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Weighted analogues of Bernstein-type inequalities on several intervals

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Abstract
We give weighted analogues of Bernstein-type inequalities for trigonometric polynomials and rational functions on several intervals.
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1 Introduction

Inequalities for polynomials have been a classical object of studies for more than one century. Modern expositions can be found in books and surveys [1–3] and [4]. Recently weighted analogues of classical polynomial inequalities were considered (see, for instance, [5, 6] and [7]). Other ways of generalizations are in replacing the domain of polynomials by more complicated (disconnected) sets and (or) in considering polynomials in more general Chebyshev systems. The main goal of the paper is to give simple proofs of weighted analogues of Bernstein-type inequalities on several intervals. They are inspired by weighted Bernstein-type inequalities from Section 5.2, E.4 in [1]. It turns out that for disconnected sets similar ideas allow to write down weighted versions with an explicit constant.

Throughout the paper, we use the notations

$$P_n^{(\mathbb{C})} = \left\{ p : p(x) = \sum_{k=0}^{n} a_k x^k, a_k \in \mathbb{R}(\mathbb{C}) \right\} \tag{1}$$

for the set of algebraic polynomials and

$$T_n^{(\mathbb{C})} = \left\{ t : t(x) = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx), a_k, b_k \in \mathbb{R}(\mathbb{C}) \right\} \tag{2}$$

for the set of trigonometric polynomials with real (complex) coefficients; as a weight w, we consider an arbitrary continuous positive function on a suitable set, $\|\cdot\|$ is the uniform norm on this set.

The first theorem is a weighted analogue of the Bernstein-type inequality on several intervals.

Theorem 1 *Let E be a set consisting of a finite number $l \geq 2$ of disjoint intervals, $E = \bigcup_{j=1}^{l}[a_j, b_j] \subset [0,1]$, $a_1 < b_1 < a_2 < \cdots < b_{2l}$, then there exists n_0 depending on w and E such* *that*

$$\left| p_n'(x)w(x)\sqrt{\prod_{j=1}^{l}\left|(x-a_j)(x-b_j)\right|}\right| \le n\|p_n w\|_E, \quad x \in E \tag{3}$$

for every polynomial $p_n \in P_n^{\mathbb{C}}$, $n \ge n_0$.

Next result is a weighted version of the Bernstein-type inequality for trigonometric polynomials on several intervals.

Theorem 2 *Let w be any function which is continuous and positive on*

$$\mathcal{E} = \bigcup_{j=1}^{l}[\theta_{2j-1}, \theta_{2j}] \subsetneq [0, 2\pi], \quad \theta_1 < \theta_2 < \cdots < \theta_{2l} < \theta_{2l+1} = \theta_1 + 2\pi \tag{4}$$

and

$$S(\theta) = \prod_{j=1}^{2l}\sin\left(\frac{\theta - \theta_j}{2}\right). \tag{5}$$

Then there exists n_0 *depending on w and* $\mathcal{E}$ *such that*

$$\left| t_n'(\theta)w(\theta)\sqrt{\left|S(\theta)\right|}\right| \le n\|t_n w\|_{\mathcal{E}}, \quad \theta \in \mathcal{E} \tag{6}$$

for every polynomial $t_n \in T_n^{\mathbb{C}}$, $n \ge n_0$. *Inequality* (6) *is sharp in the sense that it is not possible to replace n in* (6) *by* $n(1-\varepsilon)$ *for arbitrary* $\varepsilon > 0$.

Now let $0 < \alpha < \pi$, and let

$$K_\alpha = \left\{e^{i\theta} \mid \theta \in [-\alpha, \alpha]\right\} \tag{7}$$

be the circular arc on the unit circle of central angle 2α and with a midpoint at 1. Next result is a weighted version of the inequality from [8].

Theorem 3 *With the above notations and for any continuous positive function w, there exists* n_0 *depending on w and* α *such that*

$$\left| p_n'\left(e^{i\theta}\right)w\left(e^{i\theta}\right)\right|\sqrt{\left|\sin\left(\frac{\theta-\alpha}{2}\right)\sin\left(\frac{\theta+\alpha}{2}\right)\right|} \le n\|p_n w\|_{K_\alpha}, \quad \theta \in [-\alpha, \alpha] \tag{8}$$

for every polynomial $p_n \in P_n^{\mathbb{C}}$, $n \ge n_0$.

Next we recall the definition of the harmonic measure $\omega(z, G, \mathcal{D})$ of a set $G \subset \partial\mathcal{D}$ at a point $z \in \mathcal{D}$ relative to the domain $\mathcal{D}$,

$$\omega(z, G, \mathcal{D}) = \frac{1}{\pi}\int_G \frac{\partial}{\partial \mathbf{n}} g_{\mathcal{D}}(\zeta, z)|d\zeta|, \tag{9}$$

where $g_{\mathcal{D}}(\zeta, z)$ is the Green function of the domain $\mathcal{D}$ and $\mathbf{n}$ is the exterior normal at ζ (see, for example, [9]).

Our last result is an extension of Rusak's inequality [10, p.57] to the case of several intervals.

Theorem 4 *Let r_n be a complex-valued algebraic fraction*

$$r_n(x) = \frac{x^n + b_1 x^{n-1} + \cdots + b_n}{\sqrt{|\rho_\nu(x)|}}, \tag{10}$$

where $b_1, \ldots, b_n \in \mathbb{C}$, $\rho_\nu(x) = \prod_{j=1}^{\nu^*}(x - x_j)^{\nu_j}$ is a real polynomial of degree ν which is positive on $E = \bigcup_{j=1}^{l}[a_{2j-1}, a_{2j}]$, $a_1 < a_2 < \cdots < a_{2l}$ satisfying the condition

$$\left|r_n(x)\right| \le 1, \quad x \in E, \tag{11}$$

and $\gamma(x)$ is a differentiable function on E. Then the estimate

$$\left|\left(r_n(x)\gamma(x)\right)'\right| \le \sqrt{\left(\varphi_n'(x)\right)^2\gamma^2(x) + \gamma'^2(x)}, \quad x \in \operatorname{int}(E) \tag{12}$$

is valid. Here

$$\varphi_n(x) = \frac{\pi}{2}\left((2n-\nu)\varpi_E(\infty, x) + \sum_{j=1}^{\nu^*}\nu_j\varpi_E(x_j, x)\right), \tag{13}$$

$\varpi_E(z,x) = \frac{\partial}{\partial x}(\omega(z, [a_1, x] \cap E, \mathbb{C}\backslash E))$. *If x is not a multiple root of $\gamma(x)$, then the equality sign is valid only for algebraic fractions*

$$r_n(x) \equiv \varepsilon \cos\varphi_n(x), \quad |\varepsilon| = 1 \tag{14}$$

at the points x satisfying

$$\left(\gamma(x)\sin\varphi_n(x)\right)' = 0 \tag{15}$$

in the case when

$$(2n-\nu)\omega\left(\infty, [a_{2k-1}, a_{2k}], \mathbb{C}\backslash E\right) + \sum_{j=1}^{\nu^*}\nu_j\omega\left(x_j, [a_{2k-1}, a_{2k}], \mathbb{C}\backslash E\right) = q_k, \tag{16}$$

where $q_k \in \mathbb{N}$, $k = 1, \ldots, l$.

Remark 1 A Markov-type inequality, which is obtained by a similar method, was announced in the conference [11].

In the following we use several auxiliary results.

Lemma 1 [12] *Consider any algebraic fraction*

$$r_n(x) = \frac{x^n + b_1 x^{n-1} + \cdots + b_n}{\sqrt{\rho_\nu(x)}}, \tag{17}$$

where $b_1, \ldots, b_n \in \mathbb{R}$, *and* $\rho_\nu(x) = \prod_{j=1}^{\nu^*}(x - x_j)^{\nu_j}$ *is a real polynomial of degree* ν *which is positive on* $E = [a_1, a_2] \cup \cdots \cup [a_{2l-1}, a_{2l}] \subset \mathbb{R}$, $a_1 < \cdots < a_{2l}$. *Then*

$$\left(\frac{r'(x)}{\varphi_n(x)}\right)^2 + r^2(x) \leq \|r\|^2_{C(E)}, \tag{18}$$

where $\varphi_n(x)$ *is given by* (13).

For further reference, it is convenient to give a particular case of a version of Lemma 1 from [13].

Lemma 2 *The following inequality holds for any trigonometric polynomial* $t_n \in T_n$ *and* $\theta \in \operatorname{int}(\mathcal{E})$, $\mathcal{E}$ *is a real compact subset of* $[0, 2\pi]$:

$$\left(\frac{t_n'(\theta)}{2n\pi\,\varpi_{\mathcal{E}}(\infty, \theta)}\right)^2 + t_n^2(\theta) \leq \|t_n\|^2_{\mathcal{E}}. \tag{19}$$

Here,

$$\varpi_{\mathcal{E}}(z, x) = \frac{\partial}{\partial x}\omega\big(z, \Gamma_{\mathcal{E}} \cap \{e^{i\theta} : \inf\mathcal{E} \leq \theta \leq x\}, \mathbb{C}\backslash\Gamma_{\mathcal{E}}\big), \tag{20}$$

and $\Gamma_{\mathcal{E}} = \{e^{i\theta} : \theta \in \mathcal{E}\}$.

Lemma 3 [12, 14] *The following assertions are equivalent.*

1. *The trigonometric polynomial* τ_N *deviates least from zero on* $\mathcal{E} = [\theta_1, \theta_2] \cup \cdots \cup [\theta_{2l-1}, \theta_{2l}]$, $\theta_1 < \theta_2 < \cdots < \theta_{2l}$ *with respect to the sup-norm among all trigonometric polynomials of degree* $N/2$ *with leading coefficients* $\cos\psi$ *and* $\sin\psi$, *i.e.,*

$$\begin{aligned}\max_{\theta\in\mathcal{E}}\big|\tau_N(\theta)\big| = \inf_{c_j,d_j\in\mathbb{R}}\max_{\theta\in\mathcal{E}}\Bigg|&\cos\psi\cos\frac{N}{2}\theta + \sin\psi\sin\frac{N}{2}\theta \\ &+ \sum_{j=1}^{\lfloor N/2\rfloor} c_j\cos\frac{N-2j}{2}\theta + d_j\sin\frac{N-2j}{2}\theta\Bigg|\end{aligned} \tag{21}$$

   has the maximal possible number of extremum points on $\mathcal{E}$.

2. *For every* $j = 1, \ldots, l$, *the equilibrium measures of the arcs* $\Gamma_j = \{e^{i\theta} : \theta \in [\theta_{2j-1}, \theta_{2j}]\}$ *are positive rational numbers. More precisely,*

$$N\omega(\infty, \Gamma_j, \mathbb{C}\backslash\Gamma_{\mathcal{E}}) = q_j^{(N)}, \quad q_j^{(N)} \in \mathbb{N}, j = 1, \ldots, l. \tag{22}$$

3. *There is a real trigonometric polynomial* $\sigma_{N-\frac{l}{2}}$ *of order* $N - \frac{l}{2}$ *such that for a constant* $A_N > 0$,

$$\tau_N^2(\theta) - S(\theta)\sigma^2_{N-\frac{l}{2}}(\theta) = A_N^2, \tag{23}$$

   where $S(\theta)$ *is given by* (5).

If any of those assertions is valid, then

(a) *the numbers* $q_j^{(N)}$ *are equal to the number of zeros of* $\tau_N(\theta)$ *on* $\mathcal{E}_j = [\theta_{2j-1}, \theta_{2j}]$, $j = 1, \dots, l$;

(b) *the polynomial* τ_N *may also be written in terms of* $\varpi_{\mathcal{E}}(z, x)$ *as*

$$\tau_N(\theta) = A_N \varepsilon \cos\left(\pi \int_{\mathcal{E}\cap[\theta_1,\theta]} N\varpi_{\mathcal{E}}(\infty, \zeta)\, d\zeta\right), \quad \theta \in \mathcal{E}, \tag{24}$$

where $\varepsilon \in \{-1, 1\}$.

Lemma 4 [15] *The density of the equilibrium measure from* (20), $\mathcal{E} = [\theta_1, \theta_2] \cup \cdots \cup [\theta_{2l-1}, \theta_{2l}]$, $\theta_1 < \theta_2 < \cdots < \theta_{2l}$ *is given by*

$$\varpi_{\mathcal{E}}(\infty, \theta) = \frac{1}{2\pi} \frac{|Q(\theta)|}{\sqrt{|S(\theta)|}}, \tag{25}$$

where $Q(\theta) = \prod_{j=1}^{l} \sin(\frac{\theta - \xi_j}{2})$, *and* $\xi_j \in [\theta_{2j}, \theta_{2j+1}]$, $j = 1, \dots, l$, $\theta_{2l+1} := \theta_1 + 2\pi$, *are uniquely determined by*

$$\int_{\theta_{2j}}^{\theta_{2j+1}} \frac{Q(\theta)}{\sqrt{|S(\theta)|}}\, d\zeta = 0, \quad j = 1, \dots, l. \tag{26}$$

Proof We want to present here a different proof of the lemma which uses the representations of extremal polynomials in (24).

(1) Suppose firstly $\omega(\infty, \Gamma_j, \mathbb{C}\setminus\Gamma_{\mathcal{E}}) = \frac{p_j}{2N}$, $p_j \in \mathbb{N}$, $j = 1, \dots, l$. Then by Lemma 3 the function

$$\tau_N(\theta) = \cos\left(\pi \int_{\mathcal{E}\cap[\theta_1,\theta]} 2N\varpi_{\mathcal{E}}(\infty, \zeta)\, d\zeta\right), \quad \theta \in \mathcal{E} \tag{27}$$

is a real trigonometric polynomial of order N. If we take a derivative, we get

$$\tau_N'(\theta) = NQ(\theta) \prod_{j=1}^{2N-l} \sin\frac{\theta - \beta_j}{2}, \tag{28}$$

where $\beta_j, j = 1, \dots, 2N - l$, are zeros of $\sigma_{N-\frac{l}{2}}(\theta)$ and there is a real trigonometric polynomial $\sigma_{N-\frac{l}{2}}$ of order $N - \frac{l}{2}$ such that

$$\tau_N^2(\theta) - S(\theta)\sigma_{N-\frac{l}{2}}^2(\theta) = 1. \tag{29}$$

Hence

$$\sigma_{N-\frac{l}{2}}(\theta) = c \prod_{j=1}^{2N-l} \sin\frac{\theta - \beta_j}{2} = \frac{\sin(\pi \int_{\mathcal{E}\cap[\theta_1,\theta]} 2N\varpi_{\mathcal{E}}(\infty, \zeta)\, d\zeta)}{\sqrt{|S(\theta)|}}. \tag{30}$$

Moreover, $\tau_N(\theta)$ has a maximal number of deviation points, and inner zeros of its derivative coincide with zeros of $\sigma_{N-\frac{l}{2}}(\theta)$, and τ_N has one zero ξ_j at each gap $(\theta_{2j}, \theta_{2j+1})$, $j = 1, \dots, l$.

Hence

$$\begin{aligned}\tau_N'(\theta) &= \pm N\frac{\sin(\pi\int_{\mathcal{E}\cap[\theta_1,\theta]}2N\varpi_{\mathcal{E}}(\infty,\zeta)\,d\zeta)}{\sqrt{|S(\theta)|}}Q(\theta)\\ &= -\sin\left(\pi\int_{\mathcal{E}\cap[\theta_1,\theta]}2N\varpi_{\mathcal{E}}(\infty,\zeta)\,d\zeta\right)\pi 2N\varpi_{\mathcal{E}}(\infty,\theta),\end{aligned} \tag{31}$$

so we have

$$\varpi_{\mathcal{E}}(\infty,\theta) = \frac{1}{2\pi}\frac{|Q(\theta)|}{\sqrt{|S(\theta)|}}. \tag{32}$$

Now equality (26) follows from the representation (24). Uniqueness of ξ_j's follows from the uniqueness of extremal trigonometric polynomials in Lemma 3.

(2) Using density of the systems of l arcs satisfying $\omega(\infty,\Gamma_j,\mathbb{C}\setminus\Gamma_{\mathcal{E}})\in\mathbb{Q}$, $j=1,\dots,l$, among all systems of l arcs (see, for instance, [16, 17] and references therein), we obtain the lemma. □

2 Proofs

Proof of Theorem 2 First consider $t_n\in T_n$. By the Weierstrass approximation theorem, for any $\eta>0$, there is $q_k\in T_k$ such that

$$w(\theta)\le\frac{q_k(\theta)}{\prod_{j=1}^{l}|\sin(\frac{\theta-\xi_j}{2})|}\le(1+\eta)w(\theta),\quad \theta\in\mathcal{E}, \tag{33}$$

where ξ_j are given by (26) in Lemma 4. Hence

$$\begin{aligned}\left|t_n'(\theta)w(\theta)\left|S(\theta)\right|^{1/2}\right| &\le \left|t_n'(\theta)q_k(\theta)\frac{|S(\theta)|^{1/2}}{\prod_{j=1}^{l}|\sin(\frac{\theta-\xi_j}{2})|}\right|\\ &\le \left|(t_nq_k)'(\theta)\frac{|S(\theta)|^{1/2}}{\prod_{j=1}^{l}|\sin(\frac{\theta-\xi_j}{2})|}\right|\\ &\quad+\left|t_n(\theta)q_k'(\theta)\frac{|S(\theta)|^{1/2}}{\prod_{j=1}^{l}|\sin(\frac{\theta-\xi_j}{2})|}\right|,\end{aligned} \tag{34}$$

and, using Lemmas 2 and 4, we have

$$\begin{aligned}\left|t_n'(\theta)w(\theta)\left|S(\theta)\right|^{1/2}\right| &\le (n+k)\|t_nq_k\|_{\mathcal{E}}+\|t_n\|_{\mathcal{E}}k\|q_k\|_{\mathcal{E}}\\ &\le (n+k)(1+\eta)\|t_nw\|_{\mathcal{E}}\left\|\prod_{j=1}^{l}\sin\left(\frac{\theta-\xi_j}{2}\right)\right\|_{\mathcal{E}}\\ &\quad+\frac{1}{m}(1+\eta)\|t_nw\|_{\mathcal{E}}\|w\|_{\mathcal{E}}\left\|\prod_{j=1}^{l}\sin\left(\frac{\theta-\xi_j}{2}\right)\right\|_{\mathcal{E}}\\ &\le n\|t_nw\|_{\mathcal{E}}\left[1+\eta+\frac{k}{n}(1+\eta)+\frac{1}{mn}(1+\eta)\|w\|_{\mathcal{E}}\right]\\ &\quad\times\left\|\prod_{j=1}^{l}\sin\left(\frac{\theta-\xi_j}{2}\right)\right\|_{\mathcal{E}},\end{aligned} \tag{35}$$

where $m := \min\{w(\theta) : \theta \in \mathcal{E}\}$. Now, for every $t_n \in T_n$ and $\varepsilon > 0$, provided $\eta > 0$ is sufficiently small, $n \geq n_0$ such that $\varepsilon \geq \eta + \frac{(1+\eta)}{n}(k + \frac{1}{m}\|w\|_{\mathcal{E}})$, we get

$$\left|t_n'(\theta)w(\theta)\left|S(\theta)\right|^{1/2}\right| \leq n(1+\varepsilon)\|t_n w\|_{\mathcal{E}} \left\| \prod_{j=1}^{l} \sin\left(\frac{\theta - \xi_j}{2}\right)\right\|_{\mathcal{E}}, \tag{36}$$

and because of $\| \prod_{j=1}^{l} \sin(\frac{\theta-\xi_j}{2})\|_{\mathcal{E}} < 1$, we obtain, for sufficiently small $\varepsilon > 0$,

$$\left|t_n'(\theta)w(\theta)\sqrt{\left|S(\theta)\right|}\right| \leq n\|t_n w\|_{\mathcal{E}}. \tag{37}$$

The case of $t_n \in T_n^{\mathbb{C}}$ is proved then similarly to the proof of [1, Corollary 5.1.5]. The theorem is sharp even for the case $w \equiv 1$. Namely, we cannot replace the multiplier n by $n(1-\varepsilon)$ with any $\varepsilon > 0$ in the right-hand side of (6).

Take $\mathcal{E} = [-\alpha, \alpha]$, $0 < \alpha < \pi$. Then we have $S(\theta) = \sin(\frac{\theta-\alpha}{2})\sin(\frac{\theta+\alpha}{2})$, $\xi_1 = \pi$ and

$$\left|\sin\left(\frac{\theta - \xi_j}{2}\right)\right| = \left|\cos\left(\frac{\theta}{2}\right)\right|. \tag{38}$$

Consider

$$t_n(\theta) = \cos\left(2n \arccos\left(\frac{\sin\frac{\theta}{2}}{\sin\frac{\alpha}{2}}\right)\right). \tag{39}$$

Take $\theta = \theta_n = 2\arcsin(\sin\frac{\alpha}{2}\sin\frac{\pi}{4n})$, then

$$\left|t_n'(\theta_n)\right| = n\frac{\cos\frac{\theta_n}{2}}{\sqrt{|S(\theta_n)|}} \tag{40}$$

and

$$\left|t_n'(\theta_n)\sqrt{\left|S(\theta_n)\right|}\right| = n\cos\frac{\theta_n}{2} > n(1-\varepsilon) \tag{41}$$

for sufficiently large n such that

$$\sin^2\frac{\pi}{4n} < \frac{\varepsilon}{\sin^2\frac{\alpha}{2}}. \tag{42}$$

□

Proofs of Theorems 1 and 3 are quite analogous and they use related inequalities from [18, 19].

Proof of Theorem 4 Firstly we consider the case when the numerator $p_n(x)$ has real coefficients. Put $r_n(x) = \cos w = \cos(\arccos r_n(x))$; using Lemma 1, we obtain

$$\begin{aligned}
\left|\left(r_n(x)\gamma(x)\right)'\right| &= \left|\frac{\sin w r_n'(x)\gamma(x)}{1 - r_n^2(x)} + \cos w\gamma'(x)\right| \\
&\leq \sqrt{\sin^2 w + \cos^2 w}\left[\frac{(r_n'(x))^2\gamma^2(x)}{1 - r_n^2(x)} + \gamma'^2(x)\right]^{\frac{1}{2}} \\
&\leq \sqrt{\left(\varphi_n'(x)\right)^2\gamma^2(x) + \gamma'^2(x)}.
\end{aligned} \tag{43}$$

The validity of the estimate for complex-valued algebraic fractions is proved by the same trick as in [1, Corollary 5.1.5]. Equality sign in the last inequality in (43) is valid only for the function $r_n(x) \equiv \varepsilon \cos\varphi_n(x)$, $|\varepsilon| = 1$ if (16) holds [13, 20]. Equality sign in the second inequality in (43) then holds only for the same function at those points where

$$\frac{-\sin\varphi_{2n}(x)}{\varphi'_{2n}(x)\gamma(x)} = \frac{\cos\varphi_{2n}(x)}{\gamma'(x)}. \tag{44}$$

Equality (44) is equivalent to $(\gamma(x)\sin\varphi_{2n}(x))' = 0$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors jointly worked on the results and they read and approved the final manuscript.

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