m-1}} = \max_k \frac{(\mathcal{B}x^{m-1})_k}{x_k^{m-1}}.$$

Then, by Theorem 3.2, we have

$$\max_k \frac{(\mathcal{B}x^{m-1})_k}{x_k^{m-1}} \geq \rho(\mathcal{B}).$$

From the simple equality

$$(\mathcal{B} - \mathcal{A})x^{m-1} + \rho(\mathcal{A})x^{[m-1]} = \mathcal{B}x^{m-1},$$

and by considering the ith coordinate,

$$\frac{\sum_{i_2,\ldots,i_m=1}^n (b_{ii_2\cdots i_m} - a_{ii_2\cdots i_m})x_{i_2}\cdots x_{i_m}}{x_i^{m-1}} + \rho(\mathcal{A}) = \frac{(\mathcal{B}x^{m-1})_i}{x_i^{m-1}} \geq \rho(\mathcal{B}).$$

Since $\mu = \max_{i_1,\ldots,i_m}(b_{i_1,\ldots,i_m} - a_{i_1,\ldots,i_m})$ and $x_t = \min_{1\leq i\leq n} x_i$,

$$\rho(\mathcal{B}) - \rho(\mathcal{A}) \leq \frac{\mu\sum_{i_2,\ldots,i_m=1}^n x_{i_2}\cdots x_{i_m}}{x_t^{m-1}} \leq \frac{\mu[\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l(m-1)n]}{l}.$$

Similarly, we can get

$$\rho(\mathcal{B}) - \rho(\mathcal{A}) \geq \frac{\omega \sum_{i_2,\ldots,i_m=1}^{n} x_{i_2} \cdots x_{i_m}}{x_s^{m-1}} \geq \frac{\omega[\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l(m-1)n]}{\rho(\mathcal{A}) - R_{\min}(\mathcal{A}) + l}. \qquad \square$$

Remark If we let x be the Perron vector of $\mathcal{B}$, then by a similar method, we can get the following bound:

$$\frac{\omega[\rho(\mathcal{B}) - R_{\min}(\mathcal{B}) + l(m-1)n]}{\rho(\mathcal{B}) - R_{\min}(\mathcal{B}) + l} \leq \rho(\mathcal{B}) - \rho(\mathcal{A}) \leq \frac{\mu[\rho(\mathcal{B}) - R_{\min}(\mathcal{B}) + l(m-1)n]}{l}.$$

4 Conclusion

We have obtained a new and sharper bound of x_s/x_t, lower bounds for $x_t^{m-1}/\sum x_{i_2} \cdots x_{i_m}$, and upper bounds for $x_s^{m-1}/\sum x_{i_2} \cdots x_{i_m}$, where $x = (x_i) > 0$ is the Perron vector of a positive tensor $\mathcal{A}$ with $x_s = \max_{1\leq i\leq n} x_i$ and $x_t = \min_{1\leq i\leq n} x_i$. In addition we have given bounds of $\rho(\mathcal{B}) - \rho(\mathcal{A})$ when $0 < \mathcal{A} \leq \mathcal{B}$ by these bounds.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to this work. All authors read and approved the final manuscript.

Acknowledgements
This research is supported by NSFC (61170311), Chinese Universities Specialized Research Fund for the Doctoral Program (20110185110020), Sichuan Province Sci. & Tech. Research Project (12ZC1802). The first author is supported by the Fundamental Research Funds for Central Universities.

Received: 26 March 2014 Accepted: 14 August 2014 Published: 03 Sep 2014

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10.1186/1029-242X-2014-340
Cite this article as: He et al.: **Some inequalities for nonnegative tensors.** *Journal of Inequalities and Applications* 2014, 2014:340