Shatanawi et al. *Journal of Inequalities and Applications* 2013, 2013:565
http://www.journalofinequalitiesandapplications.com/content/2013/1/565

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

Almost contractive coupled mapping in ordered complete metric spaces

Wasfi Shatanawi[1], Reza Saadati[2] and Choonkil Park[3*]

*Correspondence: baak@hanyang.ac.kr
[3]Department of Mathematics, Research Institute for Natural Sciences, Hanyang University, Seoul, 133-791, Korea
Full list of author information is available at the end of the article

Abstract

In this paper, we introduce the notion of almost contractive mapping $F : X \times X \to X$ with respect to the mapping $g : X \to X$ and establish some existence and uniqueness theorems of a coupled common coincidence point in ordered complete metric spaces. Also, we introduce an example to support our main results. Our results generalize several well-known comparable results in the literature.

MSC: 54H25; 47H10; 34B15

Keywords: coupled fixed point; partially ordered set; mixed monotone property

1 Introduction and preliminaries

The existence and uniqueness theorems of a fixed point in complete metric spaces play an important role in constructing methods for solving problems in differential equations, matrix equations, and integral equations. Furthermore, the fixed point theory is a crucial method in numerical analysis to present a way for solving and approximating the roots of many equations in real analysis. One of the main theorems on a fixed point is the Banach contraction theorem [1]. Many authors generalized the Banach contraction theorem in different metric spaces in different ways. For some works on fixed point theory, we refer the readers to [2–17]. The study of a coupled fixed point was initiated by Bhaskar and Lakshmikantham [18]. Bhaskar and Lakshmikantham [18] obtained some nice results on a coupled fixed point and applied their results to solve a pair of differential equations. For some results on a coupled fixed point in ordered metric spaces, we refer the reader to [18–26].

The following definitions will be needed in the sequel.

Definition 1.1 Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$. The mapping F is said to have the mixed monotone property if $F(x,y)$ is monotone non-decreasing in x and is monotone non-increasing in y, that is, for any

$$x, y \in X, x_1, x_2 \in X, \quad x_1 \preceq x_2 \quad \Rightarrow \quad F(x_1, y) \preceq F(x_2, y)$$

and

$$y_1, y_2 \in X, \quad y_1 \preceq y_2 \quad \Rightarrow \quad F(x, y_1) \succeq F(x, y_2).$$

Definition 1.2 We call an element $(x,y) \in X \times X$ a coupled fixed point of the mapping $F : X \times X \to X$ if

$$F(x,y) = x \quad \text{and} \quad F(y,x) = y.$$

Definition 1.3 [20] Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \to X$. The mapping F is said to have the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad g(x_1) \preceq g(x_2) \quad \Rightarrow \quad F(x_1, y) \preceq F(x_2, y) \tag{1}$$

and

$$y_1, y_2 \in X, \quad g(y_1) \preceq g(y_2) \quad \Rightarrow \quad F(x, y_1) \succeq F(x, y_2). \tag{2}$$

Definition 1.4 An element $(x,y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x,y) = g(x) \quad \text{and} \quad F(y,x) = g(y).$$

The main results of Bhaskar and Lakshmikantham in [18] are the following.

Theorem 1.1 [18] *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X,d) is a complete metric space. Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists a $k \in [0,1)$ with*

$$d\bigl(F(x,y), F(u,v)\bigr) \le \frac{k}{2}\bigl[d(x,u) + d(y,v)\bigr] \quad \forall x \succeq u \text{ and } y \preceq v.$$

If there exist two elements $x_0, y_0 \in X$ with

$$x_0 \preceq F(x_0, y_0) \quad \text{and} \quad y_0 \succeq F(y_0, x_0),$$

then there exist $x, y \in X$ such that

$$x = F(x,y) \quad \text{and} \quad y = F(y,x).$$

Theorem 1.2 [18] *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X,d) is a complete metric space. Assume that X has the following property:*

(i) *if a nondecreasing sequence $\{x_n\}$ in X converges to $x \in X$, then $x_n \preceq x$ for all n,*

(ii) *if a nonincreasing sequence $\{y_n\}$ in X converges to $y \in X$, then $y_n \succeq y$ for all n.*

Let $F : X \times X \to X$ be a mapping having the mixed monotone property on X. Assume that there exists $k \in [0,1)$ with

$$d\bigl(F(x,y), F(u,v)\bigr) \le \frac{k}{2}\bigl[d(x,u) + d(y,v)\bigr] \quad \forall x \succeq u \text{ and } y \preceq v.$$

If there exist two elements $x_0, y_0 \in X$ with

$$x_0 \preceq F(x_0, y_0) \quad \textit{and} \quad y_0 \succeq F(y_0, x_0),$$

then there exist $x, y \in X$ such that

$$x = F(x, y) \quad \textit{and} \quad y = F(y, x).$$

Definition 1.5 Let (X, d) be a metric space and $F : X \times X \to X$ and $g : X \to X$ be mappings. We say that F and g commute if

$$F\big(g(x), g(y)\big) = g\big(F(x, y)\big)$$

for all $x, y \in X$.

Nashine and Shatanawi [22] proved the following coupled coincidence point theorems.

Theorem 1.3 [22] *Let $(X, d, \preceq)$ be an ordered metric space. Let $F : X \times X \to X$ and $g : X \to X$ be mappings such that F has the mixed g-monotone property on X such that there exist two elements $x_0, y_0 \in X$ with $g(x_0) \preceq F(x_0, y_0)$ and $g(y_0) \succeq F(y_0, x_0)$. Suppose that there exist non-negative real numbers α, β, L with $\alpha + \beta < 1$ such that*

$$\begin{aligned} d\big(F(x,y), F(u,v)\big) \leq{} & \alpha \min\big\{d\big(F(x,y), g(x)\big), d\big(F(u,v), g(x)\big)\big\} \\ & + \beta \min\big\{d\big(F(x,y), g(u)\big), d\big(F(u,v), g(u)\big)\big\} \\ & + L \min\big\{d\big(F(x,y), g(u)\big), d\big(F(u,v), g(x)\big)\big\} \end{aligned} \tag{3}$$

for all $(x, y), (u, v) \in X \times X$ with $g(x) \preceq g(u)$ and $g(y) \succeq g(v)$. Further suppose that $F(X \times X) \subseteq g(X)$ and $g(X)$ is a complete subspace of X. Also suppose that X satisfies the following properties:

(i) *if a nondecreasing sequence $\{x_n\}$ in X converges to $x \in X$, then $x_n \preceq x$ for all n,*
(ii) *if a nonincreasing sequence $\{y_n\}$ in X converges to $y \in X$, then $y_n \succeq y$ for all n.*

Then there exist $x, y \in X$ such that

$$F(x, y) = g(x) \quad \textit{and} \quad F(y, x) = g(y),$$

that is, F and g have a coupled coincidence point $(x, y) \in X \times X$.

Theorem 1.4 [22] *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ and $g : X \to X$ be mappings such that F has the mixed g-monotone property on X such that there exist two elements $x_0, y_0 \in X$ with $g(x_0) \preceq F(x_0, y_0)$ and $g(y_0) \succeq F(y_0, x_0)$. Suppose that there exist non-negative real numbers α, β, L with $\alpha + \beta < 1$ such that*

$$\begin{aligned} d\big(F(x,y), F(u,v)\big) \leq{} & \alpha \min\big\{d\big(F(x,y), g(x)\big), d\big(F(u,v), g(x)\big)\big\} \\ & + \beta \min\big\{d\big(F(x,y), g(u)\big), d\big(F(u,v), g(u)\big)\big\} \\ & + L \min\big\{d\big(F(x,y), g(u)\big), d\big(F(u,v), g(x)\big)\big\} \end{aligned} \tag{4}$$

for all $(x,y),(u,v) \in X \times X$ *with* $g(x) \preceq g(u)$ *and* $g(y) \succeq g(v)$. *Further suppose that* $F(X \times X) \subseteq g(X)$, g *is continuous nondecreasing and commutes with* F, *and also suppose that either*

(a) F *is continuous, or*

(b) X *has the following property*:

  (i) *if a nondecreasing sequence* $\{x_n\}$ *in* X *converges to* $x \in X$, *then* $x_n \preceq x$ *for all* n,

  (ii) *if a nonincreasing sequence* $\{y_n\}$ *in* X *converges to* $y \in X$, *then* $y_n \succeq y$ *for all* n.

Then there exist $x, y \in X$ *such that*

$$F(x,y) = g(x) \quad \textit{and} \quad F(y,x) = g(y),$$

that is, F *and* g *have a coupled coincidence point* $(x,y) \in X \times X$.

Berinde [27–30] initiated the concept of almost contractions and studied many interesting fixed point theorems for a Ćirić strong almost contraction. So, it is fundamental to recall the following definition.

Definition 1.6 [27] A single-valued mapping $f : X \times X$ is called a Ćirić strong almost contraction if there exist a constant $\alpha \in [0,1)$ and some $L \geq 0$ such that

$$d(fx,fy) \leq \alpha M(x,y) + Ld(y,fx)$$

for all $x, y \in X$, where

$$M(x,y) = \max\left\{d(x,y), d(x,fx), d(y,fy), \frac{d(x,fy) + d(y,fx)}{2}\right\}.$$

The aim of this paper is to introduce the notion of almost contractive mapping $F : X \times X \to X$ with respect to the mapping $g : X \to X$ and present some uniqueness and existence theorems of coupled fixed and coincidence point. Our results generalize Theorems 1.1-1.4.

2 Main theorems

We start with the following definition.

Definition 2.1 Let $(X, d, \preceq)$ be an ordered metric space. We say that the mapping $F : X \times X \to X$ is an almost contractive mapping with respect to the mapping $g : X \to X$ if there exist a real number $\alpha \in [0,1)$ and a nonnegative number L such that

$$\begin{aligned} &d\big(F(x,y),F(u,v)\big) \\ &\quad\leq \alpha \max\big\{d\big(g(x),g(u)\big), d\big(g(y),g(v)\big), d\big(F(x,y),g(x)\big), d\big(F(u,v),g(u)\big)\big\} \\ &\qquad + L\min\big\{d\big(F(x,y),g(u)\big), d\big(F(u,v),g(x)\big)\big\} \end{aligned} \tag{5}$$

for all $(x,y),(u,v) \in X \times X$ with $g(x) \preceq g(u)$ and $g(y) \succeq g(v)$.

Theorem 2.1 *Let* $(X, d, \preceq)$ *be an ordered metric space. Let* $F : X \times X \to X$ *and* $g : X \to X$ *be mappings such that*

(1) F *is an almost contractive mapping with respect to* g.

(2) *F has the mixed g-monotone property on X.*
(3) *There exist two elements $x_0, y_0 \in X$ with $g(x_0) \preceq F(x_0, y_0)$ and $g(y_0) \succeq F(y_0, x_0)$.*
(4) *$F(X \times X) \subseteq g(X)$ and $g(X)$ is a complete subspace of X.*

Also, suppose that X satisfies the following properties:
(i) *if a nondecreasing sequence $\{x_n\}$ in X converges to $x \in X$, then $x_n \preceq x$ for all n,*
(ii) *if a nonincreasing sequence $\{y_n\}$ in X converges to $y \in X$, then $y_n \succeq y$ for all n.*

Then there exist $x, y \in X$ such that

$$F(x,y) = g(x) \quad \text{and} \quad F(y,x) = g(y),$$

that is, F and g have a coupled coincidence point $(x, y) \in X \times X$.

Proof Let $x_0, y_0 \in X$ be such that $g(x_0) \preceq F(x_0, y_0)$ and $g(y_0) \succeq F(y_0, x_0)$. Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $g(x_1) = F(x_0, y_0)$ and $g(y_1) = F(y_0, x_0)$.

In the same way, we construct $g(x_2) = F(x_1, y_1)$ and $g(y_2) = F(y_1, x_1)$.

Continuing in this way, we construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$g(x_{n+1}) = F(x_n, y_n) \quad \text{and} \quad g(y_{n+1}) = F(y_n, x_n) \quad \forall n \in \mathbb{N} \cup \{0\}. \tag{6}$$

Since F has the mixed g-monotone property, by induction we may show that

$$g(x_0) \preceq g(x_1) \preceq g(x_2) \preceq \cdots \preceq g(x_{n+1}) \preceq \cdots$$

and

$$g(y_0) \succeq g(y_1) \succeq g(y_2) \succeq \cdots \succeq g(y_{n+1}) \succeq \cdots.$$

If $(g(x_{n+1}), g(y_{n+1})) = (g(x_n), g(y_n))$ for some $n \in \mathbb{N}$, then $F(x_n, y_n) = g(x_n)$ and $F(y_n, x_n) = g(y_n)$, that is, (x_n, y_n) is a coincidence point of F and g. So we may assume that $(g(x_{n+1}), g(y_{n+1})) \neq (g(x_n), g(y_n))$ for all $n \in \mathbb{N}$. Let $n \in \mathbb{N}$. Since $g(x_n) \succeq g(x_{n-1})$ and $g(y_n) \preceq g(y_{n-1})$, from (5) and (6), we have

$$\begin{aligned}
&d\bigl(g(x_n), g(x_{n+1})\bigr) \\
&\quad = d\bigl(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\bigr) \\
&\quad \leq \alpha \max\bigl\{d\bigl(g(x_{n-1}), g(x_n)\bigr), d\bigl(g(y_{n-1}), g(y_n)\bigr), d\bigl(F(x_n, y_n), g(x_n)\bigr), \\
&\qquad d\bigl(F(x_{n-1}, y_{n-1}), g(x_{n-1})\bigr)\bigr\} + L \min\bigl\{d\bigl(F(x_n, y_n), g(x_{n-1})\bigr), d\bigl(F(x_{n-1}, y_{n-1}), g(x_n)\bigr)\bigr\} \\
&\quad = \alpha \max\bigl\{d\bigl(g(x_{n-1}), g(x_n)\bigr), d\bigl(g(y_{n-1}), g(y_n)\bigr), d\bigl(g(x_{n+1}), g(x_n)\bigr), d\bigl(g(x_n), g(x_{n-1})\bigr)\bigr\} \\
&\qquad + L \min\bigl\{d\bigl(g(x_{n+1}), g(x_{n-1})\bigr), d\bigl(g(x_n), g(x_n)\bigr)\bigr\} \\
&\quad = \alpha \max\bigl\{d\bigl(g(x_{n-1}), g(x_n)\bigr), d\bigl(g(y_{n-1}), g(y_n)\bigr), d\bigl(g(x_{n+1}), g(x_n)\bigr)\bigr\}.
\end{aligned}$$

If $\max\{d(g(x_{n-1}), g(x_n)), d(g(y_{n-1}), g(y_n)), d(g(x_{n+1}), g(x_n))\} = d(g(x_{n+1}), g(x_n))$, then $d(g(x_{n+1}), g(x_n)) \leq \alpha d(g(x_{n+1}), g(x_n))$ and hence $d(g(x_{n+1}), g(x_n)) = 0$. Thus $d(g(x_{n-1}), g(x_n)) =$

$d(g(y_{n-1}), g(y_n)) = 0$. Therefore $d(g(x_{n-1}), g(y_{n-1})) = d(g(x_n), g(y_n))$, a contradiction. Thus

$$\begin{aligned}&\max\{d(g(x_{n-1}), g(x_n)), d(g(y_{n-1}), g(y_n)), d(g(x_{n+1}), g(x_n))\}\\&\quad= \max\{d(g(x_{n-1}), g(x_n)), d(g(y_{n-1}), g(y_n))\}.\end{aligned}$$

Therefore

$$d(g(x_{n+1}), g(x_n)) \le \alpha \max\{d(g(x_{n-1}), g(x_n)), d(g(y_{n-1}), g(y_n))\}. \tag{7}$$

Similarly, we may show that

$$d(g(y_n), g(y_{n+1})) \le \alpha \max\{d(g(x_{n-1}), g(x_n)), d(g(y_{n-1}), g(y_n))\}. \tag{8}$$

From (7) and (8), we have

$$\begin{aligned}&\max\{d(g(x_{n+1}), g(x_n)), d(g(y_n), g(y_{n+1}))\}\\&\quad\le \alpha \max\{d(g(x_{n-1}), g(x_n)), d(g(y_{n-1}), g(y_n))\}.\end{aligned} \tag{9}$$

Repeating (9) n-times, we get

$$\begin{aligned}&\max\{d(g(x_{n+1}), g(x_n)), d(g(y_n), g(y_{n+1}))\}\\&\quad\le \alpha^n \max\{d(g(x_0), g(x_1)), d(g(y_0), g(y_1))\}.\end{aligned} \tag{10}$$

Now, we shall prove that $\{g(x_n)\}$ and $\{g(y_n)\}$ are Cauchy sequences in $g(X)$.

For each $m \ge n$, we have

$$\begin{aligned}&d(g(x_m), g(x_n))\\&\quad\le d(g(x_n), g(x_{n+1})) + d(g(x_{n+1}), g(x_{n+2})) + \cdots\\&\qquad+ d(g(x_{m-1}), g(x_m))\\&\quad\le \alpha^n \max\{d(g(x_0), g(x_1)), d(g(y_0), g(y_1))\} + \cdots\\&\qquad+ \alpha^{m-1} \max\{d(g(x_0), g(x_1)), d(g(y_0), g(y_1))\}\\&\quad\le \frac{\alpha^n}{1-\alpha} \max\{d(g(x_0), g(x_1)), d(g(y_0), g(y_1))\}.\end{aligned}$$

Letting $n, m \to +\infty$ in the above inequalities, we get that $\{g(x_n)\}$ is a Cauchy sequence in $g(X)$. Similarly, we may show that $\{g(y_n)\}$ is a Cauchy sequence in $g(X)$. Since $g(X)$ is a complete subspace of X, there exists $(x, y) \in X \times X$ such that $g(x_n) \to g(x)$ and $g(y_n) \to g(y)$. Since $\{g(x_n)\}$ is a non-decreasing sequence and $g(x_n) \to g(x)$ and as $\{g(y_n)\}$ is a non-increasing sequence and $g(y_n) \to g(y)$, by the assumption we have $g(x_n) \preceq g(x)$ and $g(y_n) \succeq g(y)$ for all n. Since

$$\begin{aligned}&d(g(x_{n+1}), F(x, y))\\&\quad= d(F(x_n, y_n), F(x, y))\end{aligned}$$

$$\begin{aligned}&\le \alpha \max\bigl\{d\bigl(g(x_n),g(x)\bigr),d\bigl(g(y_n),g(y)\bigr),d\bigl(g(x_{n+1}),g(x_n)\bigr),d\bigl(F(x,y),g(x)\bigr)\bigr\}\\&\quad + L\min\bigl\{d\bigl(g(x_{n+1}),g(x)\bigr),d\bigl(F(x,y),g(x_n)\bigr)\bigr\}.\end{aligned}$$

Letting $n \to \infty$ in the above inequality, we get $d(g(x),F(x,y)) = 0$. Hence $g(x) = F(x,y)$. Similarly, one can show that $g(y) = F(y,x)$. Thus we proved that F and g have a coupled coincidence point. □

Theorem 2.2 *Let $(X,\preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X,d) is a complete metric space. Let $F : X \times X \to X$ and $g : X \to X$ be mappings such that*

(1) *F is an almost contractive mapping with respect to g.*
(2) *F has the mixed g-monotone property on X.*
(3) *There exist two elements $x_0, y_0 \in X$ with $g(x_0) \preceq F(x_0,y_0)$ and $g(y_0) \succeq F(y_0,x_0)$.*
(4) *$F(X \times X) \subseteq g(X)$.*
(5) *g is continuous nondecreasing and commutes with F.*

Also suppose that either

(a) *F is continuous, or*
(b) *X has the following property:*
 (i) *if a nondecreasing sequence $\{x_n\}$ in X converges to $x \in X$, then $x_n \preceq x$ for all n,*
 (ii) *if a nonincreasing sequence $\{y_n\}$ in X converges to $y \in X$, then $y_n \succeq y$ for all n.*

Then there exist $x,y \in X$ such that

$$F(x,y) = g(x) \quad \textit{and} \quad F(y,x) = g(y),$$

that is, F and g have a coupled coincidence point $(x,y) \in X \times X$.

Proof As in the proof of Theorem 2.1, we construct two Cauchy sequences (gx_n) and (gy_n) in X such that (gx_n) is a nondecreasing sequence in X and (gy_n) is a nonincreasing sequence in X. Since X is a complete metric space, there is $(x,y) \in X \times X$ such that $gx_n \to x$ and $gy_n \to y$. Since g is continuous, we have $g(gx_n) \to gx$ and $g(gy_n) \to gy$.

Suppose that (a) holds. Since F is continuous, we have $F(gx_n,gy_n) \to F(x,y)$ and $F(gy_n,gx_n) \to F(y,x)$. Also, since g commutes with F and g is continuous, we have $F(gx_n,gy_n) = gF(x_n,y_n) = g(gx_{n+1}) \to gx$ and $F(gy_n,gx_n) = gF(y_n,x_n) = g(gy_{n+1}) \to gy$. By uniqueness of limit, we get $gx = F(x,y)$ and $gy = F(y,x)$.

Second, suppose that (b) holds. Since $g(x_n)$ is a nondecreasing sequence such that $g(x_n) \to x$, $g(y_n)$ is a nonincreasing sequence such that $g(y_n) \to y$, and g is a nondecreasing function, we get that $g(gx_n) \preceq gx$ and $g(gy_n) \succeq g(y)$ hold for all $n \in \mathbb{N}$. By (5), we have

$$\begin{aligned}&d\bigl(g(gx_{n+1}),F(x,y)\bigr)\\&\quad = d\bigl(F(gx_n,gy_n),F(x,y)\bigr)\\&\quad \le \alpha \max\bigl\{d\bigl(g(gx_n),g(x)\bigr),d\bigl(g(gy_n),g(y)\bigr),d\bigl(g(gx_{n+1}),g(gx_n)\bigr),d\bigl(F(x,y),g(x)\bigr)\bigr\}\\&\qquad + L\min\bigl\{d\bigl(g(gx_{n+1}),g(x)\bigr),d\bigl(F(x,y),g(gx_n)\bigr)\bigr\}.\end{aligned}$$

Letting $n \to +\infty$, we get $d(g(x),F(x,y)) = 0$ and hence $g(x) = F(x,y)$. Similarly, one can show that $g(y) = F(y,x)$. Thus (x,y) is a coupled coincidence point of F and g. □

Corollary 2.1 *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ be a mapping such that F has the mixed monotone property on X such that there exist two elements $x_0, y_0 \in X$ with $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$. Suppose that there exist a real number $\alpha \in [0,1)$ and a nonnegative number L such that*

$$d\bigl(F(x,y),F(u,v)\bigr) \le \alpha \max\bigl\{d(x,u),d(y,v),d\bigl(F(x,y),x\bigr),d\bigl(F(u,v),u\bigr)\bigr\} \\ + L\min\bigl\{d\bigl(F(x,y),u\bigr),d\bigl(F(u,v),x\bigr)\bigr\} \tag{11}$$

for all $(x,y),(u,v) \in X \times X$ with $x \preceq u$ and $y \succeq v$ and also suppose that either

(a) *F is continuous, or*

(b) *X has the following property*:

 (i) *if a nondecreasing sequence $\{x_n\}$ in X converges to $x \in X$, then $x_n \preceq x$ for all n,*

 (ii) *if a nonincreasing sequence $\{y_n\}$ in X converges to $y \in X$, then $y_n \succeq y$ for all n,*

then there exist $x, y \in X$ such that

$$F(x,y) = x \quad \text{and} \quad F(y,x) = y,$$

that is, F has a coupled fixed point $(x,y) \in X \times X$.

Proof Follows from Theorem 2.2 by taking $g = I$, the identity mapping. □

Let $(X, \preceq)$ be a partially ordered set. Then we define a partial order $\preceq$ on the product space $X \times X$ as follows:

$$\text{for } (x,y),(u,v) \in X \times X, \quad (u,v) \preceq (x,y) \quad \Leftrightarrow \quad x \succeq u, \qquad y \preceq v.$$

Now, we prove some uniqueness theorem of a coupled common fixed point of mappings $F : X \times X \to X$ and $g : X \to X$.

Theorem 2.3 *In addition to the hypotheses of Theorem* 2.1, *suppose that $L = 0$, $\alpha < \frac{1}{2}$, F and g commute and for every $(x,y),(y^*,x^*) \in X \times X$, there exists $(u,v) \in X \times X$ such that $(F(u,v),F(v,u))$ is comparable to $(F(x,y),F(y,x))$ and $(F(x^*,y^*),F(y^*,x^*))$. Then F and g have a unique coupled common fixed point, that is, there exists a unique $(x,y) \in X \times X$ such that*

$$x = g(x) = F(x,y) \quad \text{and} \quad y = g(y) = F(y,x).$$

Proof The existence of coupled coincidence points of F and g follows from Theorem 2.1. To prove the uniqueness, let (x,y) and (x^*,y^*) be coupled coincidence points of F and g; that is, $g(x) = F(x,y)$, $g(y) = F(y,x)$, $g(x^*) = F(x^*,y^*)$ and $g(y^*) = F(y^*,x^*)$. Now, we prove that

$$g(x) = g\bigl(x^*\bigr) \quad \text{and} \quad g(y) = g\bigl(y^*\bigr). \tag{12}$$

By the hypotheses, there exists $(u,v) \in X \times X$ such that $(F(u,v),F(v,u))$ is comparable to $(F(x,y),F(y,x))$ and $(F(x^*,y^*),F(y^*,x^*))$. Put $u_0 = u$, $v_0 = v$. Let $u_1, v_1 \in X$ be such that

$g(u_1) = F(u_0, v_0)$ and $g(v_1) = F(v_0, u_0)$. Then as a similar proof of Theorem 2.1, we construct two sequences $\{g(u_n)\}$, $\{g(v_n)\}$ in $g(X)$, where $g(u_{n+1}) = F(u_n, v_n)$ and $g(v_{n+1}) = F(v_n, u_n)$ for all $n \in \mathbb{N}$. Further, set $x_0 = x$, $y_0 = y$, $x_0^* = x^*$, $y_0^* = y^*$. Define the sequences $\{g(x_n)\}$, $\{g(y_n)\}$ in the following way: define $gx_1 = F(x_0, y_0) = F(x, y)$ and $gy_1 = F(y_0, x_0) = F(y, x)$. Also, define $gx_2 = F(x_1, y_1)$ and $gy_2 = F(y_1, x_1)$. For each $n \in \mathbb{N}$, define $gx_{n+1} = F(x_n, y_n)$ and $gy_{n+1} = F(y_n, x_n)$. In the same way, we define the sequences $\{g(x_n^*)\}$, $\{g(y_n^*)\}$. Now, we prove that

$$g(x_n) = F(x, y) = g(x) \quad \text{and} \quad g(y_n) = F(y, x) = g(y).$$

Since (x, y) is a coupled coincidence point of F and g, we have $F(x, y) = g(x)$ and $F(y, x) = g(y)$. Thus $g(x_1) = F(x_0, y_0) = F(x, y) = g(x)$ and $g(y_1) = F(y_0, x_0) = F(y, x) = g(y)$. Therefore $g(x_1) \preceq g(x)$, $g(x) \preceq g(x_1)$, $g(y_1) \preceq g(y)$ and $g(y) \preceq g(y_1)$. Since F is monotone g-non-decreasing on its first argument, $g(x_1) \preceq g(x)$, and $g(x) \preceq g(x_1)$, we have $F(x_1, y_1) \preceq F(x, y_1)$ and $F(x, y_1) \preceq F(x_1, y_1)$. Therefore,

$$F(x_1, y_1) = F(x, y_1). \tag{13}$$

Also, since F is monotone g-non-increasing on its second argument, $g(y_1) \preceq g(y)$ and $g(y) \preceq g(y_1)$, we have $F(x, y) \preceq F(x, y_1)$ and $F(x, y_1) \preceq F(x, y)$. Therefore,

$$F(x, y) = F(x, y_1). \tag{14}$$

From (13) and (14), we have

$$g(x_2) = F(x_1, y_1) = F(x, y) = g(x).$$

Similarly, we may show that

$$g(y_2) = F(y_1, x_1) = F(y, x) = g(y).$$

Note that $g(x_2) \preceq g(x)$, $g(x) \preceq g(x_2)$, $g(y_2) \preceq g(y)$ and $g(y) \preceq g(y_2)$. Since F is monotone g-non-decreasing on its first argument, $g(x_2) \preceq g(x)$, and $g(x) \preceq g(x_2)$, we have $F(x_2, y_2) \preceq F(x, y_2)$ and $F(x, y_2) \preceq F(x_2, y_2)$. Therefore,

$$F(x_2, y_2) = F(x, y_2). \tag{15}$$

Also, since F is monotone g-non-increasing on its second argument, $g(y_2) \preceq g(y)$ and $g(y) \preceq g(y_2)$, we have $F(x, y) \preceq F(x, y_2)$ and $F(x, y_2) \preceq F(x, y)$. Therefore,

$$F(x, y) = F(x, y_2). \tag{16}$$

From (15) and (16), we have

$$g(x_3) = F(x_2, y_2) = F(x, y) = g(x).$$

Similarly, we may show that

$$g(y_3) = F(y_2, x_2) = F(y, x) = g(y).$$

Continuing in the same way, we have that

$$g(x_n) = F(x, y) = g(x) \quad \text{and} \quad g(y_n) = F(y, x) = g(y)$$

hold for all $n \in \mathbb{N}$. Similarly, we can show that

$$g(x_n^*) = F(x^*, y^*) = g(x^*) \quad \text{and} \quad g(y_n^*) = F(y^*, x^*) = g(y^*) \quad \forall n \in \mathbb{N}$$

hold for all $n \in \mathbb{N}$. Since

$$\big(F(x,y), F(y,x)\big) = \big(g(x_1), g(y_1)\big) = \big(g(x), g(y)\big)$$

and

$$\big(F(u,v), F(v,u)\big) = \big(g(u_1), g(v_1)\big)$$

are comparable, $g(x) \preceq g(u_1)$ and $g(y) \succeq g(v_1)$. Since F has the mixed g-monotone property of X, we have $g(x) \preceq g(u_n)$ and $g(y) \succeq g(v_n)$ for all $n \in \mathbb{N}$. Also, since $(g(x^*), g(y^*))$ and $(F(u,v), F(v,u)) = (g(u_1), g(v_1))$ are comparable, and F has the g-monotone property, then we can show that for $n \in \mathbb{N}$, we have that $(g(x^*), g(y^*))$ and $(g(u_n), g(v_n))$ are comparable. Now, if $(g(x), g(y)) = (g(u_k), g(v_k))$ for some $k \in \mathbb{N}$ or $(g(x^*), g(y^*)) = (g(u_k), g(v_k))$ for some $k \in \mathbb{N}$, then $(g(x), g(y))$ and $(g(x^*), g(y^*))$ are comparable, say $g(x) \preceq g(x^*)$ and $g(y) \succeq g(y^*)$. Thus from (5) we have

$$\begin{aligned}
&d\big(g(x), g(x^*)\big) \\
&\quad = d\big(F(x,y), F(x^*, y^*)\big) \\
&\quad \le \alpha \max\big\{d\big(g(x), g(x^*)\big), d\big(g(y), g(y^*)\big), d\big(F(x,y), g(x)\big), d\big(F(x^*, y^*), g(x^*)\big)\big\} \\
&\quad = \alpha \max\big\{d\big(g(x), g(x^*)\big), d\big(g(y), g(y^*)\big)\big\}
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
&d\big(g(y^*), g(y)\big) \\
&\quad = d\big(F(y^*, x^*), F(y, x)\big) \\
&\quad \le \alpha \max\big\{d\big(g(y), g(y^*)\big), d\big(g(x), g(x^*)\big), d\big(F(y^*, x^*), g(y^*)\big), d\big(F(y,x), g(y)\big)\big\} \\
&\quad = \alpha \max\big\{d\big(g(y), g(y^*)\big), d\big(g(x), g(y)\big)\big\}.
\end{aligned} \tag{18}$$

From (17) and (18), we have

$$\max\big\{d\big(g(x), g(x^*)\big), d\big(g(y), g(y^*)\big)\big\} \le \alpha \max\big\{d\big(g(y), g(y^*)\big), d\big(g(x), g(y)\big)\big\}.$$

Since $\alpha < 1$, we have $d(g(x), g(x^*)) = 0$ and $d(g(y), g(y^*)) = 0$. Therefore (12) is satisfied. Now, suppose that $(g(x), g(y)) \neq (g(u_n), g(v_n))$ for all $n \in \mathbb{N}$ and $(g(x^*), g(y^*)) \neq (g(u_n), g(v_n))$ for all $n \in \mathbb{N}$. Let $n \in \mathbb{N}$. Since $g(x) \preceq g(u_n)$ and $g(y) \succeq g(v_n)$, then from (5) we have

$$\begin{aligned}
&d\big(g(x), g(u_{n+1})\big) \\
&\quad = d\big(F(x,y), F(u_n, v_n)\big) \\
&\quad \leq \alpha \max\big\{d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big), d\big(F(x,y), g(x)\big), d\big(F(u_n, v_n), g(u_n)\big)\big\} \\
&\quad = \alpha \max\big\{d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big), d\big(g(u_{n+1}), g(u_n)\big)\big\} \\
&\quad \leq \alpha \max\big\{d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big), d\big(g(u_{n+1}), g(x)\big) + d\big(g(x), g(u_n)\big)\big\} \\
&\quad \leq \alpha \max\big\{d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big), 2d\big(g(u_{n+1}), g(x)\big), 2d\big(g(x), g(u_n)\big)\big\} \\
&\quad = \alpha \max\big\{2d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big), 2d\big(g(u_{n+1}), g(x)\big)\big\}.
\end{aligned}$$

If

$$\max\big\{2d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big), 2d\big(g(u_{n+1}), g(x)\big)\big\} = 2d\big(g(u_{n+1}), g(x)\big)$$

then $d(g(u_{n+1}), g(x)) \leq 2\alpha d(g(u_{n+1}), g(x))$. Since $2\alpha < 1$, we have $d(g(u_{n+1}), g(x)) = 0$. Therefore $d(g(x), g(u_n)) = 0$ and $d(g(y), g(v_n)) = 0$ and hence $(g(x), g(y)) = (g(u_n), g(v_n))$, a contradiction. Thus

$$\begin{aligned}
d\big(g(x), g(u_{n+1})\big) &\leq \alpha \max\big\{2d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big)\big\} \\
&\leq 2\alpha \max\big\{d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big)\big\}.
\end{aligned} \tag{19}$$

Similarly, we may show that

$$d\big(g(v_{n+1}), g(y)\big) \leq 2\alpha \max\big\{d\big(g(x), g(u_n)\big), d\big(g(y), g(v_n)\big)\big\}. \tag{20}$$

From (19) and (20), we have

$$\begin{aligned}
&\max\big\{d\big(g(x), g(u_{n+1})\big), d\big(g(v_{n+1}), g(y)\big)\big\} \\
&\quad \leq 2\alpha \max\big\{d\big(g(v_n), g(y)\big), d\big(g(u_n), g(x)\big), d(g(u_{n+1})\big\}.
\end{aligned} \tag{21}$$

By repeating (21) n-times, we have

$$\begin{aligned}
&\max\big\{d\big(g(x), g(u_{n+1})\big), d\big(g(v_{n+1}), g(y)\big)\big\} \\
&\quad \leq 2\alpha \max\big\{d\big(g(v_n), g(y)\big), d\big(g(u_n), g(x)\big)\big\} \\
&\quad \vdots \\
&\quad \leq (2\alpha)^{n+1} \max\big\{d\big(g(x), g(u_0)\big), d\big(g(v_0), g(y)\big)\big\}.
\end{aligned}$$

Letting $n \to +\infty$ in the above inequalities, we get that

$$\lim_{n\to} \max\big\{d\big(g(x), g(u_{n+1})\big), d\big(g(v_{n+1}), g(y)\big)\big\} = 0.$$

Hence

$$\lim_{n\to\infty} d\bigl(g(x), g(u_{n+1})\bigr) = 0 \tag{22}$$

and

$$\lim_{n\to\infty} d\bigl(g(y), g(v_{n+1})\bigr) = 0. \tag{23}$$

Similarly, we may show that

$$\lim_{n\to\infty} d\bigl(g(x), g(u_{n+1})\bigr) = 0 \tag{24}$$

and

$$\lim_{n\to\infty} d\bigl(g(y), g(v_{n+1})\bigr) = 0. \tag{25}$$

By the triangle inequality, (22), (23), (24) and (25),

$$\begin{aligned} &d\bigl(g(x), g\bigl(x^*\bigr)\bigr) \le d\bigl(g(x), g(u_{n+1})\bigr) + d\bigl(g\bigl(x^*\bigr), g(u_{n+1})\bigr) \to 0 \quad \text{as } n \to \infty, \\ &d\bigl(g(y), g\bigl(y^*\bigr)\bigr) \le d\bigl(g(y), g(v_{n+1})\bigr) + d\bigl(g\bigl(y^*\bigr), g(v_{n+1})\bigr) \to 0 \quad \text{as } n \to \infty, \end{aligned}$$

we have $g(x) = g(x^*)$ and $g(y) = g(y^*)$. Thus we have (12). This implies that $(g(x), g(y)) = (g(x^*), g(y^*))$.

Since $g(x) = F(x, y)$ and $g(y) = F(y, x)$, by commutativity of F and g, we have

$$g\bigl(g(x)\bigr) = g\bigl(F(x,y)\bigr) = F\bigl(g(x), g(y)\bigr) \quad \text{and} \quad g\bigl(g(y)\bigr) = g\bigl(F(y,x)\bigr) = F\bigl(g(y), g(x)\bigr). \tag{26}$$

Denote $g(x) = z$, $g(y) = w$. Then from (26)

$$g(z) = F(z, w) \quad \text{and} \quad g(w) = F(w, z). \tag{27}$$

Thus (z, w) is a coupled coincidence point. Then from (26) with $x^* = z$ and $y^* = w$ it follows $g(z) = g(x)$ and $g(w) = g(y)$, that is,

$$g(z) = z \quad \text{and} \quad g(w) = w. \tag{28}$$

From (27) and (28),

$$z = g(z) = F(z, w) \quad \text{and} \quad w = g(w) = F(w, z).$$

Therefore, (z, w) is a coupled common fixed point of F and g. To prove the uniqueness, assume that (p, q) is another coupled common fixed point. Then by (26) we have $p = g(p) = g(z) = z$ and $q = g(q) = g(w) = w$. □

Corollary 2.2 *In addition to the hypotheses of Corollary* 2.1, *suppose that* $L = 0$, $\alpha < \frac{1}{2}$, *and for every* $(x, y), (y^*, x^*) \in X \times X$, *there exists* $(u, v) \in X \times X$ *such that* $u \preceq F(u, v)$, $v \succeq F(v, u)$, *and $(F(u,v),F(v,u))$ is comparable to $(F(x,y),F(y,x))$ and $(F(x^*,y^*),F(y^*,x^*))$. Then F has a unique coupled fixed point, that is, there exist a unique $(x,y)\in X\times X$ such that*

$$x=F(x,y)\quad \textit{and}\quad y=F(y,x).$$

Proof Follows from Theorem 2.3 by taking $g=I$, the identity mapping. □

Theorem 2.4 *In addition to the hypotheses of Theorem* 2.1, *if gx_0 and gy_0 are comparable and $L=0$, then F and g have a coupled coincidence point (x,y) such that $gx=F(x,y)=F(y,x)=gy$.*

Proof Follow the proof of Theorem 2.1 step by step until constructing two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $gx_n\to gx$ and $gy_n\to gy$, where (x,y) is a coincidence point of F and g. Suppose $gx_0\preceq gy_0$, then it is an easy matter to show that

$$gx_n\preceq gy_n\quad \text{and } \forall n\in\mathbb{N}\cup\{0\}.$$

Thus, by (5) we have

$$\begin{aligned}
&d(gx_n,gy_n)\\
&\quad=d\bigl(F(x_{n-1},y_{n-1}),F(y_{n-1},x_{n-1})\bigr)\\
&\quad\le\alpha\max\bigl\{d(g(x_{n-1}),g(y_{n-1}),d\bigl(F(x_{n-1},y_{n-1}),g(x_{n-1})\bigr),d\bigl(F(y_{n-1},x_{n-1}),g(y_{n-1})\bigr)\bigr\}\\
&\quad=\alpha\max\bigl\{d\bigl(g(x_{n-1}),g(y_{n-1})\bigr),d\bigl(g(x_n),g(x_{n-1})\bigr),d\bigl(g(y_n),g(y_{n-1})\bigr)\bigr\}.
\end{aligned}$$

On taking the limit as $n\to+\infty$, we get $d(gx,gy)=0$. Hence

$$F(x,y)=gx=gy=F(y,x).$$

A similar argument can be used if $gy_0\preceq gx_0$. □

Corollary 2.3 *In addition to the hypotheses of Corollary* 2.1, *if x_0 and y_0 are comparable and $L=0$, then F has a coupled fixed point of the form (x,x).*

Proof Follows from Theorem 2.4 by taking $g=I$, the identity mapping. □

Now, we introduce the following example to support our results.

Example 2.1 Let $X=[0,1]$. Then $(X,\le)$ is a partially ordered set with the natural ordering of real numbers. Define the metric d on X by

$$d(x,y)=\begin{cases}\max\{x,y\} & \text{if } x\ne y;\\ 0 & \text{if } x=y.\end{cases}$$

Define $g:X\to X$ by $g(x)=x^2$ and $F:X\times X\to X$ by

$$F(x,y)=\begin{cases}\frac{3(x^2-y^2)}{4}, & x>y;\\ 0, & x\le y.\end{cases}$$

Then

(1) $g(X)$ is a complete subset of X.
(2) $F(X \times X) \subseteq g(X)$.
(3) X satisfies (i) and (ii) of Theorem 2.1.
(4) F has the mixed g-monotone property.
(5) For any $L \in [0, +\infty)$, F and g satisfy that

$$\begin{aligned} & d\big(F(x,y),F(u,v)\big) \\ & \quad \leq \frac{3}{4}\max\big\{d\big(g(x),g(u)\big),d\big(g(y),g(v)\big),d\big(F(x,y),g(x)\big),d\big(F(u,v),g(u)\big)\big\} \\ & \qquad + L\min\big\{d\big(F(x,y),g(u)\big),d\big(F(u,v),g(x)\big)\big\} \end{aligned}$$

for all $g(x) \leq g(u)$ and $g(y) \geq g(v)$ holds for all $x, y, u, v \in X$ with $g(x) \leq g(u)$ and $g(y) \geq g(v)$.

Thus, by Theorem 2.1, F has a coupled fixed point. Moreover, $(0,0)$ is a coupled coincidence point of F.

Proof The proof of (1)-(4) is clear. We divide the proof of (5) into the following cases.

Case 1: If $g(x) \leq g(y)$ and $g(u) \leq g(v)$, then $x \leq y$ and $u \leq v$. Hence

$$\begin{aligned} d\big(F(x,y),F(u,v)\big) &= d(0,0) = 0 \\ &\leq \frac{3}{4}\max\big\{d\big(g(x),g(u)\big),d\big(g(y),g(v)\big),d\big(F(x,y),g(x)\big),d\big(F(u,v),g(u)\big)\big\} \\ &\quad + L\min\big\{d\big(F(x,y),g(u)\big),d\big(F(u,v),g(x)\big)\big\}. \end{aligned}$$

Case 2: If $g(x) \leq g(y)$ and $g(u) > g(v)$, then $x \leq y$ and $u > v$. Hence

$$\begin{aligned} d\big(F(x,y),F(u,v)\big) &= d\left(0, \frac{3(u^2 - v^2)}{4}\right) \\ &= \frac{3}{4}\big(u^2 - v^2\big) \\ &\leq \frac{3}{4}u^2 \\ &= \frac{3}{4}\max\left\{\frac{3}{4}\big(u^2 - v^2\big), u^2\right\} \\ &= \frac{3}{4}\max\big\{F(u,v),g(u)\big\} \\ &= \frac{3}{4}d\big(F(u,v),g(u)\big) \\ &\leq \frac{3}{4}\max\big\{d\big(g(x),g(u)\big),d\big(g(y),g(v)\big),d\big(F(x,y),g(x)\big),d\big(F(u,v),g(u)\big)\big\} \\ &\quad + L\min\big\{d\big(F(x,y),g(u)\big),d\big(F(u,v),g(x)\big)\big\}. \end{aligned}$$

Case 3: If $g(x) > g(y)$ and $g(u) \leq g(v)$, then $x > y$ and $u \leq v$. Hence $v \leq y < x \leq u \leq v$. Therefore $v < v$, which is impossible.

Case 4: If $g(x) > g(y)$ and $g(u) > g(v)$, then $x > y$ and $u > v$. Thus $v \leq y < x \leq u$.

Subcase I: $x = u$ and $y = v$. Here, we have

$$\begin{aligned} d\bigl(F(x,y),F(u,v)\bigr) &= d(0,0) = 0 \\ &\le \frac{3}{4}\max\bigl\{d\bigl(g(x),g(u)\bigr),d\bigl(g(y),g(v)\bigr),d\bigl(F(x,y),g(x)\bigr),d\bigl(F(u,v),g(u)\bigr)\bigr\} \\ &\quad + L\min\bigl\{d\bigl(F(x,y),g(u)\bigr),d\bigl(F(u,v),g(x)\bigr)\bigr\}. \end{aligned}$$

Subcase II: $x \neq u$ or $y \neq v$. Here, we have $u^2 - v^2 > x^2 - y^2$. Therefore

$$\begin{aligned} d\bigl(F(x,y),F(u,v)\bigr) &= d\left(\frac{3(x^2-y^2)}{4},\frac{3(u^2-v^2)}{4}\right) \\ &= \frac{3}{4}\bigl(u^2 - v^2\bigr) \\ &\le \frac{3}{4}u^2 \\ &= \frac{3}{4}\max\left\{\frac{3}{4}\bigl(u^2-v^2\bigr),u^2\right\} \\ &= \frac{3}{4}\max\bigl\{F(u,v),g(u)\bigr\} \\ &= \frac{3}{4}d\bigl(F(u,v),g(u)\bigr) \\ &\le \frac{3}{4}\max\bigl\{d\bigl(g(x),g(u)\bigr),d\bigl(g(y),g(v)\bigr),d\bigl(F(x,y),g(x)\bigr),d\bigl(F(u,v),g(u)\bigr)\bigr\} \\ &\quad + L\min\bigl\{d\bigl(F(x,y),g(u)\bigr),d\bigl(F(u,v),g(x)\bigr)\bigr\}. \qquad \square \end{aligned}$$

Note that the mappings F and g satisfy all the hypotheses of Theorem 2.1 for $\alpha = \frac{3}{4}$ and any $L \geq 0$. Thus F and g have a coupled coincidence point. Here $(0,0)$ is a coupled coincidence point of F and g.

Remarks

(1) Theorem 1.1 is a special case of Corollary 2.1.
(2) Theorem 1.2 is a special case of Corollary 2.1.
(3) Theorem 1.3 is a special case of Theorem 2.1.
(4) Theorem 1.4 is a special case of Theorem 2.2.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, Hashemite University, P.O. Box 150459, Zarqa, 13115, Jordan. [2]Department of Mathematics, Iran University of Science and Technology, Tehran, Iran. [3]Department of Mathematics, Research Institute for Natural Sciences, Hanyang University, Seoul, 133-791, Korea.

Acknowledgements
The authors would like to thank the editor and the referees for helpful comments.

Received: 11 July 2013 Accepted: 29 October 2013 Published: 27 Nov 2013

References

1. Banach, S: Sur les opérations dans les ensembles absraites et leurs applications. Fundam. Math. **3**, 133-181 (1922)
2. Abdeljawad, T, Karapınar, E, Taş, K: A generalized contraction principle with control functions on partial metric spaces. Comput. Math. Appl. **63**, 716-719 (2012)
3. Abdeljawad, T, Karapınar, E, Taş, K: Existence and uniqueness of a common fixed point on partial metric spaces. Appl. Math. Lett. **24**, 1900-1904 (2011)
4. Agarwal, RP, El-Gebeily, MA, O'Regan, D: Generalized contractions in partially ordered metric spaces. Appl. Anal. **87**, 1-8 (2008)
5. Amini-Harandi, A, Emami, H: A fixed point theorem for contraction type maps in partially ordered metric spaces and application to ordinary differential equations. Nonlinear Anal. **72**(5), 2238-2242 (2010)
6. Je Cho, Y, Saadati, R, Wang, S: Common fixed point theorems on generalized distance in ordered cone metric spaces. Comput. Math. Appl. **61**, 1254-1260 (2011)
7. Drici, Z, McRae, FA, Vasundhara Devi, J: Fixed point theorems in partially ordered metric spaces for operators with PPF dependence. Nonlinear Anal. **67**(2), 641-647 (2007)
8. Ćirić, LB, Cakić, N, Rajović, M, Ume, JS: Monotone generalized nonlinear contractions in partially ordered metric spaces. Fixed Point Theory Appl. **2008**, Article ID 131294 (2008)
9. Nieto, JJ, Lopez, RR: Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations. Order **22**, 223-239 (2005)
10. Nieto, JJ, Lopez, RR: Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations. Acta Math. Sin. Engl. Ser. **23**(12), 2205-2212 (2007)
11. O'regan, D, Petrutel, A: Fixed point theorems for generalized contractions in ordered metric spaces. J. Math. Anal. Appl. **341**(2), 1241-1252 (2008)
12. Ran, ACM, Reurings, MCB: A fixed point theorem in partially ordered sets and some applications to matrix equations. Proc. Am. Math. Soc. **132**(5), 1435-1443 (2004)
13. Karapınar, E, Erhan, İ: Fixed point theorems for operators on partial metric spaces. Appl. Math. Lett. **24**, 1894-1899 (2011)
14. Shatanawi, W, Samet, B: On (ψ,ϕ)-weakly contractive condition in partially ordered metric spaces. Comput. Math. Appl. (2011). doi:10.1016/j.camwa.2011.08.033
15. Shatanawi, W: Fixed point theorems for nonlinear weakly *C*-contractive mappings in metric spaces. Math. Comput. Model. (2011). doi:10.1016/j.mcm.2011.06.069
16. Wu, Y: New fixed point theorems and applications of mixed monotone operator. J. Math. Anal. Appl. **341**(2), 883-893 (2008)
17. Wu, Y, Liang, Z: Existence and uniqueness of fixed points for mixed monotone operators with applications. Nonlinear Anal. **65**(10), 1913-1924 (2006)
18. Bhaskar, TG, Lakshmikantham, V: Fixed point theorems in partially ordered metric spaces and applications. Nonlinear Anal. **65**, 1379-1393 (2006)
19. Aydi, H, Damjanović, B, Samet, B, Shatanawi, W: Coupled fixed point theorems for nonlinear contractions in partially ordered *G*-metric spaces. Math. Comput. Model. (2011). doi:10.1016/j.mcm.2011.05.059
20. Lakshmikanthama, V, Ćirić, LB: Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. Nonlinear Anal. **70**, 4341-4349 (2009)
21. Karapınar, E: Coupled fixed point theorems for nonlinear contractions in cone metric spaces. Comput. Math. Appl. **59**(12), 3656-3668 (2010)
22. Nashine, HK, Shatanawi, W: Coupled common fixed point theorems for pair of commuting mappings in partially ordered complete metric spaces. Comput. Math. Appl. (2011). doi:10.1016/j.camwa.2011.06.042
23. Samet, B: Coupled fixed point theorems for a generalized Meir-Keeler contraction in partially ordered metric spaces. Nonlinear Anal. **72**, 4508-4517 (2010)
24. Shatanawi, W: Some common coupled fixed point results in cone metric spaces. Int. J. Math. Anal. **4**, 2381-2388 (2010)
25. Shatanawi, W: Partially ordered cone metric spaces and coupled fixed point results. Comput. Math. Appl. **60**, 2508-2515 (2010)
26. Sintunavarat, W, Cho, YJ, Kumam, P: Common fixed point theorems for *c*-distance in ordered cone metric spaces. Comput. Math. Appl. **62**(4), 1969-1978 (2011)
27. Berinde, V: Some remarks on a fixed point theorem for Ćirić-type almost contractions. Carpath. J. Math. **25**, 157-162 (2009)
28. Berinde, V: Approximation fixed points of weak contractions using the Picard iteration. Nonlinear Anal. Forum **9**, 43-53 (2004)
29. Berinde, V: On the approximation fixed points of weak contractive mappings. Carpath. J. Math. **19**, 7-22 (2003)
30. Berinde, V: General contractive fixed point theorems for Ćirić-type almost contraction in metric spaces. Carpath. J. Math. **24**, 10-19 (2008)

10.1186/1029-242X-2013-565
Cite this article as: Shatanawi et al.: **Almost contractive coupled mapping in ordered complete metric spaces.** *Journal of Inequalities and Applications* 2013, 2013:565