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Some results on parallel iterative algorithms for strictly pseudocontractive mappings

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Abstract

In this paper, a parallel iterative algorithm with mixed errors is investigated. Strong and weak convergence theorems of common fixed points of a finite family of strictly pseudocontractive mappings are established in a real Banach space.

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1 Introduction and preliminaries

Throughout this paper, we denote by E and E^* a real Banach space and a dual space of E, respectively. Let $\langle\cdot,\cdot\rangle$ denote the pairing between E and E^*. The normalized duality mapping $J : E \to 2^{E^*}$ is defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}, \quad \forall x \in E.$$

In the sequel, we use j to denote the single-valued normalized duality mapping. Let K be a nonempty subset of E and $T : K \to K$ be a mapping. Recall the following.

T is said to be Lipschitz if there exists a positive constant L such that

$$\|Tx - Ty\| \le L\|x - y\|, \quad \forall x, y \in K.$$

T is said to be nonexpansive if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in K.$$

T is said to be strictly pseudocontractive in the terminology of Browder and Petryshyn [1] if there exists $\lambda > 0$ such that

$$\langle Tx - Ty, j(x - y)\rangle \le \|x - y\|^2 - \lambda\|(x - Tx) - (y - Ty)\|^2, \quad \forall x, y \in K, \tag{1.1}$$

for some $j(x - y) \in J(x - y)$. It is clear that the class of strictly pseudocontractive mappings includes the class of nonexpansive mappings as a special case. It is also clear that (1.1) is equivalent to the following:

$$\langle (I - T)x - (I_T)y, j(x - y)\rangle \ge \lambda\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in K. \tag{1.2}$$

We know that strictly pseudocontractive mappings are Lipschitz continuous. Indeed, we find from (1.2) that

$$\|Tx - Ty\| \leq \frac{1+\lambda}{\lambda}\|x-y\|, \quad \forall x, y \in K.$$

T is said to be strongly pseudocontractive if there exists $k \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq k\|x-y\|^2, \quad \forall x, y \in K, \tag{1.3}$$

for some $j(x-y) \in J(x-y)$. T is said to be pseudocontractive if there exists some $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2, \quad \forall x, y \in K. \tag{1.4}$$

T is said to be Lipschitz if there exists a positive constant L such that

$$\|Tx - Ty\| \leq L\|x-y\|, \quad \forall x, y \in K.$$

It is well known that [2] (1.4) is equivalent to the following:

$$\|x-y\| \leq \big\|x - y + s\big[(I-T)x - (I-T)y\big]\big\|, \quad \forall s > 0. \tag{1.5}$$

We remark that the class of strongly pseudocontractive mappings is independent of the class of strictly pseudocontractive mappings. This can be seen from the following examples.

Example 1.1 [3] Take $K = (0,\infty)$ and define $T: K \to K$ by

$$Tx = \frac{x^2}{1+x}.$$

Then T is a strictly pseudocontractive mapping but not a strongly pseudocontractive mapping.

Example 1.2 [3] Take $K = R$ and define $T: K \to K$ by

$$Tx = \begin{cases} 1, & x \in (-\infty,-1), \\ \sqrt{1-(1+x)^2}, & x \in [-1,0), \\ -\sqrt{1-(x-1)^2}, & x \in [0,1], \\ 1, & x \in (1,\infty). \end{cases}$$

Example 1.3 [4] Take $E = R^2$, $B = \{x \in R^2 : \|x\| \leq 1\}$, $B_1 = \{x \in B : \|x\| \leq \frac{1}{2}\}$, $B_2 = \{x \in B : \frac{1}{2} \leq \|x\| \leq 1\}$. If $x = (a,b) \in E$, we define $x^\perp$ to be $(b,-a) \in E$. Define $T: B \to B$ by

$$Tx = \begin{cases} x + x^\perp, & x \in B_1, \\ \frac{x}{\|x\|} - x + x^\perp, & x \in B_2. \end{cases}$$

Then T is a Lipschitz pseudocontractive mapping but not a strictly pseudocontractive mapping.

Let $U = \{x \in E : \|x\| = 1\}$. E is said to be smooth or is said to have a Gâteaux differentiable norm if the limit

$$\lim_{t \to 0} \frac{\|x + ty\| - \|x\|}{t}$$

exists for each $x, y \in U$. E is said to have a uniformly Gâteaux differentiable norm if for each $y \in U$, the limit is attained uniformly for all $x \in U$. E is said to be uniformly smooth or is said to have a uniformly Fréchet differentiable norm if the limit is attained uniformly for $x, y \in U$. It is known that if the norm of E is uniformly Gâteaux differentiable, then the duality mapping J is single valued and uniformly norm to weak* continuous on each bounded subset of E.

In 2001, Xu and Ori [5], in the framework of Hilbert spaces, introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_1, T_2, \dots, T_N\}$ with $\{\alpha_n\}$ a real sequence in $(0,1)$ and an initial point $x_0 \in C$:

$$\begin{aligned}
&x_1 = \alpha_1 x_0 + (1 - \alpha_1) T_1 x_1, \\
&x_2 = \alpha_2 x_1 + (1 - \alpha_2) T_2 x_2, \\
&\dots \\
&x_N = \alpha_N x_{N-1} + (1 - \alpha_N) T_N x_N, \\
&x_{N+1} = \alpha_{N+1} x_N + (1 - \alpha_{N+1}) T_1 x_{N+1}, \\
&\dots
\end{aligned}$$

which can written in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n) T_n x_n, \quad \forall n \geq 1, \tag{1.6}$$

where $T_n = T_{n (\operatorname{mod} N)}$ (here the $\operatorname{mod} N$ takes values in $\{1, 2, \dots, N\}$).

They obtained the following weak convergence theorem.

Theorem XO *Let H be a real Hilbert space, K be a nonempty closed convex subset of H, and $T_i : K \to K$ be a nonexpansive mapping such that $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\{x_n\}$ be defined by* (1.6). *If $\{\alpha_n\}$ is chosen so that $\alpha_n \to 0$ as $n \to \infty$, then $\{x_n\}$ converges weakly to a common fixed point of the family of $\{T_i\}_{i=1}^N$.*

They further remarked that it is yet unclear what assumptions on the mappings and/or the parameters $\{\alpha_n\}$ are sufficient to guarantee the strong convergence of the sequence $\{x_n\}$.

In 2004, Osilike [6] further extended the above results from Hilbert spaces to Banach spaces. To be more precise, he obtain the following results.

Theorem O *Let H be a real Hilbert space, K be a nonempty closed convex subset of H, and $T_i : K \to K$ be a strictly pseudocontractive mapping such that $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\{x_n\}$*

be defined by (1.6). *If* $\{\alpha_n\}$ *is chosen so that* $\alpha_n \to 0$ *as* $n \to \infty$, *then* $\{x_n\}$ *converges weakly to a common fixed point of the family of* $\{T_i\}_{i=1}^N$.

Subsequently, many authors have investigated the fixed point problem of strictly pseudocontractive mappings based on an implicit or non-implicit iterative algorithm in Banach spaces; see [7–32] and the references therein.

In 2007, Acedo and Xu proposed a parallel iterative algorithm for strictly pseudocontractive mappings in the framework of Hilbert spaces. Weak and strong convergence theorems for common fixed points of a family of strictly pseudocontractive mappings were established; see [26] for more details and the reference therein.

In this paper, motivated by the above results, we consider an implicitly parallel iterative algorithm for a finite family of strictly pseudocontractive mappings. Weak and strong convergence theorems are established in the framework of Banach spaces.

In order to prove our main results, we need the following conceptions and lemmas.

Recall that the space E is said to satisfy Opial's condition [33] if, for each sequence $\{x_n\}$ in E, the convergence $x_n \to x$ weakly implies that

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|, \quad \forall y \in E\ (y \neq x).$$

Recall that the mapping $T : K \to K$ is semicompact if any sequence $\{x_n\}$ in K satisfying $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ has a convergent subsequence.

Lemma 1.1 [34] *Let E be a real Banach space, K be a nonempty closed convex subset of E, and $T : K \to K$ be a continuous strongly pseudocontractive mapping. Then T has a unique fixed point in K.*

Lemma 1.2 [11] *Let E be a smooth Banach space and K be a nonempty convex subset of E. Let $r \geq 1$ be some integer. Let $T_i : K \to K$ be a strictly pseudocontractive mapping. Assume that $\bigcap_{i=1}^r F(T_i)$ is not empty. Assume that $\{\mu_i\}_{i=1}^r$ is a positive sequence such that $\sum_{i=1}^r \mu_i = 1$. Then $\bigcap_{i=1}^r F(T_i) = F(\sum_{i=1}^r \mu_i T_i)$.*

Lemma 1.3 [35] *Let $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ be three nonnegative sequences satisfying the following condition*:

$$a_{n+1} \leq (1 + b_n)a_n + c_n, \quad \forall n \geq n_0,$$

where n_0 is some nonnegative integer, $\sum_{n=1}^\infty b_n < \infty$, and $\sum_{n=1}^\infty c_n < \infty$. Then the limit $\lim_{n\to\infty} a_n$ exists.

2 Main results

Theorem 2.1 *Let E be a smooth and reflexive Banach space which also satisfies Opial's condition and K be a nonempty closed convex subset of E. Let $N \geq 1$ be some positive integer. Let $T_m : K \to K$, where $m \in \{1, \dots, N\}$, be a λ_i-strictly pseudocontractive mapping and $\{u_n\}$ be a bounded sequence in K. Let $\{x_n\}_{n=0}^\infty$ be a sequence generated in the following algorithm*:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + \beta_n \sum_{m=1}^N \delta_m T_m x_n + \gamma_n u_n, \quad n \geq 1, \tag{2.1}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, *and* $\{\delta_m\}$ *are real number sequences in* $[0,1]$. *Assume that* $F := \bigcap_{m=1}^{N} F(T_m) \neq \emptyset$, *and the above control sequences satisfy the following restrictions*:

(a) $\sum_{m=1}^{N} \delta_m = \alpha_n + \beta_n + \gamma_n = 1$;

(b) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(c) $0 < a \leq \alpha_n \leq b < 1$, *where a and b are constants.*

Then $\{x_n\}$ *converges weakly to some point in* F.

Proof Put $T := \sum_{m=1}^{N} \delta_m T_m$. We show that T is λ-strictly pseudocontractive mapping, where $\lambda := \min\{\lambda_m : 1 \leq m \leq N\}$. Notice that

$$\begin{aligned}
&\langle Tx - Ty, j(x-y)\rangle \\
&\quad = \delta_1\langle T_1x - T_1y, j(x-y)\rangle + \delta_2\langle T_2x - T_2y, j(x-y)\rangle + \cdots \\
&\qquad + \delta_N\langle T_Nx - T_Ny, j(x-y)\rangle \\
&\quad \leq \delta_1\bigl(\|x-y\|^2 - \lambda_1\bigl\|(I-T_1)x - (I-T_1)y\bigr\|^2\bigr) \\
&\qquad + \delta_2\bigl(\|x-y\|^2 - \lambda_2\bigl\|(I-T_2)x - (I-T_2)y\bigr\|^2\bigr) + \cdots \\
&\qquad + \delta_N\bigl(\|x-y\|^2 - \lambda_N\bigl\|(I-T_N)x - (I-T_N)y\bigr\|^2\bigr) \\
&\quad \leq \|x-y\|^2 - \lambda\bigl(\delta_1\bigl\|(I-T_1)x - (I-T_1)y\bigr\|^2 \\
&\qquad + \delta_2\bigl\|(I-T_2)x - (I-T_2)y\bigr\|^2 + \cdots \\
&\qquad + \delta_N\bigl\|(I-T_N)x - (I-T_N)y\bigr\|^2\bigr) \\
&\quad \leq \|x-y\|^2 - \lambda\bigl\|(I-T)x - (I-T)y\bigr\|^2.
\end{aligned}$$

This proves that T is λ-strictly pseudocontractive mapping. Next, we show that the implicit iterative algorithm (2.1) is well defined for the strictly pseudocontractive mappings. Define a mapping

$$P_n(x) = \alpha_n x_{n-1} + \beta_n \sum_{m=1}^{N} \delta_m T_m x + \gamma_n u_n, \quad \forall n \geq 1.$$

It follows that

$$\begin{aligned}
&\langle P_n(x) - P_n(y), j(x-y)\rangle \\
&\quad = \beta_n\Bigl\langle \sum_{m=1}^{N} \delta_m T_m x - \sum_{m=1}^{N} \delta_m T_m y, j(x-y)\Bigr\rangle \\
&\quad = \beta_n\langle Tx - Ty, j(x-y)\rangle \\
&\quad \leq \beta_n\bigl(\|x-y\|^2 - \lambda\bigl\|(I-T)x - (I-T)y\bigr\|^2\bigr) \\
&\quad \leq \beta_n\|x-y\|.
\end{aligned}$$

This shows that P_n is strongly pseudocontractive. Since strictly pseudocontractive mappings are Lipschitz continuous, we see that P_n is also continuous. In view of Lemma 1.1, we see that P_n has a unique fixed point. This proves that the implicit iterative algorithm (2.1) is well defined. In view of Lemma 1.2, we see that $F = F(\sum_{m=1}^{N} \delta_m F(T_m)) = F(T)$. Fixing $p \in F$, we see that

$$\begin{aligned}
\|x_n - p\|^2 &= \alpha_n \langle x_{n-1} - p, j(x-p)\rangle + \beta_n \left\langle \sum_{m=1}^{N} \delta_m T_m x_n - p, j(x-y) \right\rangle + \gamma_n \langle u_n - p, j(x-y)\rangle \\
&= \alpha_n \langle x_{n-1} - p, j(x-p)\rangle + \beta_n \langle Tx_n - p, j(x-p)\rangle + \gamma_n \langle u_n - p, j(x-p)\rangle \\
&\le \alpha_n \|x_{n-1} - p\| \|x - p\| + \beta_n \|x-p\|^2 - \beta_n \lambda \|x_n - Tx_n\|^2 + \gamma_n \|u_n - p\| \|x - p\|.
\end{aligned}$$

It follows that

$$\|x_n - p\| \le \alpha_n \|x_{n-1} - p\| + \beta_n \|x_n - p\| - \beta_n \lambda \|x_n - Tx_n\|^2 + \gamma_n \|u_n - p\|.$$

This implies, from the restriction (c), that

$$\|x_n - p\| \le \|x_{n-1} - p\| + \gamma_n M - \beta_n \lambda \|x_n - Tx_n\|^2, \tag{2.2}$$

where M is an appropriate constant such that $M \ge \sup_{n\ge1}\{\frac{\|u_n - p\|}{a}\}$. In view of Lemma 1.3, we obtain that $\lim_{n\to\infty} \|x_n - p\|$ exits. Thanks to (2.2), we find from the restrictions (b) and (c) that

$$\lim_{n\to\infty} \|x_n - Tx_n\| = 0. \tag{2.3}$$

Since the space is reflexive and $\{x_n\}$ is bounded, there exists a subsequence $\{x_{n_i}\}$ of the sequence $\{x_n\}$, which weakly converges to some $x \in$. In view of Lemma 1.4, we find that $x \in F(T) = F$.

Finally, we show the sequence $\{x_n\}$ weakly converges to x. Suppose the contrary, then there exists some subsequence $\{x_{n_j}\}$ of the sequence $\{x_n\}$ which weakly converges to $x' \neq x \in C$. It also follows from Lemma 1.4 that $x' \in F$. Since $\lim_{n\to\infty} \|x_n - p\|$ exits for any $p \in F$. Put

$$\lim_{n\to\infty} \|x_n - x\| = d, \qquad \lim_{n\to\infty} \|x_n - x'\| = d'.$$

Since the space satisfies Opial's condition, we see that

$$\begin{aligned}
d &= \lim_{n\to\infty} \|x_n - x\| \\
&= \liminf_{i\to\infty} \|x_{n_i} - x\| \\
&< \liminf_{i\to\infty} \|x_{n_i} - x'\| \\
&= \lim_{n\to\infty} \|x_n - x'\| \\
&= \liminf_{j\to\infty} \|x_{n_j} - x'\| \\
&< \liminf_{j\to\infty} \|x_{n_j} - x\| \\
&= \lim_{n\to\infty} \|x_n - x\| = d.
\end{aligned}$$

This is a contradiction. This shows that $x = x'$. This proves that the sequence $\{x_n\}$ converges weakly to $x \in F$. This completes the proof. □

Corollary 2.2 *Let E be a smooth and reflexive Banach space which also satisfies Opial's condition and let K be a nonempty closed convex subset of E. Let $N \geq 1$ be some positive integer. Let $T_m : K \to K$, where $m \in \{1, \dots, N\}$, be a λ_i-strictly pseudocontractive mapping. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence generated in the following algorithm*:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + (1 - \alpha_n) \sum_{m=1}^{N} \delta_m T_m x_n, \quad n \geq 1,$$

where $\{\alpha_n\}$ and $\{\delta_m\}$ are real number sequences in $[0,1]$. Assume that $F := \bigcap_{m=1}^{N} F(T_m) \neq \emptyset$ and the above control sequences satisfy the following restrictions:
(a) $\sum_{m=1}^{N} \delta_m = 1$;
(b) $0 < a \leq \alpha_n \leq b < 1$, *where a and b are constants.*
Then $\{x_n\}$ converges weakly to some point in F.

In Hilbert spaces, we find from Theorem 2.1 the following.

Corollary 2.3 *Let E be a Hilbert space and K be a nonempty closed convex subset of E. Let $N \geq 1$ be some positive integer. Let $T_m : K \to K$, where $m \in \{1, \dots, N\}$, be a λ_i-strictly pseudocontractive mapping and $\{u_n\}$ be a bounded sequence in K. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence generated in the following algorithm*:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + \beta_n \sum_{m=1}^{N} \delta_m T_m x_n + \gamma_n u_n, \quad n \geq 1,$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, and $\{\delta_m\}$ are real number sequences in $[0,1]$. Assume that $F := \bigcap_{m=1}^{N} F(T_m) \neq \emptyset$ and the above control sequences satisfy the following restrictions:
(a) $\sum_{m=1}^{N} \delta_m = \alpha_n + \beta_n + \gamma_n = 1$;
(b) $\sum_{n=1}^{\infty} \gamma_n < \infty$;
(c) $0 < a \leq \alpha_n \leq b < 1$, *where a and b are constants.*
Then $\{x_n\}$ converges weakly to some point in F.

Next, we give a strong convergence theorem.

Theorem 2.4 *Let E be a smooth and reflexive Banach space and K be a nonempty closed convex subset of E. Let $N \geq 1$ be some positive integer. Let $T_m : K \to K$, where $m \in \{1, \dots, N\}$, be a λ_i-strictly pseudocontractive mapping and $\{u_n\}$ be a bounded sequence in K. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence generated in the following algorithm*:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + \beta_n \sum_{m=1}^{N} \delta_m T_m x_n + \gamma_n u_n, \quad n \geq 1,$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, and $\{\delta_m\}$ are real number sequences in $[0,1]$. Assume that $F := \bigcap_{m=1}^{N} F(T_m) \neq \emptyset$ and the above control sequences satisfy the following restrictions:
(a) $\sum_{m=1}^{N} \delta_m = \alpha_n + \beta_n + \gamma_n = 1$;

(b) $\sum_{n=1}^{\infty} \gamma_n < \infty$;
(c) $0 < a \leq \alpha_n \leq b < 1$, *where* a *and* b *are constants.*
If $\sum_{m=1}^{N} \delta_m T_m$ *is semicompact, then* $\{x_n\}$ *converges strongly to some point in* F.

Proof Since $\sum_{m=1}^{N} \delta_m T_m$ is semicompact, we see that there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \to x^*$. Notice that

$$\left\| x^* - \sum_{m=1}^{N} \delta_m T_m x^* \right\| \leq \| x^* - x_{n_i} \| + \left\| x_{n_i} - \sum_{m=1}^{N} \delta_m T_m x_{n_i} \right\| + \left\| \sum_{m=1}^{N} \delta_m T_m x_{n_i} - \sum_{m=1}^{N} \delta_m T_m x_{n_i} x^* \right\|.$$

Since $\sum_{m=1}^{N} \delta_m T_m$ is Lipschitz continuous, we see from (2.3) that $x^* \in F(\sum_{m=1}^{N} \delta_m T_m) = F$. From Theorem 2.1, we know that $\lim_{n\to\infty} \|x_n - p\|$ exits for any $p \in F$. That is, $\lim_{n\to\infty} \|x_n - x^*\|$ exits. In view of $x_{n_i} \to x^*$, we find that

$$\lim_{n\to\infty} \| x_n - x^* \| = 0.$$

This completes the proof. □

Corollary 2.5 *Let* E *be a smooth and reflexive Banach space and* K *be a nonempty closed convex subset of* E. *Let* $N \geq 1$ *be some positive integer. Let* $T_m : K \to K$, *where* $m \in \{1, \ldots, N\}$, *be a* λ_i*-strictly pseudocontractive mapping. Let* $\{x_n\}_{n=0}^{\infty}$ *be a sequence generated in the following algorithm*:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + (1 - \alpha_n) \sum_{m=1}^{N} \delta_m T_m x_n, \quad n \geq 1,$$

where $\{\alpha_n\}$ *and* $\{\delta_m\}$ *are real number sequences in* $[0,1]$. *Assume that* $F := \bigcap_{m=1}^{N} F(T_m) \neq \emptyset$ *and the above control sequences satisfy the following restrictions*:
(a) $\sum_{m=1}^{N} \delta_m = 1$;
(b) $0 < a \leq \alpha_n \leq b < 1$, *where* a *and* b *are constants.*
If $\sum_{m=1}^{N} \delta_m T_m$ *is semicompact, then* $\{x_n\}$ *converges strongly to some point in* F.

In Hilbert spaces, we find from Theorem 2.1 the following.

Corollary 2.6 *Let* E *be a Hilbert space and* K *be a nonempty closed convex subset of* E. *Let* $N \geq 1$ *be some positive integer. Let* $T_m : K \to K$, *where* $m \in \{1, \ldots, N\}$, *be a* λ_i*-strictly pseudocontractive mapping and* $\{u_n\}$ *be a bounded sequence in* K. *Let* $\{x_n\}_{n=0}^{\infty}$ *be a sequence generated in the following algorithm*:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + \beta_n \sum_{m=1}^{N} \delta_m T_m x_n + \gamma_n u_n, \quad n \geq 1,$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, *and* $\{\delta_m\}$ *are real number sequences in* $[0,1]$. *Assume that* $F := \bigcap_{m=1}^{N} F(T_m) \neq \emptyset$ *and the above control sequences satisfy the following restrictions*:

(a) $\sum_{m=1}^{N} \delta_m = \alpha_n + \beta_n + \gamma_n = 1$;
(b) $\sum_{n=1}^{\infty} \gamma_n < \infty$;
(c) $0 < a \le \alpha_n \le b < 1$, *where a and b are constants*.

If $\sum_{m=1}^{N} \delta_m T_m$ is semicompact, then $\{x_n\}$ converges strongly to some point in F.

Competing interests
The authors declare that they have no competing interests.

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