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On σ-type zero of Sheffer polynomials

Ajay K Shukla*, Shrinivas J Rapeli and Pratik V Shah

*Correspondence:
ajayshukla2@rediffmail.com
Department of Mathematics, S. V. National Institute of Technology, Surat, 395 007, India

Abstract
The main object of this paper is to investigate some properties of σ-type polynomials in one and two variables.
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1 Introduction

In 1945, Thorne [1] obtained an interesting characterization of Appell polynomials by means of the Stieltjes integral. Srivastava and Manocha [2] discussed the Appell sets and polynomials. Dattoli *et al.* [3] studied the properties of the Sheffer polynomials. Recently Pintér and Srivastava [4] gave addition theorems for the Appell polynomials and the associated classes of polynomial expansions and some cases have also been discussed by Srivastava and Choi [5] in their book.

Appell sets may be defined by the following equivalent condition: $\{P_n(x)\}$, $n = 0, 1, 2, \ldots$ is an Appell set [6–8] (P_n being of degree exactly n) if either

(i) $P_n'(x) = P_{n-1}(x)$, $n = 0, 1, 2, \ldots$, or

(ii) there exists a formal power series $A(t) = \sum_{n=0}^{\infty} a_n t^n$ ($a_0 \neq 0$) such that

$$A(t)\exp(xt) = \sum_{n=0}^{\infty} P_n(x)t^n.$$

Sheffer's A-type classification

Let $\phi_n(x)$ be a simple set of polynomials and let $\phi_n(x)$ belong to the operator

$$J(x, D) = \sum_{k=0}^{\infty} T_k(x)D^{k+1},$$

with $T_k(x)$ of degree $\leq k$. If the maximum degree of the coefficients $T_k(x)$ is m, then the set $\phi_n(x)$ is of Sheffer A-type m. If the degree of $T_k(x)$ is unbounded as $k \to \infty$, we say that $\phi_n(x)$ is of Sheffer A-type ∞.

Polynomials of Sheffer A-type zero

Let $\phi_n(x)$ be of Sheffer A-type zero. Then $\phi_n(x)$ belong to the operator

$$J(D) = \sum_{k=0}^{\infty} c_k D^{k+1},$$

in which c_k are constants. Here $c_0 \neq 0$ and $J\phi_n = \phi_{n-1}$. Furthermore, since c_k are independent of x for every k, a function $J(t)$ exists with the formal power series expansion

$$J(t) = \sum_{k=0}^{\infty} c_k t^{k+1}, \quad c_0 \neq 0.$$

Let $H(t)$ be the formal inverse of $J(t)$; that is,

$$H\bigl(J(t)\bigr) = J\bigl(H(t)\bigr) = t.$$

Theorem (Rainville [9]) *A necessary and sufficient condition that $\phi_n(x)$ be of Sheffer A-type zero is that $\phi_n(x)$ possess the generating function indicated in*

$$A(t)\exp\bigl(xH(t)\bigr) = \sum_{n=0}^{\infty} \phi_n(x) t^n,$$

in which $H(t)$ and $A(t)$ have (formal) expansions

$$H(t) = \sum_{n=0}^{\infty} h_n t^{n+1}, \quad h_0 \neq 0, \qquad A(t) = \sum_{n=0}^{\infty} a_n t^n, \quad a_0 \neq 0.$$

Theorem (Al-Salam and Verma [10]) *Let $\{P_n(x)\}$ be a polynomial set. In order for $\{P_n(x)\}$ to be a Sheffer A-type zero, it is necessary and sufficient that there exist (formal) power series*

$$H(t) = \sum_{j=1}^{\infty} h_j t^j, \quad h_1 \neq 0, \qquad A_s(t) = \sum_{j=0}^{\infty} a_j^{(s)} t^j \quad \bigl(\textit{not all } a_0^{(s)} \textit{ are zero}\bigr)$$

and

$$\sum_{j=1}^{r} A_j(t)\exp\bigl(xH(\varepsilon_j t)\bigr) = \sum_{n=0}^{\infty} P_n(x) t^n,$$

where

$$J(D)P_n(x) = P_{n-r}(x) \quad (n = r, r+1, \ldots) \textit{ where } J(D) = \sum_{k=0}^{\infty} a_k D^{k+r}, a_0 \neq 0$$

and r is a fixed positive integer. The function $A(t)$ may be called the determining function for the set $\{P_n(x)\}$.

Polynomial of σ-type zero [9, 11]

Let $\{p_n(x)\}$ be a simple set of polynomials that belongs to the operator

$$J(x,\sigma) = \sum_{k=0}^{\infty} T_k(x)\sigma^{k+1},$$

$$\sigma = D\prod_{i=1}^{q}(xD + b_i - 1), \qquad D = \frac{d}{dx}, \qquad \bigl(J(x,\sigma)p_n(x) = p_{n-1}(x)\bigr),$$

where b_i are constants, not equal to zero or a negative integer, and $T_k(x)$ are polynomials of degree $\leq k$. We can say that this set is of σ-type m if the maximum degree of $T_k(x)$ is m, $m = 0, 1, 2, \ldots$.

A necessary and sufficient condition that $\phi_n(x)$ be of σ-type zero, with

$$\sigma = D\prod_{i=1}^{q}(xD + b_i - 1),$$

is that $\phi_n(x)$ possess the generating function

$$A(t)_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(t)\bigr) = \sum_{n=0}^{\infty}\phi_n(x)t^n,$$

in which $H(t)$ and $A(t)$ have (formal) expansions

$$H(t) = \sum_{n=0}^{\infty}h_n t^{n+1}, \quad h_0 \neq 0,$$

and

$$A(t) = \sum_{n=0}^{\infty}a_n t^n, \quad a_0 \neq 0.$$

Since $\phi_n(x)$ belongs to the operator $J(\sigma) = \sum_{k=0}^{\infty} c_k\sigma^{k+1}$, where c_k are constant and $c_0 \neq 0$.

2 Main results

Theorem 1 *If $p_n(x)$ is a polynomial set, then $p_n(x)$ is of σ-type zero with $\sigma = D\prod_{m=1}^{q}(xD + b_m - 1)$. It is necessary and sufficient condition that there exist formal power series*

$$H(t) = \sum_{n=0}^{\infty}h_n t^{n+1}, \quad h_0 \neq 0,$$

and

$$A_i(t) = \sum_{n=0}^{\infty}a_n^{(i)}t^n \quad \bigl(\textit{not all } a_0^{(i)} \textit{ are zero}\bigr)$$

such that

$$\sum_{i=1}^{r}A_i(t)_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr) = \sum_{n=0}^{\infty}p_n(x)t^n, \tag{1}$$

where $\theta = xD$.

Proof Let $y_i = {}_0F_q(-; b_1, b_2, \ldots, b_q; z_i)$, where $i = 1, 2, \ldots, r$, be a solution of the following differential equation:

$$\left[\theta\prod_{m=1}^{q}(xD + b_m - 1) - z_i\right]y_i = 0, \qquad \theta = xD, \qquad D = \frac{d}{dx}.$$

On substituting $z_i = xH(\varepsilon_i t)$ and keeping t as a constant, where

$$\sigma = D\prod_{m=1}^{q}(xD + b_m - 1), \qquad \theta = xD,$$

we get

$$\left[xD\prod_{m=1}^{q}(\theta + b_m - 1) - xH(z_i)\right]y_i = 0.$$

This can also be written as

$$\sigma y_i = H(\varepsilon_i t)y_i$$

or

$$\sigma\, {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr) = H(\varepsilon_i t)\, {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr).$$

Operating $J(\sigma)$ on both sides of Equation (1) yields

$$\begin{aligned} J(\sigma)\sum_{n=0}^{\infty} p_n(x)t^n &= J(\sigma)\sum_{i=1}^{r} A_i(t)\, {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr) \\ &= \sum_{i=1}^{r} A_i(t) J\bigl(H(\varepsilon_i t)\bigr)\, {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr) \\ &= t\sum_{n=0}^{\infty} p_n(x)t^n \\ &= \sum_{n=1}^{\infty} p_{n-1}(x)t^n. \end{aligned}$$

Therefore, $J(\sigma)p_0(x) = 0$ and $J(\sigma)p_n(x) = p_{n-1}(x)$, $n \geq 1$.

Since $J(\sigma)$ is independent of x, using the definition of σ-type [9, 11], we arrive at the conclusion that $p_n(x)$ is σ-type zero.

Conversely, suppose $p_n(x)$ is of σ-type zero and belongs to the operator $J(\sigma)$. Now $q_n(x)$ is a simple set of polynomials, we can write

$$\sum_{i=1}^{r} {}_0F_q\bigl(-; b_1, b_2, \ldots, b_q; xH(\varepsilon_i t)\bigr) = \sum_{n=0}^{\infty} p_n(x)t^n, \tag{2}$$

where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r$ are the roots of unity.

Since $q_n(x)$ is a simple set, there exists a sequence c_k [10], independent of n, such that

$$p_n(x) = \sum_{k=0}^{n} c_{n-k}q_k(x)$$

and

$$\sum_{n=0}^{\infty} p_n(x)t^n = \sum_{n=0}^{\infty}\sum_{k=0}^{n} c_{n-k}q_k(x)t^n.$$

On replacing n by $n+k$, this becomes

$$\begin{aligned}\sum_{n=0}^{\infty} p_n(x)t^n &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} c_n q_k(x)t^{n+k} \\ &= \sum_{k=0}^{\infty} q_k(x)t^k \sum_{n=0}^{\infty} c_n t^n.\end{aligned}$$

Setting $c_n = a_n^{(i)}$ (i is independent of n, where $i = 1, 2, \dots, r$), this becomes

$$\begin{aligned}\sum_{n=0}^{\infty} p_n(x)t^n &= \sum_{k=0}^{\infty} q_k(x)t^k \sum_{n=0}^{\infty} a_n^{(i)} t^n, \quad \text{by using Equation (2), we get} \\ &= \sum_{i=1}^{r} A_i(t)_0F_q\bigl(-; b_1, b_2, \dots, b_q; xH(\varepsilon_i t)\bigr).\end{aligned}$$

This completes the proof. □

Theorem 2 *A necessary and sufficient condition that $p_n(x)$ be of σ-type zero and there exist a sequence h_k, independent of x and n, such that*

$$\sum_{i=1}^{r} \varepsilon_i^n h_{n-1}\psi(\varepsilon_i t) = \sigma p_n(x), \tag{3}$$

where $\psi(\varepsilon_i t) = A_i(t)_0F_q(-; b_1, b_2, \dots, b_q; xH(\varepsilon_i t))$.

Proof If $p_n(x)$ is of σ-type zero, then it follows from Theorem 1 that

$$\sum_{n=0}^{\infty} p_n(x)t^n = \sum_{i=1}^{r} A_i(t)_0F_q\bigl(-; b_1, b_2, \dots, b_q; xH(\varepsilon_i t)\bigr).$$

This can be written as

$$\begin{aligned}\sum_{n=0}^{\infty} \sigma p_n(x)t^n &= \sum_{i=1}^{r} A_i(t)\sigma\,{}_0F_q\bigl(-; b_1, b_2, \dots, b_q; xH(\varepsilon_i t)\bigr) \\ &= \sum_{i=1}^{r} H(\varepsilon_i t)A_i(t)_0F_q\bigl(-; b_1, b_2, \dots, b_q; xH(\varepsilon_i t)\bigr) \\ &= \sum_{i=1}^{r}\left(\sum_{n=0}^{\infty} h_n\varepsilon_i^{n+1}t^{n+1}\right)A_i(t)_0F_q\bigl(-; b_1, b_2, \dots, b_q; xH(\varepsilon_i t)\bigr) \\ &= \sum_{n=1}^{\infty}\sum_{i=1}^{r}\bigl(\varepsilon_i^n h_{n-1}\bigr)\psi(\varepsilon_i t)t^n.\end{aligned}$$

Thus

$$\sigma p_n(x) = \sum_{i=1}^{r} \varepsilon_i^n h_{n-1} \psi(\varepsilon_i t).$$

This completes the proof. □

3 Sheffer polynomials in two variables [12]

Let $p_n(x,y)$ be of σ-type zero. Then $p_n(x,y)$ belongs to an operator $J(\sigma) = \sum_{k=0}^{\infty} c_k \sigma^{k+1}$, in which c_k are constants and $c_0 \neq 0$.

Since

$$J(\sigma)p_n(x,y) = p_{n-1}(x,y), \quad n \geq 1,$$

where

$$D_x = \frac{\partial}{\partial x}, \qquad D_y = \frac{\partial}{\partial y}, \qquad \theta = x\frac{\partial}{\partial x}, \qquad \phi = y\frac{\partial}{\partial y},$$
$$\sigma_x = D_x \prod_{m=1}^{p}(\theta + b_m - 1), \qquad \sigma_y = D_y \prod_{s=1}^{q}(\theta + b_s - 1),$$

and

$$J\bigl((G+H)(t)\bigr) = \bigl((G+H)J(t)\bigr) = t, \qquad \sigma = \sigma_x + \sigma_y.$$

Theorem 3 *A necessary and sufficient condition that $p_n(x,y)$ be of σ-type zero, with*

$$\sigma_x = D_x \prod_{m=1}^{p}(\theta + b_m - 1), \qquad \sigma_y = D_y \prod_{s=1}^{q}(\theta + b_s - 1), \qquad \sigma = \sigma_x + \sigma_y,$$

is that $p_n(x,y)$ possess a generating function in

$$\sum_{i=1}^{r} A_i(t) {}_0F_p\bigl(-; b_1, b_2, \ldots, b_p; xG(\varepsilon_i t)\bigr) {}_0F_q\bigl(-; c_1, c_2, \ldots, c_q; yH(\varepsilon_i t)\bigr) = \sum_{n=0}^{\infty} p_n(x,y)t^n, \quad (4)$$

in which

$$G(t) = \sum_{n=0}^{\infty} g_n t^{n+1}, \quad g_0 \neq 0,$$
$$H(t) = \sum_{n=0}^{\infty} h_n t^{n+1}, \quad h_0 \neq 0,$$
$$A_i(t) = \sum_{n=0}^{\infty} a_n^{(i)} t^n \quad \bigl(\text{not all } a_0^{(i)} \text{ are zero}\bigr)$$

and i is independent of n.

Proof Let $u_i = {}_0F_p(-;b_1,b_2,\ldots,b_p;z_i)$ and $v_i = {}_0F_q(-;c_1,c_2,\ldots,c_q;w_i)$ be the solutions of the following differential equations:

$$\left[\theta_z\prod_{m=1}^{p}(\theta_z+b_m-1)-z_i\right]u_i=0,\qquad \theta_z=z\frac{\partial}{\partial z},$$

and

$$\left[\phi_w\prod_{s=1}^{q}(\phi_w+c_s-1)-z_i\right]w_i=0,\qquad \phi_w=w\frac{\partial}{\partial w}.$$

On substituting $z_i = xG(\varepsilon_i t)$, $w_i = yH(\varepsilon_i t)$ and keeping t as a constant, where $\theta = x\frac{\partial}{\partial x} = \theta_z$, $\phi = y\frac{\partial}{\partial y} = \phi_w$, we get

$$\theta\prod_{m=1}^{p}(\theta+b_m-1)u_i=xG(\varepsilon_i t)u_i$$

and

$$\phi\prod_{s=1}^{q}(\phi+c_s-1)w_i=yH(\varepsilon_i t)w_i.$$

This can also be written as

$$\begin{aligned}&\sigma\,{}_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr){}_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr)\\&\quad=\bigl\{G(\varepsilon_i t)+H(\varepsilon_i t)\bigr\}{}_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr){}_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr).\end{aligned}$$

Operating $J(\sigma)$ on both sides of Equation (4) yields

$$\begin{aligned}&J(\sigma)\sum_{n=0}^{\infty}p_n(x,y)t^n\\&\quad=J(\sigma)\sum_{i=1}^{r}A_i(t){}_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr){}_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr)\\&\quad=\sum_{i=1}^{r}A_i(t)J\bigl((G+H)(\varepsilon_i t)\bigr){}_0F_p\bigl[-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr]{}_0F_q\bigl[-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr]\\&\quad=t\sum_{n=0}^{\infty}p_n(x,y)t^n\\&\quad=\sum_{n=1}^{\infty}p_{n-1}(x,y)t^n.\end{aligned}$$

Therefore, $J(\sigma)p_0(x,y)=0$ and $J(\sigma)p_n(x,y)=p_{n-1}(x,y)$, $n\geq 1$.

Since $J(\sigma)$ is independent of x and y, thus we arrive at the conclusion that $p_n(x,y)$ is of σ-type zero.

Conversely, suppose $p_n(x,y)$ is of σ-type zero and belongs to the operator $J(\sigma)$. Now $q_n(x,y)$ is a simple set of polynomials. We can write

$$\sum_{i=1}^{r} {}_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr)_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr)=\sum_{n=0}^{\infty}p_n(x,y)t^n. \tag{5}$$

Since $q_n(x,y)$ is a simple set, there exists a sequence c_k, independent of n, such that

$$p_n(x,y)=\sum_{k=0}^{n}c_{n-k}q_k(x,y)$$

and

$$\sum_{n=0}^{\infty}p_n(x,y)t^n=\sum_{n=0}^{\infty}\sum_{k=0}^{n}c_{n-k}q_k(x,y)t^n.$$

On replacing n by $n+k$, this becomes

$$\begin{aligned}
&=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}c_nq_k(x,y)t^{n+k}\\
&=\sum_{k=0}^{\infty}q_k(x,y)t^k\sum_{n=0}^{\infty}c_nt^n.
\end{aligned}$$

Setting $c_n=a_n^{(i)}$ (i is independent of n, where $i=1,2,\ldots,r$), this becomes

$$\begin{aligned}
&=\sum_{k=0}^{\infty}q_k(x,y)t^k\sum_{n=0}^{\infty}a_n^{(i)}t^n\\
&=\sum_{i=1}^{r}A_i(t)_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr)_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr).
\end{aligned}$$

This completes the proof. □

Theorem 4 *A necessary and sufficient condition that $p_n(x,y)$ be of σ-type zero and there exist sequences g_k and h_k , independent of x, y and n, such that*

$$\sum_{i=1}^{r}\varepsilon_i^n(g_{n-1}+h_{n-1})\upsilon(\varepsilon_i t)=\sigma p_n(x,y), \tag{6}$$

where $\upsilon(\varepsilon_i t)=A_i(t)_0F_p(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t))_0F_q(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t))$.

Proof If $p_n(x,y)$ is of σ-type zero, then it follows from Theorem 3 that

$$\sum_{n=0}^{\infty}p_n(x,y)t^n=\sum_{i=1}^{r}A_i(t)_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr)_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr).$$

This can be written as

$$\begin{aligned}
\sum_{n=0}^{\infty} \sigma p_n(x,y)t^n &= \sum_{i=1}^{r} A_i(t)\sigma\,{}_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr)_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr)\\
&= \sum_{i=1}^{r}(G+H)(\varepsilon_i t)A_i(t)_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr)\\
&\quad \times {}_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr)\\
&= \sum_{i=1}^{r}\left(\sum_{n=0}^{\infty}(g_n+h_n)\varepsilon_i^{n+1}t^{n+1}\right)A_i(t)_0F_p\bigl(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t)\bigr)\\
&\quad \times {}_0F_q\bigl(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t)\bigr)\\
&= \sum_{n=1}^{\infty}\sum_{i=1}^{r}\bigl(\varepsilon_i^n(g_{n-1}+h_{n-1})\bigr)\upsilon(\varepsilon_i t)t^n.
\end{aligned}$$

Thus

$$\sigma p_n(x,y) = \sum_{i=1}^{r}\varepsilon_i^n(g_{n-1}+h_{n-1})\upsilon(\varepsilon_i t),$$

where $\upsilon(\varepsilon_i t) = A_i(t)_0F_p(-;b_1,b_2,\ldots,b_p;xG(\varepsilon_i t))_0F_q(-;c_1,c_2,\ldots,c_q;yH(\varepsilon_i t))$. This completes the proof. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this article. The authors read and approved the final manuscript.

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References
1. Thorne, CJ: A property of Appell sets. Am. Math. Mon. **52**, 191-193 (1945)
2. Srivastava, HM, Manocha, HL: A Treatise on Generating Functions. Wiley, New York (1984)
3. Dattoli, G, Migliorati, M, Srivastava, HM: Sheffer polynomials, monomiality principle, algebraic methods and the theory of classical polynomials. Math. Comput. Model. **45**(9-10), 1033-1041 (2007)
4. Pintér, Á, Srivastava, HM: Addition theorems for the Appell polynomials and the associated classes of polynomial expansions. Aequ. Math. (2012). doi:10.1007/s00010-012-0148-8
5. Srivastava, HM, Choi, J: Zeta and q-Zeta Functions and Associated Series and Integrals. Elsevier, Amsterdam (2012)
6. Galiffa, JD: The Sheffer B-type 1 orthogonal polynomial sequences. Ph.D. Dissertation, University of Central Florida (2009)
7. Goldberg, JL: On the Sheffer A-type of polynomials generated by $A(t)\psi(xB(t))$. Proc. Am. Math. Soc. **17**, 170-173 (1966)
8. Sheffer, IM: Note on Appell polynomials. Bull. Am. Math. Soc. **51**, 739-744 (1945)
9. Rainville, ED: Special Functions. Macmillan Co., New York (1960)
10. Al Salam, WA, Verma, A: Generalized Sheffer polynomials. Duke Math. J. **37**, 361-365 (1970)
11. Mc Bride, EB: Obtaining Generating Functions. Springer, New York (1971)
12. Alidad, B: On some problems of special functions and structural matrix analysis. Ph.D. Dissertation, Aligarh Muslim University (2008)

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