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On λ-statistical convergence of order α of sequences of function

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Abstract
In this study we introduce the concept of $S_\lambda^\alpha(f)$-statistical convergence of sequences of real valued functions. Also some relations between $S_\lambda^\alpha(f)$-statistical convergence and strong $w_{\lambda p}^\beta(f)$-summability are given.

MSC: 40A05; 40C05; 46A45

Keywords: statistical convergence; sequences of function; Cesàro summability

1 Introduction

The idea of statistical convergence was given by Zygmund [1] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [2] and Fast [3] and then reintroduced by Schoenberg [4] independently. Over the years and under different names, statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory, measure theory, trigonometric series, turnpike theory and Banach spaces. Later on, it was further investigated from the sequence space point of view and linked with summability theory by Connor [5], Edely *et al.* [6], Et *et al.* [7–10], Fridy [11], Güngör *et al.* [12–14], Kolk [15], Orhan *et al.* [16, 17], Mursaleen [18], Kumar and Mursaleen [19], Rath and Tripathy [20], Salat [21], Savaş [22] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

In the present paper, we introduce and examine the concepts of pointwise λ-statistical convergence of order α and pointwise $[V,\lambda]$-summability of order α of sequences of real valued functions. In Section 2, we give a brief overview about statistical convergence and strong p-Cesàro summability. In Section 3, we establish some inclusion relations between $w_{\lambda p}^\beta(f)$ and $S_\lambda^\alpha(f)$ and between $S_\lambda^\alpha(f)$ and $S_\lambda(f)$.

2 Definition and preliminaries

The definitions of statistical convergence and strong p-Cesàro convergence of a sequence of real numbers were introduced in the literature independently of one another and followed different lines of development since their first appearance. It turns out, however, that the two definitions can be simply related to one another in general and are equivalent for bounded sequences. The idea of statistical convergence depends on the density of subsets of the set $\mathbb{N}$ of natural numbers. The density of a subset E of $\mathbb{N}$ is defined by

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k) \quad \text{provided the limit exists,}$$

where χ_E is the characteristic function of E. It is clear that any finite subset of $\mathbb{N}$ has zero natural density and $\delta(E^c) = 1 - \delta(E)$.

The α*-density* of a subset E of $\mathbb{N}$ was defined by Çolak [23]. Let α be a real number such that $0 < \alpha \leq 1$. The α*-density* of a subset E of $\mathbb{N}$ is defined by

$$\delta_\alpha(E) = \lim_{n} \frac{1}{n^\alpha} \left|\{k \leq n : k \in E\}\right| \quad \text{provided the limit exists,}$$

where $|\{k \leq n : k \in E\}|$ denotes the number of elements of E not exceeding n.

It is clear that any finite subset of $\mathbb{N}$ has zero α density and $\delta_\alpha(E^c) = 1 - \delta_\alpha(E)$ does not hold for $0 < \alpha < 1$ in general, the equality holds only if $\alpha = 1$. Note that the α*-density* of any set reduces to the natural density of the set in case $\alpha = 1$.

The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan in [24] and after then statistical convergence of order α and strong p-Cesàro summability of order α studied by Çolak [23, 25] and generalized by Çolak and Asma [26].

Let $\lambda = (\lambda_n)$ be a nondecreasing sequence of positive real numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$. The generalized de la Vallée-Poussin mean is defined by $t_n(x) = \frac{1}{\lambda_n} \sum_{k\in I_n} x_k$, where $I_n = [n - \lambda_n + 1, n]$ for $n = 1, 2, \ldots$. A sequence $x = (x_k)$ is said to be (V, λ)-summable to a number ℓ if $t_n(x) \to \ell$ as $n \to \infty$ [27]. If $\lambda_n = n$, then (V, λ)-summability is reduced to Cesàro summability. By Λ we denote the class of all nondecreasing sequence of positive real numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$.

Throughout the paper, unless stated otherwise, by 'for all $n \in \mathbb{N}_{n_o}$' we mean 'for all $n \in \mathbb{N}$ except finite numbers of positive integers' where $\mathbb{N}_{n_o} = \{n_o, n_o + 1, n_o + 2, \ldots\}$ for some $n_o \in \mathbb{N} = \{1, 2, 3, \ldots\}$.

Let A be any non empty set, by $B(A)$ we denote the class of all bounded real valued functions defined on A.

3 Main results

In this section we give the main results of this paper. In Theorem 3.3, we give the inclusion relations between the sets of $S_\lambda^\alpha(f)$-statistically convergent sequences for different $\alpha's$ and $\mu's$. In Theorem 3.6, we give the relationship between the strong $w_{\lambda p}^\alpha(f)$-summability and the strong $w_{\mu p}^\beta(f)$-summability. In Theorem 3.9, we give the relationship between the strong $w_{\mu p}^\beta(f)$-summability and $S_\lambda^\alpha(f)$-statistical convergence.

Definition 3.1 Let the sequence $\lambda = (\lambda_n)$ be as above and $\alpha \in (0, 1]$ be any real number. A sequence of functions $\{f_k\}$ is said to be $S_\lambda^\alpha(f)$-statistical convergence (or pointwise λ-statistically convergent of order α) to the function f on a set A if, for every $\varepsilon > 0$,

$$\lim_{n} \frac{1}{\lambda_n^\alpha} \left|\left\{k \in I_n : \left|f_k(x) - f(x)\right| \geq \varepsilon \text{ for every } x \in A\right\}\right| = 0,$$

where $I_n = [n - \lambda_n + 1, n]$ and λ_n^α denote the αth power $(\lambda_n)^\alpha$ of λ_n, that is $\lambda^\alpha = (\lambda_n^\alpha) = (\lambda_1^\alpha, \lambda_2^\alpha, \ldots, \lambda_n^\alpha, \ldots)$. In this case, we write $S_\lambda^\alpha - \lim f_k(x) = f(x)$ on A. $S_\lambda^\alpha - \lim f_k(x) = f(x)$ means that for every $\delta > 0$ and $0 < \alpha \leq 1$, there is an integer N such that

$$\frac{1}{\lambda_n^\alpha} \left| \left\{ k \in I_n : \left| f_k(x) - f(x) \right| \geq \varepsilon \text{ for every } x \in A \right\} \right| < \delta,$$

for all $n > N$ $(= N(\varepsilon, \delta, x))$ and for each $\varepsilon > 0$. The set of all pointwise λ-statistically convergent function sequences of order α will be denoted by $S_\lambda^\alpha(f)$. In this case, we write $S_\lambda^\alpha - \lim f_k(x) = f(x)$ on A. For $\lambda_n = n$ for all $n \in \mathbb{N}$, we shall write $S^\alpha(f)$ instead of $S_\lambda^\alpha(f)$ and in the special case $\alpha = 1$, we shall write $S_\lambda(f)$ instead of $S_\lambda^\alpha(f)$.

The $S_\lambda^\alpha(f)$-statistical convergence is well defined for $0 < \alpha \leq 1$, but it is not well defined for $\alpha > 1$ in general. Let us define the sequence $\{f_k\}$ as follows:

$$f_k(x) = \begin{cases} 2, & \text{if } k = 3n, \\ \frac{1}{1+kx} & \text{if } k \neq 3n, \end{cases} \qquad n = 1, 2, 3, \ldots$$

then both

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left| \left\{ k \in I_n : \left| f_k(x) - 2 \right| \geq \varepsilon \text{ for every } x \in A \right\} \right| \leq \lim_{n\to\infty} \frac{[\lambda_n] + 1}{3\lambda_n^\alpha} = 0$$

and

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left| \left\{ k \in I_n : \left| f_k(x) - \frac{1}{1+kx} \right| \geq \varepsilon \text{ for every } x \in A \right\} \right| \leq \lim_{n\to\infty} \frac{2([\lambda_n] + 1)}{3\lambda_n^\alpha} = 0$$

for $\alpha > 1$, and so $S_\lambda^\alpha(f)$-statistically converges both to 2 and 0, *i.e.* $S_\lambda^\alpha - \lim f_k(x) = 2$ and $S_\lambda^\alpha - \lim f_k(x) = 0$. But this is impossible.

Definition 3.2 Let the sequence $\lambda = (\lambda_n)$ be as above, $\alpha \in (0, 1]$ be any real number and let p be a positive real number. A sequence of functions $\{f_k\}$ is said to be strongly $w_{\lambda p}^\alpha(f)$-summable (or pointwise $[V, \lambda]$-summable of order α), if there is a function f such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \sum_{\substack{k \in I_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p = 0.$$

In this case, we write $w_{\lambda p}^\alpha - \lim f_k(x) = f(x)$ on A. The set of all strongly $w_{\lambda p}^\beta(f)$-summable sequences of function will be denoted by $w_{\lambda p}^\alpha(f)$. For $\lambda_n = n$ for all $n \in \mathbb{N}$, we shall write $w_p^\alpha(f)$ instead of $w_{\lambda p}^\alpha(f)$ and in the special case $\alpha = 1$, we shall write $w_{\lambda p}(f)$ instead of $w_{\lambda p}^\alpha(f)$.

Theorem 3.3 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$, $0 < \alpha \leq \beta \leq 1$ and $\{f_k\}$ be a sequence of real valued functions defined on a set A.*

(i) *If*

$$\liminf_{n\to\infty} \frac{\lambda_n^\alpha}{\mu_n^\beta} > 0$$

then $S_\mu^\beta(f) \subseteq S_\lambda^\alpha(f)$;

(ii) *If*

$$\lim_{n\to\infty}\frac{\mu_n}{\lambda_n^{\beta}}=1$$

then $S_{\lambda}^{\alpha}(f)\subseteq S_{\mu}^{\beta}(f)$.

Proof (i) Suppose that $\lambda_n\le\mu_n$ for all $n\in\mathbb{N}_{n_o}$ and let (1) be satisfied. Then $I_n\subset J_n$ and so that $\varepsilon>0$ we may write

$$\left\{k\in J_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\supset\left\{k\in I_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}$$

and so

$$\begin{aligned}&\frac{1}{\mu_n^{\beta}}\left|\left\{k\in J_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\\&\quad\ge\frac{\lambda_n^{\alpha}}{\mu_n^{\beta}}\frac{1}{\lambda_n^{\alpha}}\left|\left\{k\in I_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\end{aligned}$$

for all $n\in\mathbb{N}_{n_o}$, where $J_n=[n-\mu_n+1,n]$. Now taking the limit as $n\to\infty$ in the last inequality and using (1), we get $S_{\mu}^{\beta}(f)\subseteq S_{\lambda}^{\alpha}(f)$.

(ii) Let $S_{\lambda}^{\alpha}-\lim f_k(x)=f(x)$ on A and (2) be satisfied. Since $I_n\subset J_n$, for $\varepsilon>0$, we may write

$$\begin{aligned}&\frac{1}{\mu_n^{\beta}}\left|\left\{k\in J_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\\&\quad=\frac{1}{\mu_n^{\beta}}\left|\left\{n-\mu_n+1\le k\le n-\lambda_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\\&\qquad+\frac{1}{\mu_n^{\beta}}\left|\left\{k\in I_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\\&\quad\le\frac{\mu_n-\lambda_n}{\mu_n^{\beta}}+\frac{1}{\mu_n^{\beta}}\left|\left\{k\in I_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\\&\quad\le\frac{\mu_n-\lambda_n^{\beta}}{\lambda_n^{\beta}}+\frac{1}{\mu_n^{\beta}}\left|\left\{k\in I_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\\&\quad\le\left(\frac{\mu_n}{\lambda_n^{\beta}}-1\right)+\frac{1}{\lambda_n^{\alpha}}\left|\left\{k\in I_n:\left|f_k(x)-f(x)\right|\ge\varepsilon\text{ for every }x\in A\right\}\right|\end{aligned}$$

for all $n\in\mathbb{N}_{n_o}$. Since $\lim_n\frac{\mu_n}{\lambda_n^{\beta}}=1$ by (2) the first term and since $S_{\lambda}^{\alpha}-\lim f_k(x)=f(x)$ on A, the second term of right-hand side of above inequality tends to 0 as $n\to\infty$. (Note that $(\frac{\mu_n}{\lambda_n^{\beta}}-1)\ge 0$ for all $n\in\mathbb{N}_{n_o}$.) This implies that $S_{\lambda}^{\alpha}(f)\subseteq S_{\mu}^{\beta}(f)$. □

From Theorem 3.3, we have the following results.

Corollary 3.4 *Let* $\lambda=(\lambda_n)$ *and* $\mu=(\mu_n)$ *be two sequences in* Λ *such that* $\lambda_n\le\mu_n$ *for all* $n\in\mathbb{N}_{n_o}$ *and* $\{f_k\}$ *be a sequence of real valued functions defined on a set* A. *If* (1) *holds then,*

(i) $S_{\mu}^{\alpha}(f)\subseteq S_{\lambda}^{\alpha}(f)$, *for each* $\alpha\in(0,1]$ *and for all* $x\in A$;

(ii) $S_{\mu}(f)\subseteq S_{\lambda}^{\alpha}(f)$, *for each* $\alpha\in(0,1]$ *and for all* $x\in A$;

(iii) $S_{\mu}(f)\subseteq S_{\lambda}(f)$ *for all* $x\in A$.

Corollary 3.5 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ and $\{f_k\}$ be a sequence of real valued functions defined on a set A. If (2) holds then,*

(i) *$S_\lambda^\alpha(f) \subseteq S_\mu^\alpha(f)$ for each $\alpha \in (0,1]$ and for all $x \in A$;*
(ii) *$S_\lambda^\alpha(f) \subseteq S_\mu(f)$, for each $\alpha \in (0,1]$ and for all $x \in A$;*
(iii) *$S_\lambda(f) \subseteq S_\mu(f)$ for all $x \in A$.*

Theorem 3.6 *Given for $\lambda = (\lambda_n)$, $\mu = (\mu_n) \in \Lambda$ suppose that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$, $0 < \alpha \leq \beta \leq 1$ and $\{f_k\}$ be a sequence of real valued functions defined on a set A. Then*

(i) *If (1) holds then $w_{\mu p}^\beta(f) \subset w_{\lambda p}^\alpha(f)$ for all $x \in A$;*
(ii) *If (2) holds and let $f(x) \in B(A)$, then $B(A) \cap w_{\lambda p}^\alpha(f) \subset w_{\mu p}^\beta(f)$ for all $x \in A$.*

Proof (i) Omitted.

(ii) Let $(f_k(x)) \in B(A) \cap w_{\lambda p}^\alpha(f)$ and suppose that (2) holds. Since $(f_k(x)) \in B(A)$, then there exists some $M > 0$ such that $|f_k(x) - f(x)| \leq M$ for all $k \in \mathbb{N}$ and for all $x \in A$. Now, since $\lambda_n \leq \mu_n$ and $I_n \subset J_n$ for all $n \in \mathbb{N}_{n_o}$, we may write

$$\begin{aligned}
\frac{1}{\mu_n^\beta} \sum_{\substack{k \in J_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p &= \frac{1}{\mu_n^\beta} \sum_{\substack{k \in J_n - I_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p + \frac{1}{\mu_n^\beta} \sum_{\substack{k \in I_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p \\
&\leq \left(\frac{\mu_n - \lambda_n}{\mu_n^\beta} \right) M^p + \frac{1}{\mu_n^\beta} \sum_{\substack{k \in I_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p \\
&\leq \left(\frac{\mu_n - \lambda_n^\beta}{\mu_n^\beta} \right) M^p + \frac{1}{\mu_n^\beta} \sum_{\substack{k \in I_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p \\
&\leq \left(\frac{\mu_n}{\lambda_n^\beta} - 1 \right) M^p + \frac{1}{\lambda_n^\alpha} \sum_{\substack{k \in I_n \\ x \in A}} \left| f_k(x) - f(x) \right|^p
\end{aligned}$$

for every $n \in \mathbb{N}_{n_o}$. Therefore, $B(A) \cap w_{\lambda p}^\alpha(f) \subset w_{\mu p}^\beta(f)$. □

From Theorem 3.6, we have the following results.

Corollary 3.7 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ and $\{f_k\}$ be a sequence of real valued functions defined on a set A. If (1) holds then:*

(i) *$w_{\mu p}^\alpha(f) \subset w_{\lambda p}^\alpha(f)$, for each $\alpha \in (0,1]$ and for all $x \in A$;*
(ii) *$w_{\mu p}(f) \subset w_{\lambda p}^\alpha(f)$, for each $\alpha \in (0,1]$ and for all $x \in A$;*
(iii) *$w_{\mu p}(f) \subset w_{\lambda p}(f)$ for all $x \in A$.*

Corollary 3.8 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ and $\{f_k\}$ be a sequence of real valued functions defined on a set A. If (2) holds then:*

(i) *$B(A) \cap w_{\lambda p}^\alpha(f) \subset w_{\mu p}^\alpha(f)$, for each $\alpha \in (0,1]$ and for all $x \in A$;*
(ii) *$B(A) \cap w_{\lambda p}^\alpha(f) \subset w_{\mu p}(f)$, for each $\alpha \in (0,1]$ and for all $x \in A$;*
(iii) *$B(A) \cap w_{\lambda p}(f) \subset w_{\mu p}(f)$ for all $x \in A$.*

Theorem 3.9 *Let α and β be fixed real numbers such that $0 < \alpha \leq \beta \leq 1$, $0 < p < \infty$, $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ and $\{f_k\}$ be a sequence of real valued functions defined on a set A. Then:*

(i) *Let* (1) *holds, if a sequence of real valued functions defined on a set A is strongly $w_{\mu p}^{\beta}(f)$-summable to f, then it is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f;*

(ii) *Let* (2) *holds, $f(x) \in B(A)$ and $\{f_k\}$ be a sequence of bounded real valued functions defined on a set A, if a sequence is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f then it is strongly $w_{\mu p}^{\beta}(f)$-summable to f.*

Proof (i) For any function sequence $(f_k(x))$ and $\varepsilon > 0$, we have

$$\begin{aligned}
\sum_{\substack{k\in J_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p &= \sum_{\substack{k\in J_n, x\in A\\ |f_k(x)-f(x)|\geq\varepsilon}} \left|f_k(x) - f(x)\right|^p + \sum_{\substack{k\in J_n, x\in A\\ |f_k(x)-f(x)|<\varepsilon}} \left|f_k(x) - f(x)\right|^p \\
&\geq \sum_{\substack{k\in I_n, x\in A\\ |f_k(x)-f(x)|\geq\varepsilon}} \left|f_k(x) - f(x)\right|^p + \sum_{\substack{k\in I_n, x\in A\\ |f_k(x)-f(x)|<\varepsilon}} \left|f_k(x) - f(x)\right|^p \\
&\geq \sum_{\substack{k\in I_n, x\in A\\ |f_k(x)-f(x)|\geq\varepsilon}} \left|f_k(x) - f(x)\right|^p \\
&\geq \left|\left\{k \in I_n : \left|f_k(x) - f(x)\right| \geq \varepsilon \text{ for every } x \in A\right\}\right| \varepsilon^p
\end{aligned}$$

and so that

$$\begin{aligned}
\frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in J_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p &\geq \frac{1}{\mu_n^{\beta}} \left|\left\{k \in I_n : \left|f_k(x) - f(x)\right| \geq \varepsilon \text{ for every } x \in A\right\}\right| \varepsilon^p \\
&\geq \frac{\lambda_n^{\alpha}}{\mu_n^{\beta}} \frac{1}{\lambda_n^{\alpha}} \left|\left\{k \in I_n : \left|f_k(x) - f(x)\right| \geq \varepsilon \text{ for every } x \in A\right\}\right| \varepsilon^p.
\end{aligned}$$

Since (1) holds, it follows that if $\{f_k\}$ is strongly $w_{\mu p}^{\beta}(f)$-summable to f, then it is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f.

(ii) Suppose that $S_{\lambda}^{\alpha} - \lim f_k(x) = f(x)$ and $(f_k(x)) \in B(A)$. Then there exists some $M > 0$ such that $|f_k(x) - f(x)| \leq M$ for all k, then for every $\varepsilon > 0$ we may write

$$\begin{aligned}
\frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in J_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p &= \frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in J_n - I_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p + \frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in I_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p \\
&\leq \left(\frac{\mu_n - \lambda_n}{\mu_n^{\beta}}\right) M^p + \frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in I_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p \\
&\leq \left(\frac{\mu_n - \lambda_n^{\beta}}{\mu_n^{\beta}}\right) M^p + \frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in I_n\\ x\in A}} \left|f_k(x) - f(x)\right|^p \\
&= \left(\frac{\mu_n}{\lambda_n^{\beta}} - \frac{\lambda_n^{\beta}}{\lambda_n^{\beta}}\right) M^p + \frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in I_n, x\in A\\ |f_k(x)-f(x)|\geq\varepsilon}} \left|f_k(x) - f(x)\right|^p \\
&\quad + \frac{1}{\mu_n^{\beta}} \sum_{\substack{k\in I_n, x\in A\\ |f_k(x)-f(x)|<\varepsilon}} \left|f_k(x) - f(x)\right|^p
\end{aligned}$$

$$\leq \left(\frac{\mu_n}{\lambda_n^{\beta}} - 1\right) M^p + \frac{M^p}{\lambda_n^{\alpha}} \left| \left\{ k \in I_n : \left| f_k(x) - f(x) \right| \geq \varepsilon \text{ for every } x \in A \right\} \right| + \varepsilon$$

for all $n \in \mathbb{N}_{n_o}$. Using (2), we obtain that $w_{\mu p}^{\beta} - \lim f_k(x) = f(x)$, whenever $S_{\lambda}^{\alpha} - \lim f_k(x) = f(x)$. □

From Theorem 3.9, we have the following results.

Corollary 3.10 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$ and $\alpha \in (0,1]$ be any real number. If* (1) *holds, then*:

(i) *If a sequence of real valued functions defined on a set A is strongly $w_{\mu p}^{\alpha}(f)$-summable to f, then it is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f*;

(ii) *If a sequence of real valued functions defined on a set A is strongly $w_{\mu p}(f)$-summable to f, then it is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f*;

(iii) *If a sequence of real valued functions defined on a set A is strongly $w_{\mu p}(f)$-summable to f, then it is $S_{\lambda}(f)$-statistically convergent to f.*

Corollary 3.11 *Let $\lambda = (\lambda_n)$ and $\mu = (\mu_n)$ be two sequences in Λ such that $\lambda_n \leq \mu_n$ for all $n \in \mathbb{N}_{n_o}$, $\alpha \in (0,1]$ be any real number. If* (2) *holds, then*:

(i) *If a sequence of bounded real valued functions defined on a set A is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f, then it is strongly $w_{\mu p}^{\alpha}(f)$-summable to f*;

(ii) *If a sequence of bounded real valued functions defined on a set is $S_{\lambda}^{\alpha}(f)$-statistically convergent to f, then it is strongly $w_{\mu p}(f)$-summable to f*;

(iii) *If a sequence of bounded real valued functions defined on a set A is $S_{\lambda}(f)$-statistically convergent to f, then it is strongly $w_{\mu p}(f)$-summable to f.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
ME, MÇ and MK have contributed to all parts of the article.

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References

1. Zygmund, A: Trigonometric Series. Cambridge University Press, Cambridge (1979)
2. Steinhaus, H: Sur la convergence ordinaire et la convergence asymptotique. Colloq. Math. **2**, 73-74 (1951)
3. Fast, H: Sur la convergence statistique. Colloq. Math. **2**, 241-244 (1951)
4. Schoenberg, IJ: The integrability of certain functions and related summability methods. Am. Math. Mon. **66**, 361-375 (1959)
5. Connor, JS: The statistical and strong *p*-Cesàro convergence of sequences. Analysis **8**, 47-63 (1988)
6. Edely, OHH, Mohiuddine, SA, Noman, AK: Korovkin type approximation theorems obtained through generalized statistical convergence. Appl. Math. Lett. **23**(11), 1382-1387 (2010)

7. Et, M, Nuray, F: Δ^m-statistical convergence. Indian J. Pure Appl. Math. **32**(6), 961-969 (2001)
8. Et, M: Strongly almost summable difference sequences of order *m* defined by a modulus. Studia Sci. Math. Hung. **40**(4), 463-476 (2003)
9. Et, M: Spaces of Cesàro difference sequences of order *r* defined by a modulus function in a locally convex space. Taiwan. J. Math. **10**(4), 865-879 (2006)
10. Et, M, Altin, Y, Choudhary, B, Tripathy, BC: On some classes of sequences defined by sequences of Orlicz functions. Math. Inequal. Appl. **9**(2), 335-342 (2006)
11. Fridy, J: On statistical convergence. Analysis **5**, 301-313 (1985)
12. Gökhan, A, Güngör, M: On pointwise statistical convergence. Indian J. Pure Appl. Math. **33**(9), 1379-1384 (2002)
13. Güngör, M, Et, M, Altn, Y: Strongly (V_σ, λ, q)-summable sequences defined by Orlicz functions. Appl. Math. Comput. **157**(2), 561-571 (2004)
14. Güngör, M, Et, M: Δ^m-strongly almost summable sequences defined by Orlicz functions. Indian J. Pure Appl. Math. **34**(8), 1141-1151 (2003)
15. Kolk, E: The statistical convergence in Banach spaces. Acta Comment. Univ. Tartu Math. **928**, 41-52 (1991)
16. Duman, O, Orhan, C: μ-statistically convergent function sequences. Czechoslov. Math. J. **54(129)**(2), 413-422 (2004)
17. Miller, HI, Orhan, C: On almost convergent and statistically convergent subsequences. Acta Math. Hung. **93**(1-2), 135-151 (2001)
18. Mursaleen, M: λ-statistical convergence. Math. Slovaca **50**(1), 111-115 (2000)
19. Kumar, V, Mursaleen, M: On (λ, μ)-statistical convergence of double sequences on intuitionistic fuzzy normed spaces. Filomat **25**(2), 109-120 (2011)
20. Rath, D, Tripathy, BC: On statistically convergent and statistically Cauchy sequences. Indian J. Pure Appl. Math. **25**(4), 381-386 (1994)
21. Šalát, T: On statistically convergent sequences of real numbers. Math. Slovaca **30**, 139-150 (1980)
22. Savaş, E: Strong almost convergence and almost λ-statistical convergence. Hokkaido Math. J. **29**(3), 531-536 (2000)
23. Çolak, R: Statistical Convergence of Order α. Modern Methods in Analysis and Its Applications, pp. 121-129. Anamaya Pub., New Delhi (2010)
24. Gadjiev, AD, Orhan, C: Some approximation theorems via statistical convergence. Rocky Mt. J. Math. **32**(1), 129-138 (2002)
25. Çolak, R: On λ-statistical convergence. In: Conference on Summability and Applications, Istanbul, Turkey, 12-13 May 2011 (2011)
26. Çolak, R, BektaşÇ, A: λ-statistical convergence of order α. Acta Math. Sci. **31**(3), 953-959 (2011)
27. Leindler, L: Über die la Vallee-Pousinsche Summierbarkeit Allgemeiner Orthogonalreihen. Acta Math. Acad. Sci. Hung. **16**, 375-387 (1965)

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