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A parallel resolvent method for solving a system of nonlinear mixed variational inequalities

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Abstract

In this paper, we introduce a system of generalized nonlinear mixed variational inequalities and obtain the approximate solvability by using the resolvent parallel technique. Our results may be viewed as an extension and improvement of the previously known results for variational inequalities.

Keywords: resolvent operator; parallel projection; relaxed cocoercive; generalized nonlinear mixed variational inequalities

1 Introduction and preliminaries

Variational inequality theory, which was introduced by Stampacchia [1] in 1964, has been witnessed as an interesting branch of mathematical and engineering sciences with a wide range of applications in industry, finance, economics and pure and applied sciences. In 2001, Verma [2] introduced a new system of strongly monotonic variational inequalities and studied the approximation solvability of the system based on the application of a projection method. The main and basic idea in this technique is to establish the equivalence between variational inequalities and fixed point problems. This alternative equivalence has been used to develop several projection iterative methods for solving variational inequalities and related optimization problems. Several extensions and generalizations of the system of strongly monotonic variational inequalities have been considered by many authors [3–12]. Inspired and motivated by research in this area, we introduce a system of generalized nonlinear mixed variational inequalities problem involving two different nonlinear operators. It is well known that if the nonlinear term in the mixed variational inequality is a proper, convex, and lower semicontinuous, then one can establish the equivalence between the mixed variational inequality and the fixed point problem. Using the parallel algorithm considered in [12], we suggest and analyze a parallel iterative method for solving this system. Our result may be viewed as an extension and improvement of the recent results.

Let $\mathcal{H}$ be a real Hilbert space whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let K be a nonempty closed convex subset of $\mathcal{H}$. Let $T_1, T_2 : K \times K \to \mathcal{H}$ be two nonlinear operators. Let $\varphi_1, \varphi_2 : \mathcal{H} \to \mathbb{R} \cup \{+\infty\}$ be proper convex lower semicontinuous functions on $\mathcal{H}$. We consider a system of generalized nonlinear mixed variational inequalities (abbreviated as SMNVI) as follows: Find $(x^*, y^*) \in K \times K$ such that

$$\begin{cases} \langle \rho T_1(y^*, x^*) + g(x^*) - g(y^*), g(x) - g(x^*) \rangle + \varphi_1(g(x)) - \varphi_1(g(x^*)) \\ \quad \geq 0, \quad \forall g(x) \in K, \\ \langle \eta T_2(x^*, y^*) + g(y^*) - g(x^*), g(x) - g(x^*) \rangle + \varphi_2(g(x)) - \varphi_2(g(y^*)) \\ \quad \geq 0, \quad \forall g(x) \in K, \end{cases} \tag{1.1}$$

where $g : K \to K$ is a mapping and $\rho, \eta > 0$.

Note that if $\varphi_1 = \varphi_2 = \delta_K$, and $g = I$, where I is the identity operator, δ_K is the indicator function of K defined by

$$\delta_K(x) = \begin{cases} 0 & \text{if } x \in K, \\ +\infty & \text{otherwise,} \end{cases}$$

then problem (1.1) reduces to the following system of nonlinear variational inequalities (SNVI) considered in [3] of finding $(x^*, y^*) \in K \times K$ such that

$$\begin{cases} \langle \rho T_1(y^*, x^*) + x^* - y^*, x - x^* \rangle \geq 0, \quad \forall x \in K, \\ \langle \eta T_2(x^*, y^*) + y^* - x^*, x - x^* \rangle \geq 0, \quad \forall x \in K. \end{cases} \tag{1.2}$$

If $T_1 = T_2 = T$ and $g = I$, where I is the identity operator, then problem (1.1) is equivalent to the following system of nonlinear mixed variational inequalities (SNVI) considered in [7, 8] of finding $(x^*, y^*) \in K \times K$ such that

$$\begin{cases} \langle \rho T(y^*, x^*) + x^* - y^*, x - x^* \rangle + \varphi_1(x) - \varphi_1(x^*) \geq 0, \quad \forall x \in K, \\ \langle \eta T(x^*, y^*) + y^* - x^*, x - x^* \rangle + \varphi_2(x) - \varphi_2(y^*) \geq 0, \quad \forall x \in K. \end{cases} \tag{1.3}$$

If $\varphi_1 = \varphi_2 = \delta_K$ and $T_1, T_2 : K \to \mathcal{H}$ are univariate mappings, then problem (1.1) is reduced to the following system of nonlinear variational inequalities (SNVI) considered in [12] of finding $(x^*, y^*) \in K \times K$ such that

$$\begin{cases} \langle \rho T_1(y^*) + g(x^*) - g(y^*), g(x) - g(x^*) \rangle \geq 0, \quad \forall g(x) \in K, \\ \langle \eta T_2(x^*) + g(y^*) - g(x^*), g(x) - g(x^*) \rangle \geq 0, \quad \forall g(x) \in K, \end{cases} \tag{1.4}$$

where $g : K \to K$ is a mapping.

If $T_1 = T_2 = T$, $g = I$ and $\varphi_1 = \varphi_2 = \delta_K$, where T is a univariate mapping defined by $T : K \to \mathcal{H}$, then problem (1.1) reduces to the following system of variational inequalities (SVI) considered in [2] of finding $(x^*, y^*) \in K \times K$ such that

$$\begin{cases} \langle \rho T(y^*) + x^* - y^*, x - x^* \rangle \geq 0, \quad \forall x \in K, \\ \langle \eta T(x^*) + y^* - x^*, x - x^* \rangle \geq 0, \quad \forall x \in K. \end{cases} \tag{1.5}$$

We also need the following well-known results.

Definition 1.1 Define the norm $\|\cdot\|$ on $\mathcal{H}\times\mathcal{H}$ by

$$\|(u,v)\| = \|u\| + \|v\|, \quad \forall(u,v)\in\mathcal{H}\times\mathcal{H}.$$

Definition 1.2 For any maximal monotone operator T, the resolvent operator associated with T, for any $\lambda > 0$, is defined by

$$J_T^{\lambda}(u) = (I+\lambda T)^{-1}(u), \quad \forall u\in\mathcal{H}.$$

Remark 1.1 It is well known that the subdifferential $\partial\varphi$ of a proper convex lower semi-continuous function $\varphi:\mathcal{H}\to\mathbb{R}\cup\{+\infty\}$ is a maximal monotone operator. We can define its resolvent operator by

$$J_{\varphi}^{\lambda}(u) = (I+\lambda\partial\varphi)^{-1}(u), \quad \forall u\in\mathcal{H},$$

where $\lambda > 0$ and J_{φ} is defined everywhere.

Lemma 1.1 [13] *For a given $u,z\in\mathcal{H}$ satisfies the inequality*

$$\langle u-z, x-u\rangle + \lambda\varphi(x) - \lambda\varphi(u) \geq 0, \quad \forall x\in\mathcal{H},$$

if and only if $u = J_{\varphi}^{\lambda}(z)$, where $J_{\varphi}^{\lambda}(u) = (I+\lambda\partial\varphi)^{-1}(u)$ is the resolvent operator and $\lambda > 0$.

If φ is the indicator function of a closed convex set $K\subseteq\mathcal{H}$, then the resolvent operator $J_{\varphi}^{\lambda}(\cdot)$ reduces to the projection operator $P_K(\cdot)$. It is well known that J_{φ}^{λ} is nonexpansive, *i.e.*,

$$\|J_{\varphi}^{\lambda}(u) - J_{\varphi}^{\lambda}(v)\| \leq \|u-v\|, \quad \forall u,v\in\mathcal{H}.$$

Based on Lemma 1.1, similar to that in [8] and [7], the following statement gives equivalent characterization of problem (1.1).

Lemma 1.2 *Problem* (1.1) *is equivalent to finding $(x^*,y^*)\in K\times K$ such that*

$$\begin{cases} g(x^*) = J_{\varphi_1}^{1}[g(y^*) - \rho T_1(y^*,x^*)], \\ g(y^*) = J_{\varphi_2}^{1}[g(x^*) - \eta T_2(x^*,y^*)], \end{cases} \tag{1.6}$$

where $J_{\varphi_i}^{1} = (I+\partial\varphi_i)^{-1}$, $i = 1,2$.

Proof Suppose that $(x^*,y^*)\in K\times K$ is a solution of the following generalized nonlinear mixed variational inequalities (abbreviated as SNMVI):

$$\begin{cases} \langle\rho T_1(y^*,x^*) + g(x^*) - g(y^*), g(x) - g(x^*)\rangle + \rho'\varphi_1(g(x)) - \rho'\varphi_1(g(x^*)) \\ \quad \geq 0, \quad \forall g(x)\in K, \\ \langle\eta T_2(x^*,y^*) + g(y^*) - g(x^*), g(x) - g(x^*)\rangle + \eta'\varphi_2(g(x)) - \eta'\varphi_2(g(y^*)) \\ \quad \geq 0, \quad \forall g(x)\in K, \end{cases} \tag{1.7}$$

where $g : K \to K$ is a mapping and $\rho > 0$, $\rho' > 0$, $\eta > 0$, $\eta' > 0$. Using Lemma 1.1, we can easily show that problem (1.7) is equivalent to

$$\begin{cases} g(x^*) = J_{\varphi_1}^{\rho'}[g(y^*) - \rho T_1(y^*, x^*)], \\ g(y^*) = J_{\varphi_2}^{\eta'}[g(x^*) - \eta T_2(x^*, y^*)], \end{cases} \tag{1.8}$$

where $J_{\varphi_1}^{\rho'} = (I + \rho'\partial\varphi_1)^{-1}$, $J_{\varphi_2}^{\eta'} = (I + \eta'\partial\varphi_2)^{-1}$. Let $\rho' = \eta' = 1$. Then problem (1.7) reduces to problem (1.1) and $J_{\varphi_i}^1 = (I + \partial\varphi_i)^{-1}$, $i = 1, 2$. This completes the proof. □

Remark 1.2 If $T_1 = T_2 = T$ and $g = I$, where I is the identity operator, then Lemma 1.2 reduces to Lemma 1.2 in [7].

Definition 1.3 A mapping $T : K \times K \to \mathcal{H}$ is said to be

(1) relaxed g-(γ, r)-cocoercive if there exist constants $\gamma > 0$ and $r > 0$ such that for all $x, y \in K$,

$$\langle T(x,u) - T(y,v), g(x) - g(y) \rangle \geq (-\gamma) \| T(x,u) - T(y,v) \|^2 + r \| g(x) - g(y) \|^2, \quad \forall u, v \in K;$$

(2) g-μ-Lipschitz continuous in the first variable if there exists a constant $\mu > 0$ such that for all $x, y \in K$,

$$\| T(x,u) - T(y,v) \| \leq \mu \| g(x) - g(y) \|, \quad \forall u, v \in K.$$

Remark 1.3 If T is a univariate mapping and $g = I$, where I is the identity operator, then Definition 1.3 reduces to the standard definition of relaxed (γ, r)-cocoercive and Lipschitz continuous, respectively.

Definition 1.4 A mapping $g : K \to \mathcal{H}$ is said to be α-expansive if there exists a constant $\alpha > 0$ such that for all $x, y \in \mathcal{H}$,

$$\| g(x) - g(y) \| \geq \alpha \| x - y \|.$$

Lemma 1.3 [14] *Suppose that* $\{\delta_n\}$ *is a nonnegative sequence satisfying the following inequality*:

$$\delta_{n+1} \leq (1 - \lambda_n)\delta_n + \sigma_n, \quad \forall n \geq n_0,$$

where n_0 *is a nonnegative number*, $\lambda_n \in [0, 1]$ *with* $\sum_{n=0}^{\infty} \lambda_n = \infty$, *and* $\sigma_n = o(\lambda_n)$. *Then* $\lim_{n\to\infty} \delta_n = 0$.

2 Algorithms

In this section, we suggest a parallel algorithm associated with the resolvent operator for solving the system of SNMVI. Our results extend and improve the corresponding results in [2, 3, 7, 11, 12]. In fact, using Lemma 1.2, we suggest the following iterative method for solving problem (1.1).

Algorithm 2.1 For arbitrarily chosen initial points $x_0, y_0 \in K$ (and $g(x_0), g(y_0) \in K$), compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} g(x_{n+1}) = (1-\alpha_n)g(x_n) + \alpha_n J^1_{\varphi_1}[g(y_n) - \rho T_1(y_n, x_n)], \\ g(y_{n+1}) = (1-\beta_n)g(y_n) + \beta_n J^1_{\varphi_2}[g(x_n) - \eta T_2(x_n, y_n)], \end{cases} \tag{2.1}$$

where $J^1_{\varphi_i} = (I + \partial\varphi_i)^{-1}$, $i = 1, 2$, is the resolvent operator, $\rho, \eta > 0$, $\alpha_n \in [0,1]$ and $\beta_n \in [0,1]$ for all $n \geq 0$.

As reported in [12], one of the attractive features of Algorithm 2.1 is that it is suitable for implementing on two different processor computers. In other words, x_{n+1} and y_{n+1} are solved in parallel, and Algorithm 2.1 is the so-called parallel resolvent method. We refer the interested reader to papers [15–17] and references therein for more examples and ideas of parallel iterative methods.

If $\varphi_1 = \varphi_2 = \delta_K$, and $g = I$, δ_K is the indicator function of K, then Algorithm 2.1 reduces to the following algorithm.

Algorithm 2.2 For arbitrarily chosen initial points $x_0, y_0 \in K$, compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n P_K[y_n - \rho T_1(y_n, x_n)], \\ y_{n+1} = (1-\beta_n)y_n + \beta_n P_K[x_n - \eta T_2(x_n, y_n)], \end{cases} \tag{2.2}$$

where $\rho, \eta > 0$, $\alpha_n \in [0,1]$ and $\beta_n \in [0,1]$ for all $n \geq 0$.

If $T_1 = T_2 = T$ and $g = I$, then Algorithm 2.1 reduces to the following algorithm.

Algorithm 2.3 For arbitrarily chosen initial points $x_0, y_0 \in K$, compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n J^1_{\varphi_1}[y_n - \rho T(y_n, x_n)], \\ y_{n+1} = (1-\beta_n)y_n + \beta_n J^1_{\varphi_2}[x_n - \eta T(x_n, y_n)], \end{cases} \tag{2.3}$$

where $J^1_{\varphi_i} = (I + \partial\varphi_i)^{-1}$, $i = 1, 2$, is the resolvent operator, $\rho, \eta > 0$, $\alpha_n \in [0,1]$ and $\beta_n \in [0,1]$ for all $n \geq 0$.

If $\varphi_1 = \varphi_2 = \delta_K$ and $T_1, T_2 : K \to \mathcal{H}$ are univariate mappings, then Algorithm 2.1 reduces to the following algorithm.

Algorithm 2.4 For arbitrarily chosen initial points $x_0, y_0 \in K$ (and $g(x_0), g(y_0) \in K$), compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} g(x_{n+1}) = (1-\alpha_n)g(x_n) + \alpha_n P_K[g(y_n) - \rho T_1(y_n)], \\ g(y_{n+1}) = (1-\beta_n)g(y_n) + \beta_n P_K[g(x_n) - \eta T_2(x_n)], \end{cases} \tag{2.4}$$

where $\rho, \eta > 0$, $\alpha_n \in [0,1]$ and $\beta_n \in [0,1]$ for all $n \geq 0$.

If $T_1 = T_2 = T$, $g = I$ and $\varphi_1 = \varphi_2 = \delta_K$, where T is a univariate mapping defined by $T: K \to \mathcal{H}$, then Algorithm 2.1 reduces to the following algorithm.

Algorithm 2.5 For arbitrarily chosen initial points $x_0, y_0 \in K$ (and $g(x_0), g(y_0) \in K$), compute the sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n P_K[y_n - \rho T(y_n)], \\ y_{n+1} = (1-\beta_n)y_n + \beta_n P_K[x_n - \eta T(x_n)], \end{cases} \tag{2.5}$$

where $\rho, \eta > 0$, $\alpha_n \in [0,1]$ and $\beta_n \in [0,1]$ for all $n \geq 0$.

3 Main results

In this section, based on Algorithm 2.1, we now present the approximation solvability of problem (1.1) involving relaxed g-(γ, r)-cocoercive and g-μ-Lipschitz continuous in the first variable mappings in Hilbert settings.

Theorem 3.1 *Let $\mathcal{H}$ be a real Hilbert space. Let K be a nonempty closed convex subset of $\mathcal{H}$, and let $T_i : K \times K \to \mathcal{H}$ be relaxed g-(γ_i, r_i)-cocoercive and g-μ_i-Lipschitz continuous in the first variable for $i = 1, 2$. Let $g : K \to K$ be an α-expansive mapping. Suppose that $(x^*, y^*) \in K \times K$ is the unique solution to problem* (1.1) *and $\{x_n\}$, $\{y_n\}$ are generated by Algorithm* 2.1. *If $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in* $[0,1]$ *satisfying the following conditions*:

(i) $\alpha_n - \theta_2\beta_n \geq 0$ *and* $\beta_n - \theta_1\alpha_n \geq 0$ *such that* $\sum_{n=0}^{\infty} \alpha_n - \theta_2\beta_n = \infty$, $\sum_{n=0}^{\infty} \beta_n - \theta_1\alpha_n = \infty$,

(ii) $\theta_1 = \sqrt{1 + 2\rho\gamma_1{\mu_1}^2 - 2\rho r_1 + \rho^2\mu_1}$ *such that* $0 < \theta_1 < 1$,

(iii) $\theta_2 = \sqrt{1 + 2\eta\gamma_2{\mu_2}^2 - 2\eta r_2 + \eta^2\mu_2}$ *such that* $0 < \theta_2 < 1$,

then the sequences $\{x_n\}$ and $\{y_n\}$ converge to x^* and y^*, respectively.

Proof Since $(x^*, y^*) \in K \times K$ is the unique solution to problem (1.1), from Lemma 1.2 it follows that

$$\begin{cases} g(x^*) = J_{\varphi_1}^1[g(y^*) - \rho T_1(y^*, x^*)], \\ g(y^*) = J_{\varphi_2}^1[g(x^*) - \eta T_2(x^*, y^*)]. \end{cases} \tag{3.1}$$

We first evaluate $\|g(x_{n+1}) - g(x^*)\|$ for all $n \geq 0$. From (2.1) and the nonexpansive property of the resolvent operator, we have

$$\begin{aligned} &\|g(x_{n+1}) - g(x^*)\| \\ &\quad = \|(1-\alpha_n)g(x_n) + \alpha_n J_{\varphi_1}^1[g(y_n) - \rho T_1(y_n, x_n)] \\ &\qquad - (1-\alpha_n)g(x^*) - \alpha_n J_{\varphi_1}^1[g(y^*) - \rho T_1(y^*, x^*)]\| \\ &\quad \leq (1-\alpha_n)\|g(x_n) - g(x^*)\| \\ &\qquad + \alpha_n \|J_{\varphi_1}^1[g(y_n) - \rho T_1(y_n, x_n)] - J_{\varphi_1}^1[g(y^*) - \rho T_1(y^*, x^*)]\| \\ &\quad \leq (1-\alpha_n)\|g(x_n) - g(x^*)\| + \alpha_n\|g(y_n) - g(y^*) - \rho(T_1(y_n, x_n) - T_1(y^*, x^*))\|. \end{aligned} \tag{3.2}$$

Notice that T_1 is relaxed g-(γ_1, r_1)-cocoercive and g-μ_1-Lipschitz continuous in the first variable. Then we have

$$\begin{aligned}
&\|g(y_n) - g(y^*) - \rho(T_1(y_n, x_n) - T_1(y^*, x^*))\|^2 \\
&\quad = \|g(y_n) - g(y^*)\|^2 - 2\rho\langle g(y_n) - g(y^*), T_1(y_n, x_n) - T_1(y^*, x^*)\rangle \\
&\qquad + \rho^2\|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\
&\quad \le \|g(y_n) - g(y^*)\|^2 + 2\rho\gamma_1\|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\
&\qquad - 2\rho r_1\|g(y_n) - g(y^*)\|^2 + \rho^2\|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\
&\quad \le \theta^2\|g(y_n) - g(y^*)\|^2,
\end{aligned} \tag{3.3}$$

where $\theta_1 = \sqrt{1 + 2\rho\gamma_1\mu_1{}^2 - 2\rho r_1 + \rho^2\mu_1} < 1$ in view of assumption (ii). Substituting (3.3) into (3.2), we have

$$\|g(x_{n+1}) - g(x^*)\| \le (1 - \alpha_n)\|g(x_n) - g(x^*)\| + \alpha_n\theta_1\|g(y_n) - g(y^*)\|. \tag{3.4}$$

Similarly, since T_2 is relaxed g-(γ_2, r_2)-cocoercive and g-μ_2-Lipschitz continuous in the first variable, we have

$$\|g(y_{n+1}) - g(y^*)\| \le (1 - \beta_n)\|g(y_n) - g(y^*)\| + \beta_n\theta_2\|g(x_n) - g(x^*)\|, \tag{3.5}$$

where $\theta_2 = \sqrt{1 + 2\eta\gamma_2\mu_2{}^2 - 2\eta r_2 + \eta^2\mu_2} < 1$ in view of assumption (iii). It follows from (3.4) and (3.5) that

$$\begin{aligned}
&\|(g(x_{n+1}), g(y_{n+1})) - (g(x^*), g(y^*))\| \\
&\quad \le [1 - (\alpha_n - \theta_2\beta_n)]\|g(x_n) - g(x^*)\| + [1 - (\beta_n - \theta_1\alpha_n)]\|g(y_n) - g(y^*)\| \\
&\quad = \max\{w_{1n}, w_{2n}\}(\|(g(x_n), g(y_n)) - (g(x^*), g(y^*))\|),
\end{aligned} \tag{3.6}$$

where $w_{1n} = 1 - (\alpha_n - \theta_2\beta_n)$ and $w_{2n} = 1 - (\beta_n - \theta_1\alpha_n)$.

From assumption (i) and Lemma 1.3, we can obtain

$$\lim_{n\to\infty}\|(g(x_{n+1}), g(y_{n+1})) - (g(x^*), g(y^*))\| = 0,$$

and so

$$\lim_{n\to\infty}\|g(x_{n+1}) - g(x^*)\| = 0 \quad \text{and} \quad \lim_{n\to\infty}\|g(y_{n+1}) - g(y^*)\| = 0,$$

which implies that sequences $\{g(x_n)\}$ and $\{g(y_n)\}$ converge to $g(x^*)$ and $g(y^*)$, respectively. Since g is α-expansive, it follows that $\{x_n\}$ and $\{y_n\}$ converge to x^* and y^*, respectively. This completes the proof. □

The following theorems can be obtained from Theorem 3.1 immediately.

Theorem 3.2 [3] *Let $\mathcal{H}$ be a real Hilbert space. Let K be a nonempty closed convex subset of $\mathcal{H}$, and let $T_i : K \times K \to \mathcal{H}$ be relaxed (γ_i, r_i)-cocoercive and μ_i-Lipschitz continuous in* *the first variable for* $i = 1, 2$. *Suppose that* $(x^*, y^*) \in K \times K$ *is the unique solution to problem* (1.2) *and* $\{x_n\}$, $\{y_n\}$ *are generated by Algorithm* 2.2. *If* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are two sequences in* $[0,1]$ *satisfying the following conditions*:

(1) $\alpha_n - \theta_2\beta_n \geq 0$ *and* $\beta_n - \theta_1\alpha_n \geq 0$ *such that* $\sum_{n=0}^{\infty} \alpha_n - \theta_2\beta_n = \infty$, $\sum_{n=0}^{\infty} \beta_n - \theta_1\alpha_n = \infty$,

(2) $\theta_1 = \sqrt{1 + 2\rho\gamma_1{\mu_1}^2 - 2\rho r_1 + \rho^2\mu_1}$ *such that* $0 < \theta_1 < 1$,

(3) $\theta_2 = \sqrt{1 + 2\eta\gamma_2{\mu_2}^2 - 2\eta r_2 + \eta^2\mu_2}$ *such that* $0 < \theta_2 < 1$,

then the sequences $\{x_n\}$ *and* $\{y_n\}$ *converge to* x^* *and* y^*, *respectively.*

Theorem 3.3 [12] *Let* $\mathcal{H}$ *be a real Hilbert space. Let* K *be a nonempty closed convex subset of* $\mathcal{H}$, *and let* $T_i : K \to \mathcal{H}$ *be relaxed* g-(γ_i, r_i)-*cocoercive and* g-μ_i-*Lipschitz continuous for* $i = 1, 2$. *Let* $g : K \to K$ *be an* α-*expansive mapping. Suppose that* $(x^*, y^*) \in K \times K$ *is the unique solution to problem* (1.4) *and* $\{x_n\}$, $\{y_n\}$ *are generated by Algorithm* 2.4. *If* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are two sequences in* $[0,1]$ *satisfying the following conditions*:

(1) $0 \leq \alpha_n, \beta_n \leq 1$, $\alpha_n - \theta_2\beta_n \geq 0$ *and* $\beta_n - \theta_1\alpha_n \geq 0$ *such that* $\sum_{n=0}^{\infty} \alpha_n - \theta_2\beta_n = \infty$, $\sum_{n=0}^{\infty} \beta_n - \theta_1\alpha_n = \infty$,

(2) $\theta_1 = \sqrt{1 + 2\rho\gamma_1{\mu_1}^2 - 2\rho r_1 + \rho^2\mu_1}$ *such that* $0 < \theta_1 < 1$,

(3) $\theta_2 = \sqrt{1 + 2\eta\gamma_2{\mu_2}^2 - 2\eta r_2 + \eta^2\mu_2}$ *such that* $0 < \theta_2 < 1$,

then the sequences $\{x_n\}$ *and* $\{y_n\}$ *converge to* x^* *and* y^*, *respectively.*

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed significantly in writing the paper. All authors read and approved the final manuscript.

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