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PPF dependent common fixed point theorems for mappings in Banach spaces

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Abstract
We prove the existence of the PPF dependent common fixed point theorems and the PPF dependent coincidence points for a pair of mappings satisfying some contractive conditions in Banach spaces where the domains and ranges of the mappings are not the same. Our results extend and generalize the results in the literature.
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1 Introduction

The fixed point theory in Banach spaces plays an important role and is useful in mathematics. It can be applied for solving various problems, for instance, variational inequalities, optimization and approximation theory. The common fixed point theorems for mappings satisfying certain contractive conditions have been continually studied for a decade (see [1–7] and references contained therein). Bernfeld *et al.* [8] proved the existence of PPF (past, present and future) dependent fixed points in the Razumikhin class of functions for mappings that have different domains and ranges. Recently, Dhage [9] extended the existence of PPF dependent fixed points to PPF common dependent fixed points for mappings satisfying the weaker contractive conditions. In this paper, the PPF dependent fixed point theorems and the PPF dependent coincidence points for a pair of mappings are proven in terms of more general contractive conditions in Banach spaces.

Suppose that E is a Banach space with the norm $\|\cdot\|_E$ and I is a closed interval $[a,b]$ in $\mathbb{R}$. Let $E_0 = C(I,E)$ be the set of all continuous E-valued functions on I equipped with the supremum norm $\|\cdot\|_{E_0}$ defined by

$$\|\phi\|_{E_0} = \sup_{t\in I}\|\phi(t)\|_E \tag{1.1}$$

for all $\phi \in E_0$. For a fixed element $c \in I$, the Razumikhin class of functions in E_0 is defined by

$$\mathcal{R}_c = \{\phi \in E_0 : \|\phi\|_{E_0} = \|\phi(c)\|_E\}. \tag{1.2}$$

Recall that a point $\phi \in E_0$ is said to be a PPF dependent fixed point or a fixed point with PPF dependence of $T : E_0 \to E$ if $T\phi = \phi(c)$ for some $c \in I$.

Definition 1.1 Let A be a subset of E. Then:

(i) A is said to be topologically closed with respect to the norm topology if for each sequence $\{x_n\}$ in A with $x_n \to x$ as $n \to \infty$ implies $x \in A$.

(ii) A is said to be algebraically closed with respect to the difference if $x - y \in A$ for all $x, y \in A$.

Definition 1.2 Let $S, T : E_0 \to E$ be two mappings. A point $\phi \in E_0$ is said to be a PPF dependent common fixed point or a common fixed point with PPF dependence of S and T if $S\phi = \phi(c) = T\phi$ for some $c \in I$.

Recently, Dhage [9] proved the existence of PPF common fixed points for mappings satisfying the condition of Cirić type generalized contraction in a Razumikhin class.

Definition 1.3 Two mappings $S, T : E_0 \to E$ are said to satisfy the condition of Cirić type generalized contraction if there exists a real number $\lambda \in [0, 1)$ satisfying

$$\|S\phi - T\alpha\| \leq \lambda \max\left\{\|\phi - \alpha\|_{E_0}, \|\phi(c) - S\phi\|_E, \|\alpha(c) - T\alpha\|_E, \frac{1}{2}\left[\|\phi(c) - T\alpha\|_E + \|\alpha(c) - S\phi\|_E\right]\right\} \tag{1.3}$$

for all $\phi, \alpha \in E_0$ and for some $c \in I$.

Theorem 1.4 (Dhage [9]) *Suppose that $S, T : E_0 \to E$ satisfy the condition of Cirić type generalized contraction. Assume that $\mathcal{R}_c$ is topologically closed with respect to the norm topology and is algebraically closed with respect to the difference, then S and T have a unique PPF dependent common fixed point in $\mathcal{R}_c$.*

Definition 1.5 Let $A : E_0 \to E$ and $S : E_0 \to E_0$. A point $\phi \in E_0$ is said to be a PPF dependent coincidence point or a coincident point with PPF dependence of A and S if $A\phi = S\phi(c)$ for some $c \in I$.

Dhage [9] also assured the existence of PPF dependent coincidence points for mappings satisfying the condition of Cirić type generalized contraction (C) in a Razumikhin class.

Definition 1.6 $A : E_0 \to E$ and $S : E_0 \to E_0$ are said to satisfy the condition of Cirić type generalized contraction (C) if there exists a real number $\lambda \in [0, 1)$ satisfying

$$\|A\phi - A\alpha\|_E \leq \lambda \max\left\{\|S\phi - S\alpha\|_{E_0}, \|S\phi(c) - A\phi\|_E, \|S\alpha(c) - A\alpha\|_E, \frac{1}{2}\left[\|S\phi(c) - A\alpha\|_E + \|S\alpha(c) - A\phi\|_E\right]\right\} \tag{1.4}$$

for all $\phi, \alpha \in E_0$ and for some $c \in I$.

Theorem 1.7 (Dhage [9]) *Let $A : E_0 \to E$ and $S : E_0 \to E_0$ be two mappings satisfying the condition of Cirić type generalized contraction* (C). *Suppose that*

(a) $A(E_0) \subseteq S(E_0)(c)$;

(b) $S(E_0)$ *is complete*;
(c) S *is continuous*.

If $\mathcal{R}_c$ *is topologically closed with respect to the norm topology and is algebraically closed with respect to the difference, then* A *and* S *have a PPF dependent coincidence point in* $\mathcal{R}_c$.

In this work, we prove the existence of PPF dependent common fixed point theorems for mappings satisfying the contractive condition which is weaker than the condition of Cirić type generalized contraction. Furthermore, we also prove the PPF dependent coincidence points for mappings satisfying the weaker contractive condition mentioned in [9] without being topologically closed with respect to the norm topology of a Razumikhin class. Our results extend Theorem 1.4 and partially generalize Theorem 1.7.

2 PPF dependent common fixed points

Let Ψ be the set of all functions ψ where $\psi : [0,+\infty) \to [0,+\infty)$ is a continuous non-decreasing function with $\psi(t) < t$ for all $t \in (0,+\infty)$ and $\psi(0) = 0$. If $\psi \in \Psi$, then ψ is called a Ψ-map. We now introduce the definition of the condition of Cirić type generalized ψ-contraction and prove our first result.

Definition 2.1 Let $S,T : E_0 \to E$. We say that S and T satisfy the condition of Cirić type generalized ψ-contraction if

$$\|S\phi - T\alpha\|_E \le \psi\left(\max\left\{\|\phi-\alpha\|_{E_0}, \|\phi(c) - S\phi\|_E, \|\alpha(c) - T\alpha\|_E, \frac{1}{2}\left[\|\phi(c) - T\alpha\|_E + \|\alpha(c) - S\phi\|_E\right]\right\}\right) \tag{2.1}$$

for all $\phi,\alpha \in E_0$ and for some $c \in I$.

Theorem 2.2 *Suppose that* $S,T : E_0 \to E$ *satisfy the condition of Cirić type generalized* ψ*-contraction. Assume that* $\mathcal{R}_c$ *is topologically closed with respect to norm topology and is algebraically closed with respect to the difference, then* S *and* T *have a unique PPF dependent common fixed point in* $\mathcal{R}_c$.

Proof Let $\phi_0 \in \mathcal{R}_c$. Since $S\phi_0 \in E$, there exists $x_1 \in E$ such that $S\phi_0 = x_1$. Choose $\phi_1 \in \mathcal{R}_c$ such that

$$x_1 = \phi_1(c) \quad \text{and} \quad \|\phi_1 - \phi_0\|_{E_0} = \|\phi_1(c) - \phi_0(c)\|_E.$$

Since $\phi_1 \in \mathcal{R}_c$ and by assumption, we have $T\phi_1 \in E$. This implies that there exists $x_2 \in E$ such that $T\phi_1 = x_2$. Therefore, we can choose $\phi_2 \in \mathcal{R}_c$ such that

$$x_2 = \phi_2(c) \quad \text{and} \quad \|\phi_2 - \phi_1\|_{E_0} = \|\phi_2(c) - \phi_1(c)\|_E.$$

By continuing the process as before, we can construct the sequence $\{\phi_n\}$ such that

$$S\phi_{2n} = \phi_{2n+1}(c), \qquad T\phi_{2n+1} = \phi_{2n+2}(c)$$

and

$$\|\phi_n - \phi_{n+1}\|_{E_0} = \left\|\phi_n(c) - \phi_{n+1}(c)\right\|_E$$

for all $n \in \mathbb{N} \cup \{0\}$. We will show that $\{\phi_n\}$ is a Cauchy sequence in E_0. Assume that $\phi_{N-1} = \phi_N$ for some $N \in \mathbb{N}$. If N is even, then we have $N = 2m$ for some $m \in \mathbb{N}$. Therefore

$$\begin{aligned}
&\|\phi_{2m} - \phi_{2m+1}\|_{E_0} \\
&\quad = \left\|\phi_{2m}(c) - \phi_{2m+1}(c)\right\|_E \\
&\quad = \|S\phi_{2m} - T\phi_{2m-1}\|_E \\
&\quad \leq \psi\Bigg(\max\Bigg\{\|\phi_{2m} - \phi_{2m-1}\|_{E_0}, \left\|\phi_{2m}(c) - S\phi_{2m}\right\|_E, \left\|\phi_{2m-1}(c) - T\phi_{2m-1}\right\|_E, \\
&\qquad \frac{1}{2}\left[\left\|\phi_{2m}(c) - T\phi_{2m-1}\right\|_E + \left\|\phi_{2m-1}(c) - S\phi_{2m}\right\|_E\right]\Bigg\}\Bigg) \\
&\quad \leq \psi\Bigg(\max\Bigg\{\|\phi_{2m} - \phi_{2m-1}\|_{E_0}, \left\|\phi_{2m}(c) - \phi_{2m+1}(c)\right\|_E, \left\|\phi_{2m-1}(c) - \phi_{2m}(c)\right\|_E, \\
&\qquad \frac{1}{2}\left[\left\|\phi_{2m}(c) - \phi_{2m}(c)\right\|_E + \left\|\phi_{2m-1}(c) - \phi_{2m+1}(c)\right\|_E\right]\Bigg\}\Bigg) \\
&\quad \leq \psi\Bigg(\max\Bigg\{\|\phi_{2m} - \phi_{2m-1}\|_{E_0}, \|\phi_{2m} - \phi_{2m+1}\|_{E_0}, \frac{1}{2}\|\phi_{2m-1} - \phi_{2m+1}\|_{E_0}\Bigg\}\Bigg) \\
&\quad \leq \psi\Bigg(\max\Bigg\{\|\phi_{2m} - \phi_{2m-1}\|_{E_0}, \|\phi_{2m} - \phi_{2m+1}\|_{E_0}, \\
&\qquad \frac{1}{2}\|\phi_{2m-1} - \phi_{2m}\|_{E_0} + \|\phi_{2m} - \phi_{2m+1}\|_{E_0}\Bigg\}\Bigg) \\
&\quad \leq \psi\left(\max\left\{\|\phi_{2m} - \phi_{2m-1}\|_{E_0}, \|\phi_{2m} - \phi_{2m+1}\|_{E_0}\right\}\right) \\
&\quad \leq \psi\left(\max\left\{\|\phi_{2m} - \phi_{2m+1}\|_{E_0}\right\}\right).
\end{aligned}$$

This implies that $\|\phi_{2m} - \phi_{2m+1}\|_{E_0} = 0$ and so $\phi_{2m} = \phi_{2m+1}$. Similarly, we can prove that $\phi_{2m+1} = \phi_{2m+2}$. Therefore $\phi_N = \phi_{N+1}$. By mathematical induction, we can conclude that $\phi_{N-1} = \phi_{N+k}$ for all $k \geq 0$. If N is odd, then by the same argument we also obtain that $\phi_{N-1} = \phi_{N+k}$ for all $k \geq 0$. Therefore $\{\phi_n\}$ is a constant sequence for all $n \geq N - 1$. This implies that $\{\phi_n\}$ is a Cauchy sequence in E_0. Suppose that $\phi_{n-1} \neq \phi_n$ for all $n \in \mathbb{N}$. For each $n \in \mathbb{N}$, we obtain that

$$\begin{aligned}
\|\phi_{2n} - \phi_{2n+1}\|_{E_0} &= \left\|\phi_{2n}(c) - \phi_{2n+1}(c)\right\|_E \\
&= \|S\phi_{2n} - T\phi_{2n-1}\|_E \\
&\leq \psi\Bigg(\max\Bigg\{\|\phi_{2n} - \phi_{2n-1}\|_{E_0}, \left\|\phi_{2n}(c) - S\phi_{2n}\right\|_E, \left\|\phi_{2n-1}(c) - T\phi_{2n-1}\right\|_E, \\
&\quad \frac{1}{2}\left[\left\|\phi_{2n}(c) - T\phi_{2n-1}\right\|_E + \left\|\phi_{2n-1}(c) - S\phi_{2n}\right\|_E\right]\Bigg\}\Bigg) \\
&\leq \psi\Bigg(\max\Bigg\{\|\phi_{2n} - \phi_{2n-1}\|_{E_0}, \left\|\phi_{2n}(c) - \phi_{2n+1}(c)\right\|_E, \left\|\phi_{2n-1}(c) - \phi_{2n}(c)\right\|_E,
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}\bigl[\bigl\|\phi_{2n}(c)-\phi_{2n}(c)\bigr\|_E+\bigl\|\phi_{2n-1}(c)-\phi_{2n+1}(c)\bigr\|_E\bigr]\biggr\}\biggr) \\
&\leq \psi\left(\max\left\{\|\phi_{2n}-\phi_{2n-1}\|_{E_0},\|\phi_{2n}-\phi_{2n+1}\|_{E_0},\frac{1}{2}\|\phi_{2n-1}-\phi_{2n+1}\|_{E_0}\right\}\right) \\
&\leq \psi\left(\max\left\{\|\phi_{2n}-\phi_{2n-1}\|_{E_0},\|\phi_{2n}-\phi_{2n+1}\|_{E_0},\right.\right. \\
&\left.\left.\frac{1}{2}\|\phi_{2n-1}-\phi_{2n}\|_{E_0}+\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\right\}\right) \\
&\leq \psi\bigl(\max\bigl\{\|\phi_{2n}-\phi_{2n-1}\|_{E_0},\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\bigr\}\bigr).
\end{aligned}$$

If $\max\{\|\phi_{2n}-\phi_{2n-1}\|_{E_0},\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\}=\|\phi_{2n}-\phi_{2n+1}\|_{E_0}$, then

$$\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\leq\psi\bigl(\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\bigr)<\|\phi_{2n}-\phi_{2n+1}\|_{E_0}.$$

This leads to a contradiction. Therefore

$$\begin{aligned}
\|\phi_{2n}-\phi_{2n+1}\|_{E_0} &\leq \psi\bigl(\|\phi_{2n}-\phi_{2n-1}\|_{E_0}\bigr) \\
&< \|\phi_{2n}-\phi_{2n-1}\|_{E_0}.
\end{aligned}$$

Thus $\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\leq\|\phi_{2n}-\phi_{2n-1}\|_{E_0}$. Similarly, we can prove that

$$\|\phi_{2n+1}-\phi_{2n+2}\|_{E_0}\leq\|\phi_{2n}-\phi_{2n+1}\|_{E_0}.$$

It follows that $\|\phi_n-\phi_{n+1}\|_{E_0}\leq\|\phi_{n-1}-\phi_n\|_{E_0}$ for all $n\in\mathbb{N}$. Since the sequence $\{\|\phi_n-\phi_{n+1}\|_{E_0}\}$ is a nonincreasing sequence of nonnegative real numbers, we obtain that it is a convergent sequence. Suppose that

$$\lim_{n\to\infty}\|\phi_n-\phi_{n+1}\|_{E_0}=\alpha$$

for some nonnegative real number α. We will prove that $\alpha=0$. Suppose that $\alpha>0$. Since

$$\|\phi_{2n}-\phi_{2n+1}\|_{E_0}\leq\psi\bigl(\|\phi_{2n}-\phi_{2n-1}\|_{E_0}\bigr)$$

for all $n\in\mathbb{N}$ and the continuity of ψ, we have

$$\alpha\leq\psi(\alpha)<\alpha,$$

which leads to a contradiction. This implies that $\alpha=0$. We next prove that the sequence $\{\phi_n\}$ is a Cauchy sequence in E_0. It suffices to prove that the sequence $\{\phi_{2n}\}$ is a Cauchy sequence in E_0. Assume that $\{\phi_{2n}\}$ is not a Cauchy sequence. It follows that there exist $\varepsilon>0$ and two sequences of even positive integers $\{2m_k\}$ and $\{2n_k\}$ satisfying $2m_k>2n_k>k$ for each $k\in\mathbb{N}$ and

$$\|\phi_{2m_k}-\phi_{2n_k}\|_{E_0}\geq\varepsilon. \tag{2.2}$$

Let $\{2m_k\}$ be the sequence of the least positive integers exceeding $\{2n_k\}$ which satisfies (2.2) and

$$\|\phi_{2m_k-2} - \phi_{2n_k}\|_{E_0} < \varepsilon. \tag{2.3}$$

We will prove that $\lim_{k\to\infty} \|\phi_{2m_k} - \phi_{2n_k}\|_{E_0} = \varepsilon$. Since $\|\phi_{2m_k} - \phi_{2n_k}\|_{E_0} \geq \varepsilon$ for all $k \in \mathbb{N}$, we have

$$\lim_{k\to\infty} \|\phi_{2m_k} - \phi_{2n_k}\|_{E_0} \geq \varepsilon.$$

For each $k \in \mathbb{N}$, we obtain that

$$\begin{aligned}\|\phi_{2m_k} - \phi_{2n_k}\|_{E_0} &\leq \|\phi_{2m_k} - \phi_{2m_k-1}\|_{E_0} + \|\phi_{2m_k-1} - \phi_{2m_k-2}\|_{E_0} + \|\phi_{2m_k-2} - \phi_{2n_k}\|_{E_0} \\ &\leq \|\phi_{2m_k} - \phi_{2m_k-1}\|_{E_0} + \|\phi_{2m_k-1} - \phi_{2m_k-2}\|_{E_0} + \varepsilon.\end{aligned}$$

This implies that $\lim_{k\to\infty} \|\phi_{2m_k} - \phi_{2n_k}\|_{E_0} \leq \varepsilon$. Therefore

$$\lim_{k\to\infty} \|\phi_{2m_k} - \phi_{2n_k}\|_{E_0} = \varepsilon.$$

Similarly, we can prove that

$$\lim_{k\to\infty} \|\phi_{2m_k+1} - \phi_{2n_k}\|_{E_0} = \varepsilon, \qquad \lim_{k\to\infty} \|\phi_{2m_k} - \phi_{2n_k-1}\|_{E_0} = \varepsilon$$

and

$$\lim_{k\to\infty} \|\phi_{2m_k+1} - \phi_{2n_k-1}\|_{E_0} = \varepsilon.$$

For each $k \in \mathbb{N}$, we obtain that

$$\begin{aligned}\|\phi_{2n_k} - \phi_{2m_k+1}\|_{E_0} &\leq \left\|\phi_{2n_k}(c) - \phi_{2m_k+1}(c)\right\|_E \\ &\leq \|S\phi_{2m_k} - T\phi_{2n_k-1}\|_E \\ &\leq \psi\Bigg(\max\Bigg\{\|\phi_{2m_k} - \phi_{2n_k-1}\|_{E_0}, \left\|\phi_{2m_k}(c) - S\phi_{2m_k}\right\|_E, \\ &\quad \left\|\phi_{2n_k-1}(c) - T\phi_{2n_k-1}\right\|_E, \\ &\quad \frac{1}{2}\left[\left\|\phi_{2m_k}(c) - T\phi_{2n_k-1}\right\|_E + \left\|\phi_{2n_k-1}(c) - S\phi_{2n_k}\right\|_E\right]\Bigg\}\Bigg) \\ &\leq \psi\Bigg(\max\Bigg\{\|\phi_{2m_k} - \phi_{2n_k-1}\|_{E_0}, \left\|\phi_{2m_k}(c) - \phi_{2m_k+1}(c)\right\|_E, \\ &\quad \left\|\phi_{2n_k-1}(c) - \phi_{2n_k}(c)\right\|_E, \\ &\quad \frac{1}{2}\left[\left\|\phi_{2m_k}(c) - \phi_{2n_k}(c)\right\|_E + \left\|\phi_{2n_k-1}(c) - \phi_{2n_k+1}(c)\right\|_E\right]\Bigg\}\Bigg) \\ &\leq \psi\Bigg(\max\Bigg\{\|\phi_{2m_k} - \phi_{2n_k-1}\|_{E_0}, \|\phi_{2m_k} - \phi_{2m_k+1}\|_{E_0}, \\ &\quad \|\phi_{2n_k-1} - \phi_{2n_k}\|_{E_0},\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}\bigl[\|\phi_{2m_k}-\phi_{2n_k}\|_{E_0}+\|\phi_{2n_k-1}-\phi_{2n_k+1}\|_{E_0}\bigr]\biggr\}\biggr)\\
&\leq \psi\biggl(\max\biggl\{\|\phi_{2m_k}-\phi_{2n_k-1}\|_{E_0},\|\phi_{2m_k}-\phi_{2m_k+1}\|_{E_0},\\
&\|\phi_{2n_k-1}-\phi_{2n_k}\|_{E_0},\\
&\frac{1}{2}\bigl[\|\phi_{2m_k}-\phi_{2n_k}\|_{E_0}+\|\phi_{2n_k-1}-\phi_{2n_k}\|_{E_0}+\|\phi_{2n_k}-\phi_{2n_k+1}\|_{E_0}\bigr]\biggr\}\biggr).
\end{aligned}$$

By taking the limit of both sides, we have

$$\varepsilon \leq \psi(\varepsilon) < \varepsilon,$$

which leads to a contradiction. It follows that the sequence $\{\phi_{2n}\}$ is a Cauchy sequence and so $\{\phi_n\}$ is a Cauchy sequence. By the completeness of E_0, we have $\{\phi_n\}$ is a convergent sequence. Suppose that $\lim_{n\to\infty}\phi_n = \phi$ for some $\phi \in E_0$. Since $\mathcal{R}_c$ is algebraically closed with respect to the norm topology, we have $\phi \in \mathcal{R}_c$. Moreover, we also obtain that

$$\lim_{n\to\infty}\phi_{2n+1} = \phi = \lim_{n\to\infty}\phi_{2n+2}.$$

We will prove that ϕ is a PPF dependent fixed point of S. By using (2.1), we obtain that

$$\begin{aligned}
\bigl\|S\phi-\phi(c)\bigr\|_E &\leq \bigl\|S\phi-\phi_{2n+2}(c)\bigr\|_E+\bigl\|\phi_{2n+2}(c)-\phi(c)\bigr\|_E\\
&\leq \|S\phi-T\phi_{2n+1}\|_E+\|\phi_{2n+2}-\phi\|_{E_0}\\
&\leq \psi\biggl(\max\biggl\{\|\phi-\phi_{2n+1}\|_{E_0},\bigl\|\phi(c)-S\phi\bigr\|_E,\bigl\|\phi_{2n+1}(c)-T\phi_{2n+1}\bigr\|_E,\\
&\quad\frac{1}{2}\bigl[\bigl\|\phi(c)-T\phi_{2n+1}\bigr\|_E+\bigl\|\phi_{2n+1}(c)-S\phi\bigr\|_E\bigr]\biggr\}\biggr)+\|\phi_{2n+2}-\phi\|_{E_0}\\
&\leq \psi\biggl(\max\biggl\{\|\phi-\phi_{2n+1}\|_{E_0},\bigl\|\phi(c)-S\phi\bigr\|_E,\bigl\|\phi_{2n+1}(c)-\phi_{2n+2}(c)\bigr\|_E,\\
&\quad\frac{1}{2}\bigl[\bigl\|\phi(c)-\phi_{2n+2}(c)\bigr\|_E+\bigl\|\phi_{2n+1}(c)-S\phi\bigr\|_E\bigr]\biggr\}\biggr)+\|\phi_{2n+2}-\phi\|_{E_0}\\
&\leq \psi\biggl(\max\biggl\{\|\phi-\phi_{2n+1}\|_{E_0},\bigl\|\phi(c)-S\phi\bigr\|_E,\|\phi_{2n+1}-\phi_{2n+2}\|_{E_0},\\
&\quad\frac{1}{2}\bigl[\|\phi-\phi_{2n+2}\|_{E_0}+\bigl\|\phi_{2n+1}(c)-S\phi\bigr\|_E\bigr]\biggr\}\biggr)+\|\phi_{2n+2}-\phi\|_{E_0}.
\end{aligned}$$

Letting $n\to\infty$, we obtain that $\|S\phi-\phi(c)\|_E = 0$. Therefore $S\phi = \phi(c)$. Similarly, we can prove that $T\phi = \phi(c)$. This implies that ϕ is a PPF dependent common fixed point of S and T. We finally prove the uniqueness of the PPF dependent fixed point of S and T in $\mathcal{R}_c$. Let $\alpha \in \mathcal{R}_c$ be any PPF dependent common fixed point of S and T. Therefore

$$\begin{aligned}
\|\phi-\alpha\|_{E_0} &= \bigl\|\phi(c)-\alpha(c)\bigr\|_E\\
&\leq \|S\phi-T\alpha\|_E\\
&\leq \psi\biggl(\max\biggl\{\|\phi-\alpha\|_{E_0},\bigl\|\phi(c)-S\phi\bigr\|_E,\bigl\|\alpha(c)-T\alpha\bigr\|_E,
\end{aligned}$$

$$\frac{1}{2}\big[\|\phi(c) - T\alpha\|_E + \|\alpha(c) - S\phi\|_E\big]\bigg\}\bigg)$$
$$\leq \psi\bigg(\max\bigg\{\|\phi - \alpha\|_{E_0}, \frac{1}{2}\big[\|\phi(c) - \alpha(c)\|_E + \|\alpha(c) - \phi(c)\|_E\big]\bigg\}\bigg)$$
$$\leq \psi\big(\max\{\|\phi - \alpha\|_{E_0}, \|\phi - \alpha\|_{E_0}\}\big)$$
$$= \psi\big(\|\phi - \alpha\|_{E_0}\big).$$

It follows that $\phi = \alpha$. Hence S and T have a unique PPF dependent common fixed point in $\mathcal{R}_c$. □

Remark 2.3 From the proof of Theorem 2.2, we obtain the following observations:

(1) We assume that the Razumikhin class $\mathcal{R}_c$ is algebraically closed with respect to difference, that is, $\phi - \alpha \in \mathcal{R}_c$ for all $\phi, \alpha \in \mathcal{R}_c$, in order to construct the sequence $\{\phi_n\}$ satisfying

$$\|\phi - \alpha\|_{E_0} = \|\phi(c) - \alpha(c)\|_E.$$

(2) If the Razumikhin class $\mathcal{R}_c$ is not assumed to be topologically closed, then the limit of the sequence $\{\phi_n\}$ may be outside of $\mathcal{R}_c$.

By applying Theorem 2.2, we obtain the following corollary which is Theorem 3.3 in [9].

Corollary 2.4 *Suppose that $S, T : E_0 \to E$ satisfy the condition of Cirić type generalized contraction. Assume that $\mathcal{R}_c$ is topologically and algebraically closed with respect to the difference, then S and T have a unique PPF dependent fixed point in $\mathcal{R}_c$.*

Proof Define a function $\psi : [0, +\infty) \to [0, +\infty)$ by $\psi(t) = \lambda t$ for all $t \in [0, +\infty)$. Therefore ψ is a continuous nondecreasing function and

$$\psi(t) < t \quad \text{for all } t \in (0, +\infty) \text{ and } \psi(0) = 0.$$

This implies that all assumptions in Theorem 2.2 are satisfied. Hence the proof is complete. □

3 PPF dependent coincidence points

Definition 3.1 Let $A : E_0 \to E$ and $S : E_0 \to E_0$. A point $\phi \in E_0$ is said to be a PPF dependent coincidence point or a coincidence point with PPF dependence of A and S if $A\phi = S\phi(c)$ for some $c \in I$.

Definition 3.2 Let $A : E_0 \to E$ and $S : E_0 \to E_0$. We say that A and S satisfy the condition of Cirić type generalized contraction (C) if there exists a real number $\lambda \in [0, 1)$ satisfying

$$\|A\phi - A\alpha\|_E \leq \lambda \max\bigg\{\|S\phi - S\alpha\|_{E_0}, \|S\phi(c) - A\phi\|_E, \|S\alpha(c) - A\alpha\|_E, \frac{1}{2}\big[\|S\phi(c) - A\alpha\|_E + \|S\alpha(c) - A\phi\|_E\big]\bigg\} \tag{3.1}$$

for all $\phi, \alpha \in E_0$ and for some $c \in I$.

We introduce the following contractive condition which is weaker than the condition of Cirić type generalized contraction (C).

Definition 3.3 Let $A : E_0 \to E$ and $S : E_0 \to E_0$. We say that A and S satisfy the condition of Cirić type generalized ψ-contraction (C) if

$$\begin{aligned}\|A\phi - A\alpha\|_E \le \psi\Bigg(\max\bigg\{&\|S\phi - S\alpha\|_{E_0}, \big\|S\phi(c) - A\phi\big\|_E, \big\|S\alpha(c) - A\alpha\big\|_E, \\ &\frac{1}{2}\big[\big\|S\phi(c) - A\alpha\big\|_E + \big\|S\alpha(c) - A\phi\big\|_E\big]\bigg\}\Bigg) \qquad (3.2)\end{aligned}$$

for all $\phi, \alpha \in E_0$ and for some $c \in I$.

We now prove the existence of PPF dependent coincidence points for mappings satisfying the weaker contractive condition assumed in [9] without being topologically closed of the Razumikhin class.

Theorem 3.4 *Let $A : E_0 \to E$ and $S : E_0 \to E_0$ be two mappings satisfying the condition of Cirić type generalized ψ-contraction* (C). *Suppose that*

(a) $A(E_0) \subseteq S(E_0)(c)$;
(b) $S(\mathcal{R}_c)$ *is complete*;
(c) S *is continuous*.

If $\mathcal{R}_c$ is algebraically closed with respect to the difference, then A and S have a PPF dependent coincidence point in $\mathcal{R}_c$.

Proof Let $\phi_0 \in \mathcal{R}_c$. Since $A\phi_0 \in E$, there exists $x_1 \in E$ such that $A\phi_0 = x_1$. Because $A(E_0) \subseteq S(E_0)(c)$, we can choose $\phi_1 \in \mathcal{R}_c$ such that

$$x_1 = S\phi_1(c) = \alpha_1(c) \quad \text{and} \quad \|\alpha_1 - \alpha_0\|_{E_0} = \big\|\alpha_1(c) - \alpha_0(c)\big\|_E.$$

Since $A\phi_1 \in E_0$ and by assumption, we have $A\phi_1 = x_2$ for some $x_2 \in E$. Because $A(E_0) \subseteq S(E_0)(c)$, we can choose $\phi_2 \in \mathcal{R}_c$ such that

$$x_2 = S\phi_2(c) = \alpha_2(c) \quad \text{and} \quad \|\alpha_2 - \alpha_1\|_{E_0} = \big\|\alpha_2(c) - \alpha_1(c)\big\|_E.$$

By continuing the process as before, we can construct the sequence $\{\alpha_n\}$ such that

$$A\phi_n = S\phi_{n+1}(c), \qquad S\phi_{n+1} = \alpha_{n+1}$$

and

$$\|\alpha_n - \alpha_{n+1}\|_{E_0} = \big\|\alpha_n(c) - \alpha_{n+1}(c)\big\|_E$$

for all $n \in \mathbb{N} \cup \{0\}$. We will show that $\{\alpha_n\}$ is a Cauchy sequence in E_0. If $\alpha_N = \alpha_{N+1}$ for some $N \in \mathbb{N}$, then we have

$$\begin{aligned}\|\alpha_{N+1} - \alpha_{N+2}\|_{E_0} &= \big\|\alpha_{N+1}(c) - \alpha_{N+2}(c)\big\|_E \\ &= \|A\phi_N - A\phi_{N+1}\|_E\end{aligned}$$

$$\begin{aligned}
&\le \psi\Bigg(\max\bigg\{\|S\phi_N - S\phi_{N+1}\|_{E_0}, \big\|S\phi_N(c) - A\phi_N\big\|_E, \\
&\quad \big\|S\phi_{N+1}(c) - A\phi_{N+1}\big\|_E, \\
&\quad \frac{1}{2}\big[\big\|S\phi_N(c) - A\phi_{N+1}\big\|_E + \big\|S\phi_{N+1}(c) - A\phi_N\big\|_E\big]\bigg\}\Bigg) \\
&\le \psi\Bigg(\max\bigg\{\|\alpha_N - \alpha_{N+1}\|_{E_0}, \big\|\alpha_N(c) - \alpha_{N+1}(c)\big\|_E, \\
&\quad \big\|\alpha_{N+1}(c) - \alpha_{N+2}(c)\big\|_E, \\
&\quad \frac{1}{2}\big[\big\|\alpha_N(c) - \alpha_{N+2}(c)\big\|_E + \big\|\alpha_{N+1}(c) - \alpha_{N+1}(c)\big\|_E\big]\bigg\}\Bigg) \\
&\le \psi\Bigg(\max\bigg\{\|\alpha_N - \alpha_{N+1}\|_{E_0}, \|\alpha_N - \alpha_{N+1}\|_{E_0}, \\
&\quad \|\alpha_{N+1} - \alpha_{N+2}\|_{E_0}, \frac{1}{2}\|\alpha_N - \alpha_{N+2}\|_{E_0}\bigg\}\Bigg) \\
&\le \psi\Bigg(\max\bigg\{\|\alpha_N - \alpha_{N+1}\|_{E_0}, \|\alpha_{N+1} - \alpha_{N+2}\|_{E_0}, \frac{1}{2}\big[\|\alpha_N - \alpha_{N+2}\|_{E_0}\big]\bigg\}\Bigg) \\
&\le \psi\Bigg(\max\bigg\{\|\alpha_N - \alpha_{N+1}\|_{E_0}, \|\alpha_{N+1} - \alpha_{N+2}\|_{E_0}, \\
&\quad \frac{1}{2}\big[\|\alpha_N - \alpha_{N+1}\|_{E_0} + \|\alpha_{N+1} - \alpha_{N+2}\|_{E_0}\big]\bigg\}\Bigg) \\
&\le \psi\big(\max\big\{\|\alpha_N - \alpha_{N+1}\|_{E_0}, \|\alpha_{N+1} - \alpha_{N+2}\|_{E_0}\big\}\big) \\
&\le \psi\big(\|\alpha_{N+1} - \alpha_{N+2}\|_{E_0}\big).
\end{aligned}$$

Therefore $\alpha_{N+1} = \alpha_{N+2}$. By mathematical induction, we obtain that

$$\alpha_N = \alpha_{N+k} \quad \text{for all } k \in \mathbb{N}.$$

This implies that $\{\alpha_n\}$ is a constant sequence for $n \ge N$. Thus $\{\alpha_n\}$ is a Cauchy sequence in E_0. Suppose that $\alpha_n \ne \alpha_{n+1}$ for all $n \in \mathbb{N}$. For each $n \in \mathbb{N}$, we have

$$\begin{aligned}
\|\alpha_{n+1} - \alpha_{n+2}\|_{E_0} &= \big\|\alpha_{n+1}(c) - \alpha_{n+2}(c)\big\|_E \\
&= \|A\phi_n - A\phi_{n+1}\|_E \\
&\le \psi\Bigg(\max\bigg\{\|S\phi_n - S\phi_{n+1}\|_{E_0}, \big\|S\phi_n(c) - A\phi_n\big\|_E, \\
&\quad \big\|S\phi_{n+1}(c) - A\phi_{n+1}\big\|_E, \\
&\quad \frac{1}{2}\big[\big\|S\phi_n(c) - A\phi_{n+1}\big\|_E + \big\|S\phi_{n+1}(c) - A\phi_n\big\|_E\big]\bigg\}\Bigg) \\
&\le \psi\Bigg(\max\bigg\{\|\alpha_n - \alpha_{n+1}\|_{E_0}, \big\|\alpha_n(c) - \alpha_{n+1}(c)\big\|_E, \\
&\quad \big\|\alpha_{n+1}(c) - \alpha_{n+2}(c)\big\|_E, \\
&\quad \frac{1}{2}\big[\big\|\alpha_n(c) - \alpha_{n+2}(c)\big\|_E + \big\|\alpha_{n+1}(c) - \alpha_{n+1}(c)\big\|_E\big]\bigg\}\Bigg)
\end{aligned}$$

$$\begin{aligned}
&\leq \psi\Big(\max\Big\{\|\alpha_n-\alpha_{n+1}\|_{E_0},\|\alpha_n-\alpha_{n+1}\|_{E_0},\\
&\quad \|\alpha_{n+1}-\alpha_{n+2}\|_{E_0},\frac{1}{2}\|\alpha_n-\alpha_{n+2}\|_{E_0}\Big\}\Big)\\
&\leq \psi\Big(\max\Big\{\|\alpha_n-\alpha_{n+1}\|_{E_0},\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0},\frac{1}{2}\big[\|\alpha_n-\alpha_{n+2}\|_{E_0}\big]\Big\}\Big)\\
&\leq \psi\Big(\max\Big\{\|\alpha_n-\alpha_{n+1}\|_{E_0},\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0},\\
&\quad \frac{1}{2}\big[\|\alpha_n-\alpha_{n+1}\|_{E_0}+\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0}\big]\Big\}\Big)\\
&\leq \psi\big(\max\big\{\|\alpha_n-\alpha_{n+1}\|_{E_0},\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0}\big\}\big).
\end{aligned}$$

If $\max\{\|\alpha_n-\alpha_{n+1}\|_{E_0},\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0}\} = \|\alpha_{n+1}-\alpha_{n+2}\|_{E_0}$, then

$$\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0} \leq \psi\big(\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0}\big) < \|\alpha_{n+1}-\alpha_{n+2}\|_{E_0}.$$

This leads to a contradiction. Therefore

$$\begin{aligned}
\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0} &\leq \psi\big(\|\alpha_n-\alpha_{n+1}\|_{E_0}\big)\\
&< \|\alpha_n-\alpha_{n+1}\|_{E_0}.
\end{aligned}$$

It follows that $\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0} \leq \|\alpha_n-\alpha_{n+1}\|_{E_0}$ for all $n \in \mathbb{N}$. Since the sequence $\{\|\alpha_n - \alpha_{n+1}\|_{E_0}\}$ is a nonincreasing sequence of real numbers, we obtain that it is a convergent sequence. Suppose that

$$\lim_{n\to\infty}\|\alpha_n-\alpha_{n+1}\|_{E_0} = \alpha$$

for some nonnegative real number α. We will prove that $\alpha = 0$. Assume that $\alpha > 0$. Since

$$\|\alpha_{n+1}-\alpha_{n+2}\|_{E_0} \leq \psi\big(\|\alpha_n-\alpha_{n+1}\|_{E_0}\big)$$

for all $n \in \mathbb{N}$ and the continuity of ψ, we have

$$\alpha \leq \psi(\alpha) < \alpha,$$

which leads to a contradiction. This implies that $\alpha = 0$. We will prove that $\{\alpha_n\}$ is a Cauchy sequence in E_0. It suffices to prove that the sequence $\{\alpha_{2n}\}$ is a Cauchy sequence in E_0. Assume that $\{\alpha_{2n}\}$ is not a Cauchy. It follows that there exist $\varepsilon > 0$ and two sequences of even positive integers $\{2m_k\}$ and $\{2n_k\}$ satisfying $2m_k > 2n_k > k$ for each $k \in \mathbb{N}$ and

$$\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0} \geq \varepsilon. \tag{3.3}$$

Let $\{2m_k\}$ be the sequence of the least positive integers exceeding $\{2n_k\}$ which satisfies (3.3) and

$$\|\alpha_{2m_k-2}-\alpha_{2n_k}\|_{E_0} < \varepsilon. \tag{3.4}$$

We will prove that $\lim_{k\to\infty}\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}=\varepsilon$. Since $\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}\geq\varepsilon$ for all $k\in\mathbb{N}$, we have

$$\lim_{k\to\infty}\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}\geq\varepsilon.$$

For each $k\in\mathbb{N}$, we obtain that

$$\begin{aligned}\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}&\leq\|\alpha_{2m_k}-\alpha_{2m_k-1}\|_{E_0}+\|\alpha_{2m_k-1}-\alpha_{2m_k-2}\|_{E_0}+\|\alpha_{2m_k-2}-\alpha_{2n_k}\|_{E_0}\\&\leq\|\alpha_{2m_k}-\alpha_{2m_k-1}\|_{E_0}+\|\alpha_{2m_k-1}-\alpha_{2m_k-2}\|_{E_0}+\varepsilon.\end{aligned}$$

This implies that $\lim_{k\to\infty}\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}\leq\varepsilon$. Therefore

$$\lim_{k\to\infty}\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}=\varepsilon.$$

Similarly, we can prove that

$$\lim_{k\to\infty}\|\alpha_{2m_k+1}-\alpha_{2n_k}\|_{E_0}=\varepsilon,\qquad\lim_{k\to\infty}\|\alpha_{2m_k}-\alpha_{2n_k-1}\|_{E_0}=\varepsilon$$

and

$$\lim_{k\to\infty}\|\alpha_{2m_k+1}-\alpha_{2n_k-1}\|_{E_0}=\varepsilon.$$

Since

$$\begin{aligned}\|\alpha_{2n_k}-\alpha_{2m_k+1}\|_{E_0}&\leq\left\|\alpha_{2n_k}(c)-\alpha_{2m_k+1}(c)\right\|_E\\&=\|A\phi_{2n_k-1}-A\phi_{2m_k}\|_E\\&\leq\psi\Bigg(\max\Bigg\{\|S\phi_{2n_k-1}-S\phi_{2m_k}\|_{E_0},\left\|S\phi_{2n_k-1}(c)-A\phi_{2n_k-1}\right\|_E,\\&\qquad\left\|S\phi_{2m_k}(c)-A\phi_{2m_k}\right\|_E,\\&\qquad\frac{1}{2}\Big[\left\|S\phi_{2n_k-1}(c)-A\phi_{2m_k}\right\|_E+\left\|S\phi_{2m_k}(c)-A\phi_{2n_k-1}\right\|_E\Big]\Bigg\}\Bigg)\\&\leq\psi\Bigg(\max\Bigg\{\|\alpha_{2n_k-1}-\alpha_{2m_k}\|_{E_0},\left\|\alpha_{2n_k-1}(c)-\alpha_{2n_k}(c)\right\|_E,\\&\qquad\left\|\alpha_{2m_k}(c)-\alpha_{2m_k+1}(c)\right\|_E,\\&\qquad\frac{1}{2}\Big[\left\|\alpha_{2n_k-1}(c)-\alpha_{2m_k+1}(c)\right\|_E+\left\|\alpha_{2m_k}(c)-\alpha_{2n_k}(c)\right\|_E\Big]\Bigg\}\Bigg)\\&\leq\psi\Bigg(\max\Bigg\{\|\alpha_{2n_k-1}-\alpha_{2m_k}\|_{E_0},\|\alpha_{2n_k-1}-\alpha_{2n_k}\|_{E_0},\\&\qquad\|\alpha_{2m_k}-\alpha_{2m_k+1}\|_{E_0},\frac{1}{2}\|\alpha_{2n_k-1}-\alpha_{2m_k+1}\|_{E_0}+\|\alpha_{2m_k}-\alpha_{2n_k}\|_{E_0}\Bigg\}\Bigg),\end{aligned}$$

by taking the limit of both sides, we have

$$\varepsilon\leq\psi(\varepsilon)<\varepsilon,$$

which leads to a contradiction. It follows that the sequence $\{\alpha_{2n}\}$ is a Cauchy sequence and so $\{\alpha_n\}$ is a Cauchy sequence in E_0. Therefore $\{S\phi_n\}$ is a Cauchy sequence in $S(\mathcal{R}_c)$. By the completeness of $S(\mathcal{R}_c)$, we have $\{S\phi_n\}$ is a convergent sequence. Suppose that $\lim_{n\to\infty} S\phi_n = \phi^*$ for some $\phi^* \in S(\mathcal{R}_c)$. Therefore $\phi^* = S\phi$ for some $\phi \in \mathcal{R}_c$. Moreover, we also have

$$\lim_{n\to\infty} A\phi_n = \lim_{n\to\infty} S\phi_{n+1}(c) = S\phi(c).$$

We will prove that ϕ is a PPF dependent coincidence fixed point of A and S. By using (3.2), we obtain that

$$\begin{aligned}
\|A\phi - S\phi(c)\|_E &\le \|A\phi - A\phi_n\|_E + \|A\phi_n - S\phi(c)\|_E \\
&\le \|A\phi - A\phi_n\|_E + \|S\phi_{n+1}(c) - S\phi(c)\|_E \\
&\le \psi\Bigl(\max\Bigl\{\|S\phi - S\phi_n\|_{E_0}, \|S\phi(c) - A\phi\|_E, \|S\phi_n(c) - A\phi_n\|_E, \\
&\quad \frac{1}{2}\bigl[\|S\phi(c) - A\phi_n\|_E + \|S\phi_n(c) - A\phi\|_E\bigr]\Bigr\}\Bigr) + \|S\phi_{n+1}(c) - S\phi(c)\|_E.
\end{aligned}$$

By taking $n \to \infty$, we obtain that $A\phi = S\phi(c)$. Hence ϕ is a PPF dependent coincidence point of A and S. □

By applying Theorem 3.4, we obtain the following corollary.

Corollary 3.5 *Let $A : E_0 \to E$ and $S : E_0 \to E_0$ be two mappings satisfying the condition of Cirić type generalized contraction* (C). *Suppose that*

(a) $A(E_0) \subseteq S(E_0)(c)$;
(b) $S(\mathcal{R}_c)$ *is complete*;
(c) S *is continuous*.

If $\mathcal{R}_c$ is algebraically closed with respect to the difference, then A and S have a PPF dependent coincidence point in $\mathcal{R}_c$.

Proof Define a function $\psi : [0, +\infty) \to [0, +\infty)$ by $\psi(t) = \lambda t$ for all $t \in [0, +\infty)$. Therefore $A : E_0 \to E$ and $S : E_0 \to E_0$ satisfy the condition of Cirić type generalized ψ-contraction (C). This implies that all assumptions in Theorem 3.4 are now satisfied. Hence the proof is complete. □

Questions

(i) Are the results in Theorem 2.2 and Theorem 3.4 still true when the norm closedness for $\mathcal{R}_c$ is replaced by weak closedness or weak* closedness (for dual Banach spaces)?
(ii) Is there some way to improve the results to more than two mappings or a family of mappings as in the case of nonexpansive mappings (see, for example, [10] and references contained therein)?

Competing interests
The author declares that they have no completing interests.

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