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On sufficient conditions for Carathéodory functions with applications

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Abstract
In the present paper, we derive some interesting relations associated with the Carathéodory functions which yield sufficient conditions for the Carathéodory functions in the open unit disk $\mathbb{U} = \{z : |z| < 1\}$. Some interesting applications of the main results are also obtained.

MSC: Primary 30C45; 30C80

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1 Introduction

Let P denote the class of functions of the form

$$p(z) = \sum_{n=0}^{\infty} p_n z^n,$$

which are analytic in the unit disc $\mathbb{U} = \{z : |z| < 1\}$. The function $p(z)$ is called a Carathéodory function if it satisfies the condition

$$\operatorname{Re}\big(p(z)\big) > 0.$$

Moreover, let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the unit disc $\mathbb{U}$.

A function $f(z) \in A$ is in K, the class of convex functions, if it satisfies

$$\operatorname{Re}\left(1 + \frac{zf''(z)}{f(z)}\right) > 0 \quad (z \in \mathbb{U}). \tag{1.2}$$

Also, a function $f(z) \in A$ is in S^{λ} ($|\lambda| < \frac{\pi}{2}$), the class of λ-spirallike functions, if it satisfies

$$\operatorname{Re}\left(e^{i\lambda} \frac{zf'(z)}{f(z)}\right) > 0 \quad (z \in \mathbb{U}). \tag{1.3}$$

Moreover, we denote by $S^* = S^0$ the class of starlike functions in $\mathbb{U}$.

Definition 1.1 Let $f(z)$ and $F(z)$ be analytic functions. The function $f(z)$ is said to be *subordinate* to $F(z)$, written $f(z) \prec F(z)$, if there exists a function $w(z)$ analytic in $\mathbb{U}$, with $w(0) = 0$ and $|w(z)| \leq 1$, and such that $f(z) = F(w(z))$. If $F(z)$ is univalent, then $f(z) \prec F(z)$ *if and only if* $f(0) = F(0)$ and $f(\mathbb{U}) \subset F(\mathbb{U})$.

Definition 1.2 Let $\mathbb{D}$ be the set of analytic functions $q(z)$ and injective on $\bar{\mathbb{U}}\backslash E(q)$, where

$$E(q) = \left\{\zeta \in \partial\mathbb{U} : \lim_{z\to\zeta} q(z) = \infty\right\}$$

and $q'(\zeta) \neq 0$ for $\zeta \in \partial\mathbb{U}\backslash E(q)$. Further, let $\mathbb{D}_a = \{q(z) \in \mathbb{D} : q(0) = a\}$.

Many authors have obtained several relations of Carathéodory functions, *e.g.*, see ([1–13]).

In the present paper, we derive some relations associated with the Carathéodory functions which yield the sufficient conditions for Carathéodory functions in $\mathbb{U}$. Some applications of the main results are also obtained.

2 Main results

To prove our results, we need the following lemma due to Miller and Mocanu [14, p.24]

Lemma 2.1 *Let* $q(z) \in \mathbb{D}_a$ *and let*

$$p(z) = b + b_n z^n + \cdots \tag{2.1}$$

be analytic in $\mathbb{U}$ *with* $p(z) \neq b$. *If* $p(z) \not\prec q(z)$, *then there exist points* $z_0 \in \mathbb{U}$ *and* $\zeta_0 \in \partial\mathbb{U}\backslash E(q)$ *and on* $m \geq n \geq 1$ *for which*

(i) $p(z_0) = q(\zeta_0)$,

(ii) $z_0 p'(z_0) = m\zeta_0 q'(\zeta_0)$.

Theorem 2.1 *Let*

$$P : \mathbb{U} \to \mathbb{C}$$

with

$$\operatorname{Re}\bigl(\bar{a}P(z)\bigr) > 0 \quad (a \in \mathbb{C}).$$

If $p(z)$ *is an analytic function in* $\mathbb{U}$ *with* $p(0) = 1$ *and*

$$\operatorname{Re}\bigl(p(z) + P(z)zp'(z)\bigr) > \frac{E}{2|a|^2 \operatorname{Re}(\bar{a}P(z))}, \tag{2.2}$$

then

$$\operatorname{Re}\bigl(ap(z)\bigr) > 0,$$

where

$$E = -\bigl(\operatorname{Re}(a)\bigr)\bigl(\operatorname{Re}\bigl(\bar{a}P(z)\bigr)\bigr)^2 + 2\operatorname{Re}\bigl(\bar{a}P(z)\bigr)\bigl(\operatorname{Im}(a)\bigr)^2 + \bigl(\operatorname{Re}(a)\bigr)\bigl(\operatorname{Im}(a)\bigr)^2 \tag{2.3}$$

with $\operatorname{Re}(a) > 0$.

Proof Let us define both $q(z)$ and $h(z)$ as follows:

$$q(z) = ap(z)$$

and

$$h(z) = \frac{a + \bar{a}z}{1 - z} \quad \bigl(\operatorname{Re}(a) > 0\bigr),$$

where $p(z)$ is defined by (2.1) since $q(z)$ and $h(z)$ are analytic functions in $\mathbb{U}$ with $q(0) = h(0) = a \in \mathbb{C}$ with

$$h(\mathbb{U}) = \bigl\{w : \operatorname{Re}(w) > 0\bigr\}.$$

Now, we suppose that $q(z) \not\prec h(z)$. Therefore, by using Lemma 2.1, there exist points

$$z_0 \in \mathbb{U} \quad \text{and} \quad \zeta_0 \in \partial\mathbb{U}\backslash\{1\}$$

such that $q(z_0) = h(\zeta_0)$ and $z_0 q'(z_0) = m\zeta_0 h'(\zeta_0)$, $m \geq n \geq 1$.

We note that

$$\zeta_0 = h^{-1}\bigl(q(z_0)\bigr) = \frac{q(z_0) - a}{q(z_0) + \bar{a}} \tag{2.4}$$

and

$$\varsigma_0 h'(\varsigma_0) = -\frac{|q(z_0) - a|^2}{2\operatorname{Re}(a - q(z_0))}. \tag{2.5}$$

We have $h(\zeta_0) = \rho i$ ($\rho \in \mathbb{R}$); therefore,

$$\begin{aligned}
&\operatorname{Re}\bigl(p(z_0) + P(z_0)zp'(z_0)\bigr) \\
&\quad = \operatorname{Re}\left(\frac{1}{a}h(\zeta_0) + \frac{1}{a}P(z_0)m\zeta_0 h'(\zeta_0)\right) \\
&\quad = \operatorname{Re}\left(\frac{\rho i}{a}\right) - m\frac{|\rho i - a|^2}{2\operatorname{Re}(a)}\operatorname{Re}\left(\frac{P(z_0)}{a}\right) \\
&\quad \leq \operatorname{Re}\left(\frac{\rho i}{a}\right) - \frac{|\rho i - a|^2}{2\operatorname{Re}(a)}\operatorname{Re}\left(\frac{P(z_0)}{a}\right) \\
&\quad = A\rho^2 + B\rho + C \\
&\quad = g(\rho),
\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}
A &= -\frac{\operatorname{Re}(\bar{a}p(z_0))}{2|a|^2 \operatorname{Re}(a)}, \\
B &= \frac{\operatorname{Im}(a)}{|a|^2}\left(1 + \frac{\operatorname{Re}(\bar{a}p(z_0))}{\operatorname{Re}(a)}\right)
\end{aligned}$$

and

$$C = -\frac{\operatorname{Re}(\bar{a}p(z_0))}{2\operatorname{Re}(a)}.$$

We can see that the function $g(\rho)$ in (2.6) takes the maximum value at ρ_1 given by

$$\rho_1 = \operatorname{Im}(a)\left(1 + \frac{\operatorname{Re}(a)}{\operatorname{Re}(\bar{a}p(z_0))}\right).$$

Hence, we have

$$\begin{aligned}\operatorname{Re}\bigl(p(z_0) + P(z_0)zp'(z_0)\bigr) &\le g(\rho_1)\\ &= \frac{E}{2|a|^2\operatorname{Re}(\bar{a}P(z))},\end{aligned}$$

where E is defined by (2.3). This is a contradiction to (2.2). Then we obtain $\operatorname{Re}(ap(z)) > 0$. □

Theorem 2.2 *Let $p(z)$ be a nonzero analytic function in $\mathbb{U}$ and $p(0) = 1$. If*

$$\gamma_1 < \operatorname{Im}\left(p(z) + \frac{zp'(z)}{p(z)}\right) < \gamma_2, \tag{2.7}$$

where

$$\gamma_1 = -\frac{\sqrt{|a|^2 + 2(\operatorname{Re}(a))^2} - \operatorname{Im}(a)}{\operatorname{Re} a}$$

and

$$\gamma_2 = \frac{\sqrt{|a|^2 + 2(\operatorname{Re}(a))^2} + \operatorname{Im}(a)}{\operatorname{Re}(a)},$$

then

$$\operatorname{Re}\bigl(ap(z)\bigr) > 0,$$

where $\operatorname{Re}(a) > 0$.

Proof Let us define both $q(z)$ and $h(z)$ as follows:

$$q(z) = ap(z)$$

and

$$h(z) = \frac{a + \bar{a}z}{1 - z} \quad \bigl(\operatorname{Re}(a) > 0\bigr),$$

where $p(z)$ is defined by (2.1) since $q(z)$ and $h(z)$ are analytic functions in $\mathbb{U}$ with $q(0) = h(0) = a \in \mathbb{C}$ with

$$h(\mathbb{U}) = \bigl\{w : \operatorname{Re}(w) > 0\bigr\}.$$

Now, we suppose that $q(z) \not\prec h(z)$. Therefore, by using Lemma 2.1, there exist points

$$z_0 \in \mathbb{U} \quad \text{and} \quad \zeta_0 \in \partial\mathbb{U}\backslash\{1\}$$

such that $q(z_0) = h(\zeta_0)$ and $z_0 q'(z_0) = m\zeta_0 h'(\zeta_0)$, $m \geq n \geq 1$.

We note that

$$\zeta_0 h'(\zeta_0) = -\frac{|q(z_0) - a|^2}{2\operatorname{Re}(a - q(z_0))}. \tag{2.8}$$

We have $h(\zeta_0) = \rho i$ ($\rho \in \mathbb{R}$); therefore,

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &= \operatorname{Im}\left(q(z_0) + \frac{z_0 q'(z_0)}{q(z_0)}\right) \\
&= \operatorname{Im}\left(\frac{h(\zeta_0)}{a} + \frac{m\zeta_0 h'(\zeta_0)}{h(\zeta_0)}\right) \\
&= \operatorname{Im}\left(\frac{\rho i}{a} - \frac{m|\rho i - a|^2}{2\operatorname{Re}(a)\rho i}\right) \\
&= \frac{\rho}{|a|^2}\operatorname{Re}(a) + \frac{m|\rho i - a|^2}{2\rho\operatorname{Re}(a)}.
\end{aligned}$$

For the case $\rho > 0$, we obtain

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\geq \frac{\rho}{|a|^2}\operatorname{Re}(a) + \frac{|\rho i - a|^2}{2\rho\operatorname{Re}(a)} \\
&= \frac{1}{2\rho\operatorname{Re}(a)}\left[\left(1 + 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2\right)\rho^2 + 2\operatorname{Im}(a)\rho + |a|\right] \\
&= g(\rho).
\end{aligned} \tag{2.9}$$

We can see that the function $g(\rho)$ in (2.9) takes the minimum value at ρ_1 given by

$$\rho_1 = \frac{|a|^2}{\sqrt{|a|^2 + 2(\operatorname{Re}(a))^2}}.$$

Hence, we have

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\geq g(\rho_1) \\
&= \gamma_2.
\end{aligned}$$

This is a contradiction to (2.7). Then we obtain $\operatorname{Re}(ap(z)) > 0$.

For the case $\rho < 0$, we obtain

$$\begin{aligned}
\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\leq \frac{\rho}{|a|^2}\operatorname{Re}(a) + \frac{|\rho i - a|^2}{2\rho\operatorname{Re}(a)} \\
&= \frac{1}{2\rho\operatorname{Re}(a)}\left[\left(1 + 2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2\right)\rho^2 + 2\operatorname{Im}(a)\rho + |a|^2\right] \\
&= g(\rho).
\end{aligned} \tag{2.10}$$

We can see that the function $g(\rho)$ in (2.10) takes the maximum value at ρ_2 given by

$$\rho_2 = -\frac{|a|^2}{\sqrt{|a|^2 + 2(\operatorname{Re}(a))^2}}.$$

Hence, we have

$$\begin{aligned}\operatorname{Im}\left(p(z_0) + \frac{z_0 p'(z_0)}{p(z_0)}\right) &\leq g(\rho_2)\\ &= \gamma_1.\end{aligned}$$

This is a contradiction to (2.7). Then we obtain $\operatorname{Re}(ap(z)) > 0$. □

Theorem 2.3 *Let $p(z)$ be a nonzero analytic function in $\mathbb{U}$ with $p(0) = 1$. If*

$$\left|p(z) + \frac{zp'(z)}{p(z)} - 1\right| < \frac{3\operatorname{Re}(a)}{2|a|},$$

then

$$\operatorname{Re}\left(\frac{a}{p(z)}\right) > 0,$$

where $\operatorname{Re}(a) > 0$.

Proof Let us define both $q(z)$ and $h(z)$ as follows:

$$q(z) = ap(z)$$

and

$$h(z) = \frac{a + \bar{a}z}{1 - z} \quad \bigl(\operatorname{Re}(a) > 0\bigr),$$

where $p(z)$ is defined by (2.1) since $q(z)$ and $h(z)$ are analytic functions in $\mathbb{U}$ with $q(0) = h(0) = a \in \mathbb{C}$ with

$$h(\mathbb{U}) = \{w : \operatorname{Re} w > 0\}.$$

Now, we suppose that $q(z) \not\prec h(z)$. Therefore, by using Lemma 2.1, there exist points

$$z_0 \in \mathbb{U} \quad \text{and} \quad \zeta_0 \in \partial\mathbb{U}\backslash\{1\}$$

such that $q(z_0) = h(\zeta_0)$ and $z_0 q'(z_0) = m\zeta_0 h'(\zeta_0)$, $m \geq n \geq 1$.

We note that

$$\zeta_0 h'(\zeta_0) = -\frac{|q(z_0) - a|^2}{2\operatorname{Re}(a - q(z_0))}. \tag{2.11}$$

We have $h(\zeta_0) = \rho i$ ($\rho \in \mathbb{R}$).

Therefore,

$$\begin{aligned}\frac{|p(z_0)+\frac{zp'(z_0)}{p(z_0)}-1|}{|p(z_0)|} &= \left|\frac{\rho i}{a}-\frac{m}{a}\frac{|a-i\rho|^2}{2\operatorname{Re}(a)}-1\right| \\ &\geq \frac{1}{|a|}\left|\frac{m|a-i\rho|^2}{2\operatorname{Re}(a)}+\operatorname{Re}(a)\right| \\ &\geq \frac{1}{|a|}\left(\frac{|a-i\rho|^2}{2\operatorname{Re}(a)}+\operatorname{Re}(a)\right) \\ &\geq \frac{1}{2|a|\operatorname{Re}(a)}\left(3\left(\operatorname{Re}(a)\right)^2+\left(\operatorname{Im}(a)-\rho\right)^2\right) \\ &\geq \frac{3\operatorname{Re}(a)}{2|a|}.\end{aligned}$$

This is a contradiction to (2.7). Then we obtain $\operatorname{Re}(\frac{a}{p(z)})>0$. □

3 Applications and examples

Putting $P(z)=\beta$ ($\beta>0$; real) in Theorem 2.1, we have the following corollary.

Corollary 3.1 *If $p(z)$ is an analytic function in $\mathbb{U}$ with $p(0)=1$ and*

$$\operatorname{Re}\left(p(z)+\beta zp'(z)\right)>\frac{E}{2\beta|a|^2\operatorname{Re}(a)},$$

then

$$\operatorname{Re}\left(ap(z)\right)>0,$$

where

$$E=-\left(\operatorname{Re}(a)\right)\left[\beta^2\left(\operatorname{Re}(a)\right)^2+(1+2\beta)\left(\operatorname{Im}(a)\right)^2\right]$$

with $\operatorname{Re}(a)>0$.

Putting $\beta=1$ in Corollary 3.1, we obtain the following corollary.

Corollary 3.2 *If $p(z)$ is an analytic function in $\mathbb{U}$ with $p(0)=1$ and*

$$\operatorname{Re}\left(p(z)+zp'(z)\right)>\frac{3}{2}-2\left(\frac{\operatorname{Re}(a)}{|a|}\right)^2,$$

then

$$\operatorname{Re}\left(ap(z)\right)>0,$$

where $\operatorname{Re}(a)>0$.

Putting $p(z)=\frac{f(z)}{g(z)}$ and $P(z)=\frac{g(z)}{zg'(z)}$ in Theorem 2.1, we have the following corollary.

Corollary 3.3 *Let* $f(z) \in A$, $g(z) \in S^*$ *and*

$$\mathrm{Re}\left(\frac{f'(z)}{g'(z)}\right) > \frac{3}{2} - 2\left(\frac{\mathrm{Re}(a)}{|a|}\right)^2.$$

Then

$$\mathrm{Re}\left(a\frac{f(z)}{g(z)}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Example 3.1 Let $f(z) \in A$ satisfy the following relation:

$$\mathrm{Re}\big(f'(z)\big) > \frac{3}{2} - 2\left(\frac{\mathrm{Re}(a)}{|a|}\right)^2.$$

Then

$$\mathrm{Re}\left(a\frac{f(z)}{z}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Example 3.2 Let $f(z) \in A$ satisfy the following relation:

$$\mathrm{Re}\left(\left(2 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)}\right)\frac{zf'(z)}{f(z)}\right) > \frac{3}{2} - 2\left(\frac{\mathrm{Re}(a)}{|a|}\right)^2.$$

Then

$$\mathrm{Re}\left(a\frac{zf'(z)}{f(z)}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Remark 3.1

(i) Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$) in Theorem 2.1, we have Theorem 1 due to Kim and Cho [3].

(ii) Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$), $P(z) = \beta$ ($\beta > 0$; real) in Theorem 2.1, we have Corollary 1 due to Kim and Cho [3].

(iii) Putting $a = 0$ and $P(z) = 1$ in Theorem 2.1, we have the result due to Nunokawa *et al.* [15].

(iv) Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$), $P(z) = 1$ in Theorem 2.1, we have Corollary 2 due to Kim and Cho [3].

Putting $p(z) = \frac{zf'(z)}{f(z)}$ in Theorem 2.2, we have the following corollary.

Corollary 3.4 *Let* $f(z) \in A$. *If*

$$\gamma_1 < \mathrm{Im}\left(1 + \frac{zf''(z)}{f'(z)}\right) < \gamma_2,$$

where

$$\gamma_1 = -\frac{\sqrt{|a|^2 + 2(\mathrm{Re}(a))^2} - \mathrm{Im}(a)}{\mathrm{Re}(a)}$$

and

$$\gamma_2 = \frac{\sqrt{|a|^2 + 2(\mathrm{Re}(a))^2} + \mathrm{Im}(a)}{\mathrm{Re}(a)},$$

then

$$\mathrm{Re}\left(a\frac{zf'(z)}{f(z)}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Putting $p(z) = \frac{zf'(z)}{f(z)}$ in Theorem 2.3, we have the following corollary.

Corollary 3.5 *Let* $p(z)$ *be a nonzero analytic function in* $\mathbb{U}$ *with* $p(0) = 1$. *If*

$$\left|\frac{zf''(z)}{f'(z)}\right| < \frac{3\,\mathrm{Re}(a)}{2|a|},$$

then

$$\mathrm{Re}\left(\frac{1}{a}\frac{zf'(z)}{f(z)}\right) > 0,$$

where $\mathrm{Re}(a) > 0$.

Remark 3.2 Putting $a = e^{i\lambda}$ ($|\lambda| < \frac{\pi}{2}$) in Corollary 3.5, we have the result due to Kim and Cho [3].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the paper. Also, all authors have read and approved the final version of the paper.

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