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Unified fixed point theorems in fuzzy metric spaces via common limit range property

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Abstract

The object of this paper is to utilize the notion of common limit range property to prove unified fixed point theorems for weakly compatible mappings in fuzzy metric spaces satisfying an implicit relation due to Rao *et al.* (Hacet. J. Math. Stat. 37(2):97-106, 2008). Some illustrative examples are furnished which demonstrate the validity of the hypotheses and degree of utility of our results. As an application to our main result, we prove an integral type fixed point theorem in fuzzy metric space.

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Keywords: fuzzy metric space; compatible mappings; weakly compatible mappings; common limit range property; fixed point

1 Introduction

The concept of a fuzzy set was introduced by Zadeh [1] in his seminal paper in 1965. In the last two decades, there has been a tremendous development and growth in fuzzy mathematics. In 1975, Kramosil and Michalek [2] introduced the concept of fuzzy metric space, which opened an avenue for further development of analysis in such spaces. Further, George and Veeramani [3] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [2] with a view to obtain a Hausdorff topology on it. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication, *etc.*

Mishra *et al.* [4] extended the notion of compatible mappings to fuzzy metric spaces and proved common fixed point theorems in presence of continuity of at least one of the mappings, completeness of the underlying space and containment of the ranges amongst involved mappings. Further, Singh and Jain [5] weakened the notion of compatibility by using the notion of weakly compatible mappings in fuzzy metric spaces and showed that every pair of compatible mappings is weakly compatible but reverse is not true. Many mathematicians used different contractive conditions on self-mappings and proved several fixed point theorems in fuzzy metric spaces (see [5–13]). However, the study of common fixed points of non-compatible mappings is also of great interest due to Pant [14]. In 2002, Aamri and Moutawakil [15] defined the notion of property (E.A) for a pair of self-mappings which contains the class of non-compatible mappings. In an interesting paper of Ali and Imdad [16], it was pointed out that property (E.A) allows replacing the completeness requirement of the space with a more natural condition of closedness of the range.

Afterward, Liu *et al.* [17] defined a new property which contains the property (E.A) and proved some common fixed point theorems under hybrid contractive conditions. It was observed that the notion of common property (E.A) relatively relaxes the requirement on containment of the range of one mapping into the range of other which is utilized to construct the sequence of joint iterates. Subsequently, there are a number of results proved for contraction mappings satisfying property (E.A) and common property (E.A) in fuzzy metric spaces (see [18–26]). In 2011, Sintunavarat and Kumam [27] coined the idea of 'common limit range property' (also see [28–32]), which relaxes the condition of closedness of the underlying subspace. Recently, Imdad *et al.* [33] extended the notion of common limit range property to two pairs of self-mappings without any requirement on closedness of the underlying subspaces. Several common fixed point theorems have been proved by many researchers in framework of fuzzy metric spaces via implicit relations (see [5, 18, 23, 34]).

In this paper, by using an implicit relation due to Rao *et al.* [12], we prove some common fixed point theorems for weakly compatible mappings with common limit range property. Some related results are also derived besides furnishing illustrative examples. We also present an integral type common fixed point theorem in fuzzy metric space. Our results improve, extend and generalize a host of previously known results including the ones contained in Rao *et al.* [12].

2 Preliminaries

Definition 2.1 [35] A triangular norm $*$ (shortly t-norm) is a binary operation on the unit interval $[0,1]$ such that for all $a,b,c,d \in [0,1]$ and the following conditions are satisfied:

(1) $a * 1 = a$,

(2) $a * b = b * a$,

(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,

(4) $(a * b) * c = a * (b * c)$.

Two typical examples of continuous t-norms are $a * b = \min\{a,b\}$ and $a * b = ab$.

Definition 2.2 [3] A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions: for all $x,y,z \in X$, $t,s > 0$,

(1) $M(x,y,t) > 0$,

(2) $M(x,y,t) = 1$ iff $x = y$,

(3) $M(x,y,t) = M(y,x,t)$,

(4) $M(x,z,t+s) \geq M(x,y,t) * M(y,z,s)$,

(5) $M(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous.

Then M is called a fuzzy metric on X whereas $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $\mathcal{B}(x,r,t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$\mathcal{B}(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}.$$

Now let $(X, M, *)$ be a fuzzy metric space and τ the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $\mathcal{B}(x, r, t) \subset A$. Then τ is a topology on X induced by the fuzzy metric M.

In the following example (see [3]), we know that every metric induces a fuzzy metric.

Example 2.1 Let (X, d) be a metric space. Denote $a * b = ab$ (or $a * b = \min\{a, b\}$) for all $a, b \in [0, 1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{(t + d(x, y))}.$$

Then $(X, M_d, *)$ is a fuzzy metric space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Lemma 2.1 [36] *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing for all $x, y \in X$.*

Definition 2.3 [4] A pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to be compatible iff $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Sx_n \to z$ for some $z \in X$ as $n \to \infty$.

Definition 2.4 [37] A pair (A, S) of self-mappings of a non-empty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is, if $Az = Sz$ some $z \in X$, then $ASz = SAz$.

Remark 2.1 [37] Two compatible self-mappings are weakly compatible but the converse is not true. Hence, the notion of weak compatibility is more general than compatibility.

Definition 2.5 [19] A pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to satisfy the property (E.A), if there exists a sequence $\{x_n\}$ in X for some $z \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$.

Note that weak compatibility and property (E.A) are independent to each other (see [38, Examples 2.1-2.2]).

Remark 2.2 From Definition 2.3, it is inferred that a pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to be non-compatible iff there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$, but for some $t > 0$, $\lim_{n\to\infty} M(ASx_n, SAx_n, t)$ is either less than 1 or non-existent.

Therefore, in view of Definition 2.5, a pair of non-compatible mappings of a fuzzy metric space $(X, M, *)$ satisfies the property (E.A) but the converse need not be true (see [38, Remark 4.8]).

Definition 2.6 [19] Two pairs (A, S) and (B, T) of self-mappings of a fuzzy metric space $(X, M, *)$ are said to satisfy the common property (E.A), if there exist two sequences $\{x_n\}$,

$\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$.

Definition 2.7 [27] A pair (A, S) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to satisfy the common limit range property with respect to mapping S (briefly, (CLR_S) property), if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

where $z \in S(X)$.

Definition 2.8 [28] Two pairs (A, S) and (B, T) of self-mappings of a fuzzy metric space $(X, M, *)$ are said to satisfy the common limit range property with respect to mappings S and T (briefly, (CLR_{ST}) property), if there exist two sequences $\{x_n\}$, $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$.

Definition 2.9 [39] Two families of self-mappings $\{A_i\}_{i=1}^m$ and $\{S_k\}_{k=1}^n$ are said to be pairwise commuting if

(1) $A_iA_j = A_jA_i$ for all $i, j \in \{1, 2, \dots, m\}$,
(2) $S_kS_l = S_lS_k$ for all $k, l \in \{1, 2, \dots, n\}$,
(3) $A_iS_k = S_kA_i$ for all $i \in \{1, 2, \dots, m\}$ and $k \in \{1, 2, \dots, n\}$.

Lemma 2.2 [4] *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$. If there exists a constant $k \in (0, 1)$ such that*

$$M(x, y, kt) \geq M(x, y, t),$$

for all $x, y \in X$, then $x = y$.

3 Implicit function

Following by Rao *et al.* [12], let Φ_6 denote the set of all continuous functions $\phi : [0, 1]^6 \to \mathbb{R}$ satisfying the conditions:

(ϕ_1) ϕ is decreasing in t_2, t_3, t_4, t_5 and t_6.
(ϕ_2) $\phi(u, v, v, v, v, v) \geq 0$ implies $u \geq v$ for all $u, v \in [0, 1]$.

Example 3.1 $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2, t_3, t_4, t_5, t_6\}$.

Example 3.2 $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^2 - \min\{t_it_j : i, j \in \{2, 3, 4, 5, 6\}\}$.

Example 3.3 $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^3 - \min\{t_it_jt_k : i, j, k \in \{2, 3, 4, 5, 6\}\}$.

4 Results

We begin with the following observation.

Lemma 4.1 *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself satisfying*:

(1) *The pair (A, S) (or (B, T)) satisfies the (CLR_S) (or (CLR_T)) property,*
(2) *$A(X) \subset T(X)$ (or $B(X) \subset S(X)$),*
(3) *$T(X)$ (or $S(X)$) is a closed subset of X,*
(4) *$B(y_n)$ converges for every sequence $\{y_n\}$ in X whenever $T(y_n)$ converges (or $A(x_n)$ converges for every sequence $\{x_n\}$ in X whenever $S(x_n)$ converges),*
(5) *there exists a constant $k \in (0,1)$ such that*

$$\phi\begin{pmatrix} M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, t), \\ M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t) \end{pmatrix} \geq 0 \tag{4.1}$$

for all $x, y \in X$, $t > 0$ and $\phi \in \Phi_6$.

Then the pairs (A, S) and (B, T) share the (CLR_{ST}) property.

Proof Since the pair (A, S) enjoys the (CLR_S) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

where $z \in S(X)$. By (2), $A(X) \subset T(X)$, and for each sequence $\{x_n\}$ there exists a sequence $\{y_n\}$ in X such that $Ax_n = Ty_n$. Therefore, due to the closedness of $T(X)$,

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ax_n = z,$$

where $z \in S(X) \cap T(X)$. Thus, we have $Ax_n \to z$, $Sx_n \to z$ and $Ty_n \to z$ as $n \to \infty$. By (4), the sequence $\{By_n\}$ converges and in all we need to show that $By_n \to z$ as $n \to \infty$. Let, on the contrary that $By_n \to z'$ $(\neq z)$ as $n \to \infty$. On using inequality (4.1) with $x = x_n$, $y = y_n$, we have

$$\phi\begin{pmatrix} M(Ax_n, By_n, kt), M(Sx_n, Ty_n, t), M(Ax_n, Sx_n, t), \\ M(By_n, Ty_n, t), M(Ax_n, Ty_n, t), M(By_n, Sx_n, t) \end{pmatrix} \geq 0.$$

Taking the limit as $n \to \infty$, we have

$$0 \leq \phi\begin{pmatrix} M(z, z', kt), M(z, z, t), M(z, z, t), \\ M(z', z, t), M(z, z, t), M(z', z, t) \end{pmatrix},$$

or, equivalently,

$$0 \leq \phi\bigl(M(z, z', kt), 1, 1, M(z', z, t), 1, M(z', z, t)\bigr).$$

Since ϕ is decreasing in $t_2, \ldots, t_6$, we get

$$0 \leq \phi\bigl(M(z, z', kt), M(z, z', t), M(z, z', t), M(z', z, t), M(z, z', t), M(z', z, t)\bigr).$$

From (ϕ_2), we have $M(z,z',kt) \geq M(z,z',t)$. Appealing to Lemma 2.2, we obtain $z = z'$. Therefore, $\lim_{n\to\infty} By_n = z$ and hence the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property. □

Remark 4.1 The converse of Lemma 4.1 is not true in general. For a counter example, we refer to Example 4.1.

Theorem 4.1 *Let $(X,M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself satisfying inequality* (4.1). *Suppose that the pairs (A,S) and (B,T) enjoy the (CLR_{ST}) property. Then the pairs (A,S) and (B,T) have a coincidence point each. Moreover, A,B, S and T have a unique common fixed point provided both the pairs (A,S) and (B,T) are weakly compatible.*

Proof Since the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$. Since $z \in S(X)$, there exists a point $u \in X$ such that $Su = z$. First we show that $Au = Su$, by putting $x = u$ and $y = y_n$ in (4.1), we get

$$\phi\begin{pmatrix} M(Au,By_n,kt), M(Su,Ty_n,t), M(Au,Su,t), \\ M(By_n,Ty_n,t), M(Au,Ty_n,t), M(By_n,Su,t) \end{pmatrix} \geq 0,$$

which on making $n \to \infty$, reduces to

$$\phi\begin{pmatrix} M(Au,z,kt), M(z,z,t), M(Au,z,t), \\ M(z,z,t), M(Au,z,t), M(z,z,t) \end{pmatrix} \geq 0,$$

and so,

$$\phi\big(M(Au,z,kt), 1, M(Au,z,t), 1, M(Au,z,t), 1\big) \geq 0.$$

From (ϕ_1) and (ϕ_2), we have $M(Au,z,kt) \geq M(Au,z,t)$. In view of Lemma 2.2, $Au = z$, and hence $Au = Su = z$ which shows that u is a coincidence point of the pair (A,S).

Also $z \in T(X)$, there exists a point $v \in X$ such that $Tv = z$. Now we assert that $Bv = Tv$. On using (4.1) with $x = u$, $y = v$, we have

$$\phi\begin{pmatrix} M(Au,Bv,kt), M(Su,Tv,t), M(Au,Su,t), \\ M(Bv,Tv,t), M(Au,Tv,t), M(Bv,Su,t) \end{pmatrix} \geq 0,$$

which reduces to

$$\phi\begin{pmatrix} M(z,Bv,kt), M(z,z,t), M(z,z,t), \\ M(Bv,z,t), M(z,z,t), M(Bv,z,t) \end{pmatrix} \geq 0,$$

or, equivalently,

$$\phi\big(M(z,Bv,kt), 1, 1, M(Bv,z,t), 1, M(Bv,z,t)\big) \geq 0.$$

From (ϕ_1) and (ϕ_2), we have $M(z,Bv,kt) \geq M(z,Bv,t)$. On employing Lemma 2.2, we have $z = bv$. Hence, $Bv = Tv = z$ which shows that v is a coincidence point of the pair (B,T).

Since the pair (A,S) is weakly compatible and $Au = Su$, hence $Az = ASu = SAu = Sz$. Now, we show that z is a common fixed point of the pair (A,S). Putting $x = z$ and $y = v$ in (4.1), we have

$$\phi\begin{pmatrix} M(Az,Bv,kt), M(Sz,Tv,t), M(Az,Sz,t), \\ M(Bv,Tv,t), M(Az,Tv,t), M(Bv,Sz,t) \end{pmatrix} \geq 0,$$

and so

$$\phi\begin{pmatrix} M(Az,z,kt), M(Az,z,t), M(Az,Az,t), \\ M(z,z,t), M(Az,z,t), M(z,Az,t) \end{pmatrix} \geq 0,$$

or, equivalently,

$$\phi\bigl(M(Az,z,kt), M(Az,z,t), 1, 1, M(Az,z,t), M(z,Az,t)\bigr) \geq 0.$$

From (ϕ_1) and (ϕ_2), we have $M(Az,z,kt) \geq M(Az,z,t)$. Owing Lemma 2.2, $Az = z = Sz$, which shows that z is a common fixed point of the pair (A,S).

Also the pair (B,T) is weakly compatible and $Bv = Tv$, then $Bz = BTv = TBv = Tz$. On using (4.1) with $x = u$, $y = z$, we have

$$\phi\begin{pmatrix} M(Au,Bz,kt), M(Su,Tz,t), M(Au,Su,t), \\ M(Bz,Tz,t), M(Au,Tz,t), M(Bz,Su,t) \end{pmatrix} \geq 0,$$

which reduces to

$$\phi\begin{pmatrix} M(z,Bz,kt), M(z,Bz,t), M(z,z,t), \\ M(Bz,Bz,t), M(z,Bz,t), M(Bz,z,t) \end{pmatrix} \geq 0,$$

and so

$$\phi\bigl(M(z,Bz,kt), M(z,Bz,t), 1, 1, M(z,Bz,t), M(Bz,z,t)\bigr) \geq 0.$$

From (ϕ_1) and (ϕ_2), we have $M(z,Bz,kt) \geq M(z,Bz,t)$. In view of Lemma 2.2, $Bz = z = Tz$. Hence, z is a common fixed point of the pair (B,T). Therefore, z is a common fixed point of the mappings A, B, S and T. The uniqueness of common fixed point is an easy consequence of the inequality (4.1) in view of (ϕ_1) and (ϕ_2). □

Remark 4.2 Theorem 4.1 improves the results of Aalam *et al.* [18, Theorem 3.1], Rao *et al.* [12, Theorem 3.1] and Chauhan and Kumar [20, Theorem 3.1] as Theorem 4.1 never requires conditions on completeness (or closedness) of the underlying space (or subspace), continuity of the involved mappings and containment amongst range sets of the involved mappings.

Example 4.1 Let $(X,M,*)$ be a fuzzy metric space, where $X = [1,15)$, with t-norm defined $a * b = \min\{a,b\}$ for all $a,b \in [0,1]$ and

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases}$$

for all $t > 0$ and $x,y \in X$. Define the self-mappings A, B, S and T by

$$A(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3,15); \\ 14, & \text{if } x \in (1,3], \end{cases} \qquad B(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3,15); \\ 5, & \text{if } x \in (1,3], \end{cases}$$

$$S(x) = \begin{cases} 1, & \text{if } x = 1; \\ \frac{7}{2}, & \text{if } x \in (1,3]; \\ \frac{x+1}{4}, & \text{if } x \in (3,15), \end{cases} \qquad T(x) = \begin{cases} 1, & \text{if } x = 1; \\ 9+x, & \text{if } x \in (1,3]; \\ x-2, & \text{if } x \in (3,15). \end{cases}$$

We have $A(X) = \{1,14\} \nsubseteq [1,13) = T(X)$ and $B(X) = \{1,5\} \nsubseteq [1,4) = S(X)$ which shows that $S(X)$ and $T(X)$ are not closed subsets of X. Consider $\phi : [0,1]^6 \to \mathbb{R}$ as defined in Examples 3.1-3.3. If we choose two sequences $\{x_n\} = \{3 + \frac{1}{n}\}$, $\{y_n\} = \{1\}$ (or $\{x_n\} = \{1\}$, $\{y_n\} = \{3 + \frac{1}{n}\}$), then clearly

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 1 \in S(X) \cap T(X).$$

Hence, both the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property. By a routine calculation, one can verify the inequality (4.1). Thus, all the conditions of Theorem 4.1 are satisfied for some fixed $k \in (0,1)$ and 1 is a unique common fixed point of the pairs (A,S) and (B,T), which also remains a point of coincidence as well. Also, all the involved mappings are even discontinuous at their unique common fixed point 1.

Theorem 4.2 *Let $(X,M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself. Suppose that the inequality* (4.1) *and the following hypotheses hold*:
(1) *the pairs (A,S) and (B,T) satisfy the common property $(E.A)$*,
(2) *$S(X)$ and $T(X)$ are closed subsets of X.*
Then the pairs (A,S) and (B,T) have a coincidence point each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A,S) and (B,T) are weakly compatible.

Proof Since the pairs (A,S) and (B,T) enjoy the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

for some $z \in X$. Since $S(X)$ is a closed subset of X, hence $\lim_{n\to\infty} Sx_n = z \in S(X)$. Therefore, there exists a point $u \in X$ such that $Su = z$. Also, $T(X)$ is a closed subset of X, $\lim_{n\to\infty} Ty_n = z \in T(X)$. Therefore, there exists a point $v \in X$ such that $Tv = z$. The rest of the proof runs on the lines of the proof of Theorem 4.1. □

Example 4.2 In the setting of Example 4.1, replace the self-mappings A, B, S and T by the following besides retaining the rest:

$$A(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3,15); \\ 14, & \text{if } x \in (1,3], \end{cases} \qquad B(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3,15); \\ 5, & \text{if } x \in (1,3], \end{cases}$$

$$S(x) = \begin{cases} 1, & \text{if } x = 1; \\ 4, & \text{if } x \in (1,3]; \\ \frac{x+1}{4}, & \text{if } x \in (3,15), \end{cases} \qquad T(x) = \begin{cases} 1, & \text{if } x = 1; \\ 10 + x, & \text{if } x \in (1,3]; \\ x - 2, & \text{if } x \in (3,15). \end{cases}$$

Then we have $A(X) = \{1,14\} \nsubseteq [1,13] = T(X)$ and $B(X) = \{1,5\} \nsubseteq [1,4] = S(X)$, hence $S(X)$ and $T(X)$ are closed subsets of X.

Consider two sequences as in Example 4.1, one can see that both the pairs (A,S) and (B,T) enjoy the common property (E.A), that is,

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = 1 \in X.$$

Therefore, all the conditions of Theorem 4.2 are satisfied for some fixed $k \in (0,1)$ and 1 is a unique common fixed point of the pairs (A,S) and (B,T), which also remains a point of coincidence as well.

Theorem 4.3 *Let $(X,M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself satisfying all the hypotheses of Lemma* 4.1. *Then the pairs (A,S) and (B,T) have a coincidence point each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A,S) and (B,T) are weakly compatible.*

Proof In view of Lemma 4.1, the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$. The rest of the proof can be completed on the lines of the proof of Theorem 4.1. □

Example 4.3 In the setting of Example 4.1, replace the self-mappings A, B, S and T by the following besides retaining the rest:

$$A(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3,15); \\ 5, & \text{if } x \in (1,3], \end{cases} \qquad B(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3,15); \\ 4, & \text{if } x \in (1,3], \end{cases}$$

$$S(x) = \begin{cases} 1, & \text{if } x = 1; \\ 4, & \text{if } x \in (1,3]; \\ \frac{x+1}{4}, & \text{if } x \in (3,15), \end{cases} \qquad T(x) = \begin{cases} 1, & \text{if } x = 1; \\ 10 + x, & \text{if } x \in (1,3]; \\ x - 2, & \text{if } x \in (3,15). \end{cases}$$

We obtain $A(X) = \{1,5\} \subset [1,13] = T(X)$ and $B(X) = \{1,4\} \subset [1,4] = S(X)$, which shows that $S(X)$ and $T(X)$ are closed subsets of X. By a routine calculation, one can easily verify the inequality (4.1) for all $x, y \in X$ and for some fixed $k \in (0,1)$. Thus, all the conditions of Theorem 4.3 are satisfied and 1 is a unique common fixed point of (A,S) and (B,T), which also remains a point of coincidence as well.

Here, it is worth noting that the majority of earlier proved results Theorem 4.1 cannot be used in the context of this example as Theorem 4.1 never requires any condition on the containment of ranges amongst involved mappings.

Remark 4.3 The conclusions of Lemma 4.1, Theorem 4.1, Theorem 4.2, Corollary 4.1 and Theorem 4.3 remain true if we define $\phi(t_1, t_2, t_3, t_4, t_5, t_6)$ as in Examples 3.1-3.3.

Remark 4.4 Theorem 4.1 and Theorem 4.3 (in view of Example 3.1) improve and generalize the results of Mishra *et al.* [4, Theorem 1].

By choosing A, B, S and T suitably, we can drive a multitude of common fixed point theorems for a pair or triod of mappings. As a sample, we deduce the following natural result for a pair of self-mappings.

Corollary 4.1 *Let $(X,M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A and S be mappings from X into itself satisfying the following conditions*:

(1) *the pair (A,S) satisfies the (CLR_S) property,*
(2) *there exists a constant $k \in (0,1)$ such that*

$$\phi\begin{pmatrix} M(Ax,Ay,kt), M(Sx,Sy,t), M(Ax,Sx,t), \\ M(Ay,Sy,t), M(Ax,Sy,t), M(Ay,Sx,t) \end{pmatrix} \geq 0 \tag{4.2}$$

for all $x, y \in X$, $t > 0$ and $\phi \in \Phi_6$.

Then A and S have a coincidence point. Moreover, if the pair (A,S) is weakly compatible, then A and S have a unique common fixed point.

Remark 4.5 Corollary 4.1 (in view of Example 3.1) generalizes the results of Miheţ [25, Theorem 3.1].

As an application of Theorem 4.2, we have the following result involving four finite families of self-mappings.

Theorem 4.4 *Let $(X,M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$ and $\{T_g\}_{g=1}^q$ be four finite families from X into itself such that $A = A_1A_2 \cdots A_m$, $B = B_1B_2 \cdots B_n$, $S = S_1S_2 \cdots S_p$ and $T = T_1T_2 \cdots T_q$ which satisfy the inequality (4.1). If the pairs (A,S) and (B,T) satisfy the (CLR_{ST}) property, then (A,S) and (B,T) have a point of coincidence each.*

Moreover, $\{A_i\}_{i=1}^m$, $\{B_r\}_{r=1}^n$, $\{S_k\}_{k=1}^p$ and $\{T_g\}_{g=1}^q$ have a unique common fixed point provided the pairs of families $(\{A_i\},\{S_k\})$ and $(\{B_r\},\{T_g\})$ are commute pairwise, where $i \in \{1,2,\ldots,m\}$, $k \in \{1,2,\ldots,p\}$, $r \in \{1,2,\ldots,n\}$ and $g \in \{1,2,\ldots,q\}$.

Proof The proof of this theorem is similar to that of Theorem 3.1 contained in Imdad *et al.* [39], hence details are avoided. □

Now, we indicate that Theorem 4.4 can be utilized to derive common fixed point theorems for any finite number of mappings. As a sample for five mappings, we can derive the following by setting one family of two members while the remaining three of single members.

Corollary 4.2 *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, R, S and T be mappings from X into itself and satisfy the following conditions*:

(1) *the pairs (A, SR) and (B, T) share the $(CLR_{(SR)(T)})$ property,*
(2) *there exists a constant $k \in (0,1)$ such that*

$$\phi\begin{pmatrix} M(Ax, By, kt), M(SRx, THy, t), M(Ax, SRx, t), \\ M(By, THy, t), M(Ax, THy, t), M(By, SRx, t) \end{pmatrix} \geq 0 \tag{4.3}$$

for all $x, y \in X$, $t > 0$ and $\phi \in \Phi_6$.

Then the pairs (A, SR) and (B, T) have a coincidence point each. Moreover, A, B, R, S and T have a unique common fixed point provided the pairs (A, SR) and (B, T) commute pairwise (*that is, $AS = SA$, $AR = RA$, $SR = RS$ and $BT = TB$*).

Similarly, we can derive a common fixed point theorem for six mappings by setting two families of two members while the rest two of single members.

Corollary 4.3 *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, R, S, H and T be mappings from X into itself and satisfy the following conditions*:

(1) *the pairs (A, SR) and (B, TH) enjoy the $(CLR_{(SR)(TH)})$ property,*
(2) *there exists a constant $k \in (0,1)$ such that*

$$\phi\begin{pmatrix} M(Ax, By, kt), M(SRx, THy, t), M(Ax, SRx, t), \\ M(By, THy, t), M(Ax, THy, t), M(By, SRx, t) \end{pmatrix} \geq 0 \tag{4.4}$$

for all $x, y \in X$, $t > 0$ and $\phi \in \Phi_6$.

Then the pairs (A, SR) and (B, TH) have a coincidence point each. Moreover, A, B, R, S, H and T have a unique common fixed point provided the pairs (A, SR) and (B, TH) commute pairwise (*that is, $AS = SA$, $AR = RA$, $SR = RS$, $BT = TB$, $BH = HB$ and $TH = HT$*).

By setting $A_1 = A_2 = \cdots = A_m = A$, $B_1 = B_2 = \cdots = B_n = B$, $S_1 = S_2 = \cdots = S_p = S$ and $T_1 = T_2 = \cdots = T_q = T$ in Theorem 4.4, we deduce the following.

Corollary 4.4 *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself such that the pairs (A^m, S^p) and (B^n, T^q) share the (CLR_{S^p,T^q}) property. Suppose that there exists a constant $k \in (0,1)$ such that*

$$\phi\begin{pmatrix} M(A^m x, B^n y, kt), M(S^p x, T^q y, t), M(A^m x, S^p x, t), \\ M(B^n y, T^q y, t), M(A^m x, T^q y, t), M(B^n y, S^p x, t) \end{pmatrix} \geq 0 \tag{4.5}$$

for all $x, y \in X$, $t > 0$, $\phi \in \Phi_6$ and m, n, p, q are fixed positive integers.

Then the pairs (A, S) and (B, T) have a point of coincidence each. Further, A, B, S and T have a unique common fixed point provided both the pairs (A^m, S^p) and (B^n, T^q) commute pairwise.

Remark 4.6 Theorem 4.4 and Corollary 4.4 generalize and extend the results of Abbas *et al.* [19, Theorem 2.3, Corollary 2.5] to finite families of weakly compatible mappings.

5 Related results for contractive conditions of integral type

A study of contractive conditions of integral type has been initiated by Branciari [40], giving an integral version of the Banach contraction principle. Subsequently, several authors proved common fixed point theorems satisfying contractive conditions of integral type (*e.g.* [41–44]). In this section, we state and prove an integral analogue of Theorem 4.1. First, we need to prove the following lemma in fuzzy metric spaces which is motivated by Altun *et al.* [45].

Lemma 5.1 *Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. If there exists a constant $k \in (0,1)$ such that for all $x, y \in X$ and all $t > 0$,*

$$\int_0^{M(x,y,kt)} \varphi(s)\,ds \geq \int_0^{M(x,y,t)} \varphi(s)\,ds, \tag{5.1}$$

where $\varphi : [0,\infty) \to [0,\infty)$ is a summable non-negative Lebesgue integrable function such that

$$\int_\epsilon^1 \varphi(s)\,ds > 0,$$

for each $\epsilon \in [0,1)$, then $x = y$.

Proof From (5.1), we have

$$\int_0^{M(x,y,kt)} \varphi(s)\,ds \geq \int_0^{M(x,y,t)} \varphi(s)\,ds,$$

implies

$$\int_0^{M(x,y,t)} \varphi(s)\,ds \geq \int_0^{M(x,y,k^{-1}t)} \varphi(s)\,ds,$$

similarly, we can inductively write for $n \in \mathbb{N}$

$$\begin{aligned}\int_0^{M(x,y,t)} \varphi(s)\,ds &\geq \int_0^{M(x,y,k^{-1}t)} \varphi(s)\,ds \\ &\geq \int_0^{M(x,y,k^{-2}t)} \varphi(s)\,ds \\ &\geq \cdots \\ &\geq \int_0^{M(x,y,k^{-n}t)} \varphi(s)\,ds \to \int_0^1 \varphi(s)\,ds,\end{aligned}$$

as $n \to \infty$. Therefore,

$$\int_0^{M(x,y,t)} \varphi(s)\,ds - \int_0^1 \varphi(s)\,ds \geq 0,$$

and so

$$\int_0^{M(x,y,t)} \varphi(s)\,ds - \left(\int_0^{M(x,y,t)} \varphi(s)\,ds + \int_{M(x,y,t)}^{1} \varphi(s)\,ds\right) \geq 0,$$

or, equivalently,

$$\int_{M(x,y,t)}^{1} \varphi(s)\,ds \leq 0,$$

implying $M(x,y,t) \geq 1$ for all $t \geq 0$. Then we get $x = y$. □

Remark 5.1 By setting $\varphi(s) = 1$ for each $s \geq 0$ in (5.1), we have

$$\int_0^{M(x,y,kt)} ds = [s]_0^{M(x,y,kt)} = M(x,y,kt) \geq M(x,y,t) = \int_0^{M(x,y,t)} ds,$$

which shows that Lemma 5.1 is a generalization of the Lemma 2 contained in [4].

Theorem 5.1 *Let $(X,M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$. Let A, B, S and T be mappings from X into itself. Suppose that there exists a function $\phi \in \Phi_6$ satisfying*

$$\int_0^{\phi(u,v,v,v,v,v)} \varphi(s)\,ds \geq 0, \tag{5.2}$$

where $\varphi : [0,\infty) \to [0,\infty)$ is a summable non-negative Lebesgue integrable function such that for each $\epsilon \in [0,1)$

$$\int_\epsilon^1 \varphi(s)\,ds > 0,$$

implies $u \geq v$ for all $u,v \in [0,1]$. Suppose that the pairs (A,S) and (B,T) share the (CLR_{ST}) property. There exists a constant $k \in (0,1)$ such that

$$\int_0^{\phi(M(Ax,By,kt),M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t))} \varphi(s)\,ds \geq 0, \tag{5.3}$$

for all $x,y \in X$ and $t > 0$, then the pairs (A,S) and (B,T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A,S) and (B,T) are weakly compatible.

Proof Since the pairs (A,S) and (B,T) share the (CLR_{ST}) property, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z,$$

where $z \in S(X) \cap T(X)$. Since $z \in S(X)$, there exists a point $u \in X$ such that $Su = z$. Now we assert that $Au = Su$. On using (5.3) with $x = u$, $y = y_n$, we have

$$\int_0^{\phi(M(Au,By_n,kt),M(Su,Ty_n,t),M(Au,Su,t),M(By_n,Ty_n,t),M(Au,Ty_n,t),M(By_n,Su,t))} \varphi(s)\,ds \geq 0.$$

Taking the limit as $n \to \infty$, we have

$$0 \leq \int_0^{\phi(M(Au,z,kt),M(z,z,t),M(Au,z,t),M(z,z,t),M(Au,z,t),M(z,z,t))} \varphi(s)\,ds,$$

and so

$$0 \leq \int_0^{\phi(M(Au,z,kt),1,M(Au,z,t),1,M(Au,z,t),1)} \varphi(s)\,ds.$$

Since ϕ is decreasing in $t_2, \ldots, t_6$, we get

$$0 \leq \int_0^{\phi(M(Au,z,kt),M(z,Au,t),M(Au,z,t),M(z,Au,t),M(Au,z,t),M(z,Au,t))} \varphi(s)\,ds.$$

From (5.2), we have $M(Au, z, kt) \geq M(Au, z, t)$. Now appealing to Lemma 4.1, $Au = z$ and hence $Au = Su = z$ which shows that u is a coincidence point of the pair (A, S).

Also $z \in T(X)$, therefore, there exists a point $v \in X$ such that $Tv = z$. Now we show that $Bv = Tv$. On using (5.3) with $x = u$, $y = v$, we have

$$\int_0^{\phi(M(Au,Bv,kt),M(Su,Tv,t),M(Au,Su,t),M(Bv,Tv,t),M(Au,Tv,t),M(Bv,Su,t))} \varphi(s)\,ds \geq 0,$$

or, equivalently,

$$0 \leq \int_0^{\phi(M(z,Bv,kt),M(z,z,t),M(z,z,t),M(Bv,z,t),M(z,z,t),M(Bv,z,t))} \varphi(s)\,ds,$$

and so

$$0 \leq \int_0^{\phi(M(z,Bv,kt),1,1,M(Bv,z,t),1,M(Bv,z,t))} \varphi(s)\,ds.$$

Since ϕ is decreasing in $t_2, \ldots, t_6$, we get

$$0 \leq \int_0^{\phi(M(z,Bv,kt),M(z,Bv,kt),M(z,Bv,kt),M(Bv,z,t),M(z,Bv,kt),M(Bv,z,t))} \varphi(s)\,ds.$$

From (5.2), we have $M(z, Bv, kt) \geq M(z, Bv, t)$. Owing Lemma 4.1, $Bv = z$ and henceforth $Bv = Tv = z$. Therefore, v is a coincidence point of the pair (B, T).

Since the pair (A, S) is weakly compatible and $Au = Su$, therefore, $Az = ASu = SAu = Sz$. Putting $x = z$, $y = v$ in (5.3), we get

$$\int_0^{\phi(M(Az,Bv,kt),M(Sz,Tv,t),M(Az,Sz,t),M(Bv,Tv,t),M(Az,Tv,t),M(Bv,Sz,t))} \varphi(s)\,ds \geq 0,$$

and so

$$0 \leq \int_0^{\phi(M(Az,z,kt),M(z,z,t),M(Az,Az,t),M(z,z,t),M(Az,z,t),M(z,Az,t))} \varphi(s)\,ds,$$

or, equivalently,

$$0 \leq \int_0^{\phi(M(Az,z,kt),1,1,1,M(Az,z,t),M(z,Az,t))} \varphi(s)\,ds.$$

Since ϕ is decreasing in $t_2, \dots, t_6$, we obtain

$$0 \leq \int_0^{\phi(M(Az,z,kt),M(Az,z,t),M(Az,z,t),M(Az,z,t),M(Az,z,t),M(z,Az,t))} \varphi(s)\,ds.$$

From (5.2), we have $M(Az,z,kt) \geq M(Az,z,t)$. Using Lemma 4.1, we have $Az = z = Sz$ which shows that z is a common fixed point of the pair (A,S).

Also the pair (B,T) is weakly compatible and $Bv = Tv$, hence $Bz = BTv = TBv = Tz$. Now we show that z is a common fixed point of the pair (B,T). In order to accomplish this, putting $x = u$, $y = z$ in (5.3), we have

$$\int_0^{\phi(M(Au,Bz,kt),M(Su,Tz,t),M(Au,Su,t),M(Bz,Tz,t),M(Au,Tz,t),M(Bz,Su,t))} \varphi(s)\,ds \geq 0,$$

and so

$$0 \leq \int_0^{\phi(M(z,Bz,kt),M(z,Bz,t),M(z,z,t),M(Bz,Bz,t),M(z,Bz,t),M(Bz,z,t))} \varphi(s)\,ds,$$

which reduces to

$$0 \leq \int_0^{\phi(M(z,Bz,kt),M(z,Bz,t),1,1,M(z,Bz,t),M(Bz,z,t))} \varphi(s)\,ds.$$

Since ϕ is decreasing in $t_2, \dots, t_6$, we get

$$0 \leq \int_0^{\phi(M(z,Bz,kt),M(z,Bz,t),M(z,Bz,t),M(z,Bz,t),M(z,Bz,t),M(Bz,z,t))} \varphi(s)\,ds.$$

From (5.2), we have $M(z,Bz,kt) \geq M(z,Bz,t)$. Appealing to Lemma 4.1, we have $Bz = z = Tz$ which shows that z is a common fixed point of the pair (B,T). Hence z is a common fixed point of mappings A, B, S and T. Uniqueness of common fixed point is an easy consequence of the inequality (5.3). □

Remark 5.2 Results similar to Theorem 4.2, Theorem 4.3, Theorem 4.4, Corollary 4.1, Corollary 4.2 and Corollary 4.3 can be outlined in views of Theorem 5.1, but we do not include the details due to repetition. Earlier proved results can also be described in view of Examples 3.1-3.3 (using Lemma 5.1) which improve the results of Chauhan and Kumar [20, Theorem 4.1].

Remark 5.3 Theorem 5.1 (in view of Remark 5.2) improves the results of Bhatia [41, Theorem 3.1], Shao [43, Theorem 3.2], Murthy *et al.* [42, Theorems 2, 3, 5] and extend the result of Sedghi and Shobe [44, Theorem 2.2].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
SC, MAK and SK contributed equally. All authors read and approved the final manuscript.

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