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Correction: Cesàro summable difference sequence space

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Abstract

Theorem 3.7 of Bhardwaj and Gupta, Cesàro summable difference sequence space, J. Inequal. Appl. 2013:315, 2013, is incorrect as it stands. The corrected version of this theorem is given here.

MSC: 40C05; 40A05; 46A45

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In [1], Bhardwaj and Gupta have introduced the Cesàro summable difference sequence space $C_1(\Delta)$ as the set of all complex sequences $x = (x_k)$ with $(x_k - x_{k+1}) \in C_1$, where C_1 is the linear space of all $(C,1)$ summable sequences.

Unfortunately, Theorem 3.7 of [1] is incorrect, as it stands. Consequently the assertions of Corollaries 3.8 and 3.9 of [1] remain actually open. The corrected version of Theorem 3.7 of [1] is obtained here as Corollary 2 to Theorem 1, which is itself a negation of Corollary 3.8 of [1]. Finally Corollary 3.9 of [1] is proved as Theorem 3.

It is well known that C_1 is separable (see, for example, Theorem 4 of [2]). In view of the fact [3, Theorem 3] that 'if a normed space X is separable, then so is $X(\Delta)$', it follows that Theorem 3.7 of [1] is untrue. The mistake lies in the third line of the proof where it is claimed that A is uncountable. In fact, A is countable.

The following theorem provides a Schauder basis for $C_1(\Delta)$ and hence negates Corollary 3.8 of [1].

Theorem 1 *$C_1(\Delta)$ has Schauder basis namely $\{\bar{e}, e, e_1, e_2, \ldots\}$, where $\bar{e} = (0,1,2,3,\ldots)$, $e = (1,1,1,\ldots)$ and $e_k = (0,0,0,\ldots,1,0,0,\ldots)$, 1 is in the kth place and 0 elsewhere for $k = 1,2,\ldots$.*

Proof Let $x = (x_k) \in C_1(\Delta)$ with $\frac{1}{k}\sum_{i=1}^{k} \Delta x_i \to \ell$, *i.e.*, $\lim_k \frac{1}{k}(x_1 - x_{k+1}) = \ell$. We have

$$\left\| x - \left\{ x_1 e - \ell\bar{e} + \sum_{k=1}^{n} \bigl(x_k - x_1 + (k-1)\ell\bigr) e_k \right\} \right\|_\Delta = \sup_{k \ge n} \left| \frac{1}{k}(x_1 - x_{k+1}) - \ell \right| \to 0 \quad \text{as } n \to \infty$$

so that $x = x_1 e - \ell\bar{e} + \sum_k (x_k - x_1 + (k-1)\ell)e_k$. If also we had $x = ae + b\bar{e} + \sum_k a_k e_k$, then

$$s_n = (x_1 - a)e - (\ell + b)\bar{e} + \sum_{k=1}^{n} \bigl(x_k - x_1 + (k-1)\ell - a_k\bigr)e_k \to 0 \quad \text{as } n \to \infty.$$

But for all $n \in \mathbb{N}$, $|x_1 - a - a_1| \leq \|s_n\|_\Delta$, $|\frac{kb-x_{k+1}+x_1+a_{k+1}-a_1}{k}| \leq \|s_n\|_\Delta$ for $1 \leq k \leq n-1$ and $|\frac{-a_1+k(\ell+b)}{k}| \leq \|s_n\|_\Delta$ for all $k \geq n$. Letting $n \to \infty$, we see that $x_1 = a$, $b = -\ell$, $a_1 = 0$ and $a_{k+1} = x_{k+1} - kb - x_1 + a_1 = k\ell - x_1 + x_{k+1}$, for $k \geq 1$, so that the representation $x = x_1 e - \ell\bar{e} + \sum_k (x_k - x_1 + (k-1)\ell)e_k$ is unique. □

The following is a correction of Theorem 3.7 of [1].

Corollary 2 *$C_1(\Delta)$ is separable.*

The result follows from the fact that if a normed space has a Schauder basis, then it is separable.

Finally, we prove a theorem which is in fact Corollary 3.9 of [1].

Theorem 3 *$C_1(\Delta)$ does not have the AK property.*

Proof Let $x = (x_k) = (1, 2, 3, \ldots) \in C_1(\Delta)$. Consider the nth section of the sequence (x_k) written as $x^{[n]} = (1, 2, 3, \ldots, n, 0, 0, \ldots)$. Then

$$\begin{aligned} \left\| x - x^{[n]} \right\|_\Delta &= \left\| (0, 0, 0, \ldots, n+1, n+2, \ldots) \right\|_\Delta \\ &= \sup_{k \geq n} \left| \frac{0 - (k+1)}{k} \right| \\ &= 1 + \frac{1}{n} \end{aligned}$$

which does not tend to 0 as $n \to \infty$. □

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