RESEARCH

Open Access

$(\varphi, \alpha, \delta, \lambda, \Omega)_p$ -Neighborhood for some classes of multivalent functions

Fatma Sağsöz^{1*} and Muhammet Kamali²

*Correspondence:

faltuntas@atauni.edu.tr

¹Department of Mathematics,
Faculty of Science, Atatürk
University, Erzurum, 25240, Turkey
Full list of author information is
available at the end of the article

Abstract

In the present paper, we obtain some interesting results for neighborhoods of multivalent functions. Furthermore, we give an application of Miller and Mocanu's lemma.

MSC: 30C45

Keywords: neighborhood; multivalent function; Miller and Mocanu's lemma

1 Introduction and definitions

Let A denote the class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic in the open unit disk

$$U = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

We denote by $A(p, n)$ the class of functions f of the form

$$f(z) = z^p + \sum_{k=n}^{\infty} a_{k+p} z^{k+p} \quad (n, p \in \mathbb{N} = \{1, 2, \dots\})$$

which are analytic and multivalent in the open unit disk U .

The concept of neighborhood for $f \in A$ was first given by Goodman [1]. The concept of δ -neighborhoods $N_\delta(f)$ of analytic functions $f \in A$ was first introduced by Ruscheweyh [2]. Walker [3] defined a neighborhood of analytic functions having positive real part. Owa *et al.* [4] generalized of the results given by Walker. In 1996, Altıntaş and Owa [5] gave (n, δ) -neighborhoods for functions $f \in A$ with negative coefficients. In 2007, new definitions for neighborhoods of analytic functions $f \in A$ were considered by Orhan *et al.* [6]. The authors gave the following definition of neighborhoods:

For $f, g \in A$, f is said to be (α, δ) -neighborhood for g if it satisfies

$$|f'(z) - e^{i\alpha} g'(z)| < \delta \quad (z \in U)$$

for some $-\pi \leq \alpha \leq \pi$ and $\delta > \sqrt{2(1 - \cos \alpha)}$. They denote this neighborhood by $(\alpha, \delta) - N(g)$.

Also, they saw that $f \in (\alpha, \delta) - M(g)$ if it satisfies

$$\left| \frac{f(z)}{z} - e^{i\alpha} \frac{g(z)}{z} \right| < \delta \quad (z \in U)$$

for some $-\pi \leq \alpha \leq \pi$ and $\delta > \sqrt{2(1 - \cos \alpha)}$.

In 2009, Altuntaş *et al.* [7] gave the following definition for neighborhood of analytic functions $f \in A(p, n)$.

For $f, g \in A(p, n)$, f is said to be $(\alpha, \delta)_p$ -neighborhood for g if it satisfies

$$\left| \frac{f'(z)}{z^{p-1}} - e^{i\alpha} \frac{g'(z)}{z^{p-1}} \right| < \delta \quad (z \in U)$$

for some $-\pi \leq \alpha \leq \pi$ and $\delta > p\sqrt{2(1 - \cos \alpha)}$. They denote this neighborhood by $(\alpha, \delta)_p - N(g)$.

Also, they saw that $f \in (\alpha, \delta)_p - M(g)$ if it satisfies

$$\left| \frac{f(z)}{z^p} - e^{i\alpha} \frac{g(z)}{z^p} \right| < \delta \quad (z \in U)$$

for some $-\pi \leq \alpha \leq \pi$ and $\delta > \sqrt{2(1 - \cos \alpha)}$.

Recently, Frasin [8] introduced the following definition of (α, β, δ) -neighborhood for analytic function f in the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0). \quad (1.1)$$

Let f be defined by (1.1). Then f is said to be (α, β, δ) -neighborhood for $g = z - \sum_{n=2}^{\infty} b_n z^n$ ($b_n \geq 0$) if it satisfies

$$\left| e^{i\alpha} (D^k f(z))' - e^{i\beta} (D^k g(z))' \right| < \delta$$

for some $-\pi \leq \alpha, \beta \leq \pi$ and $\delta > \sqrt{2(1 - \cos(\alpha - \beta))}$.

The differential operator D^k was introduced by Salagean [9].

Now, we give the following equalities for the functions $f \in A(p, n)$

$$D^0 f(z) = f(z),$$

$$D^1 f(z) = z(D^0 f(z))' = pz^p + \sum_{k=n}^{\infty} (p+k)a_{k+p}z^{k+p},$$

$$\vdots$$

$$D^{\Omega} f(z) = D(D^{\Omega-1} f(z)) = p^{\Omega} z^p + \sum_{k=n}^{\infty} (p+k)^{\Omega} a_{k+p} z^{k+p}.$$

We define $\wp : A(p, n) \rightarrow A(p, n)$ such that

$$\wp(f(z)) = \left(\frac{1}{p^\Omega} - \lambda \right) D^\Omega f(z) + \frac{\lambda}{p} z (D^\Omega f(z))' \quad \left(0 \leq \lambda \leq \frac{1}{p^\Omega}, \Omega \in \mathbb{N} \cup \{0\} \right). \quad (1.2)$$

We denote by $\wp_{(\Omega, \lambda)}$ the class of analytic functions of the form (1.2) in U .

For $f, g \in \wp_{(\Omega, \lambda)}$, f is said to be $(\varphi, \alpha, \delta, \lambda, \Omega)_p$ -neighborhood for g if it satisfies

$$\left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| < \delta \quad (z \in U)$$

for some $-\pi \leq \varphi - \alpha \leq \pi$ and $\delta > p\sqrt{2(1 - \cos(\varphi - \alpha))}$. We denote this neighborhood by $(\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$.

Also, we say that $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{M}(g)$ if it satisfies

$$\left| e^{i\varphi} \frac{\wp(f(z))}{z^p} - e^{i\alpha} \frac{\wp(g(z))}{z^p} \right| < \delta \quad (z \in U)$$

for some $-\pi \leq \varphi - \alpha \leq \pi$ and $\delta > \sqrt{2(1 - \cos(\varphi - \alpha))}$.

We discuss some properties of f belonging to $(\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$ and $(\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{M}(g)$.

2 Main results

Theorem 2.1 *If $f \in \wp_{(\Omega, \lambda)}$ satisfies*

$$\begin{aligned} & \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| \\ & \leq \delta - p\sqrt{2\{1 - \cos(\varphi - \alpha)\}} \end{aligned} \quad (2.1)$$

for some $-\pi \leq \varphi - \alpha \leq \pi$ and $\delta > p\sqrt{2(1 - \cos(\varphi - \alpha))}$, then $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$.

Proof By virtue of (1.2), we can write

$$\begin{aligned} & \left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| \\ & = \left| p e^{i\varphi} + e^{i\varphi} \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) a_{k+p} z^k - p e^{i\alpha} \right. \\ & \quad \left. - \sum_{k=n}^{\infty} e^{i\alpha} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) b_{k+p} z^k \right| \\ & < p\sqrt{2\{1 - \cos(\varphi - \alpha)\}} + \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}|. \end{aligned}$$

If

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |a_{k+p} - e^{i\alpha} b_{k+p}| \leq \delta - p\sqrt{2\{1 - \cos(\varphi - \alpha)\}},$$

then we see that

$$\left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| < \delta \quad (z \in U).$$

Thus, $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$. □

Example 2.2 For given

$$g(z) = z^p + \sum_{k=n}^{\infty} B_{k+p}(\varphi, \alpha, \delta, \lambda, \Omega) z^{k+p} \in \wp(\Omega, \lambda) \quad (n, p \in \mathbb{N} = \{1, 2, \dots\})$$

we consider

$$f(z) = z^p + \sum_{k=n}^{\infty} A_{k+p}(\varphi, \alpha, \delta, \lambda, \Omega) z^{k+p} \in \wp(\Omega, \lambda) \quad (n, p \in \mathbb{N} = \{1, 2, \dots\})$$

with

$$A_{k+p} = \frac{p^{\Omega}(\delta - p\sqrt{2(1 - \cos(\varphi - \alpha))})}{(k+p)^{\Omega+2}(1 + \lambda kp^{\Omega-1})(k+p-1)} (n+p-1)e^{-i\varphi} + e^{i(\alpha-\varphi)} B_{k+p}.$$

Then we have that

$$\begin{aligned} & \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda kp^{\Omega-1}) |e^{i\varphi} A_{k+p} - e^{i\alpha} B_{k+p}| \\ &= (n+p-1)(\delta - p\sqrt{2(1 - \cos(\varphi - \alpha))}) \sum_{k=n}^{\infty} \frac{1}{(k+p)(k+p-1)}. \end{aligned} \quad (2.2)$$

Finally, in view of the telescopic sum, we can write

$$\begin{aligned} \sum_{k=n}^{\infty} \frac{1}{(k+p)(k+p-1)} &= \lim_{q \rightarrow \infty} \sum_{k=n}^q \left\{ \frac{1}{(k+p-1)} - \frac{1}{(k+p)} \right\} \\ &= \lim_{q \rightarrow \infty} \left\{ \frac{1}{(n+p-1)} - \frac{1}{(p+q)} \right\} \\ &= \frac{1}{n+p-1}. \end{aligned} \quad (2.3)$$

Using (2.3) in (2.2), we have

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda kp^{\Omega-1}) |e^{i\varphi} A_{k+p} - e^{i\alpha} B_{k+p}| = (\delta - p\sqrt{2(1 - \cos(\varphi - \alpha))}).$$

Therefore, $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$.

Corollary 2.3 If $f \in \wp(\Omega, \lambda)$ satisfies

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda kp^{\Omega-1}) |a_{k+p} - b_{k+p}| \leq \delta - p\sqrt{2(1 - \cos(\varphi - \alpha))}$$

for some $-\pi \leq \varphi - \alpha \leq \pi$, $\delta > p\sqrt{2[1 - \cos(\varphi - \alpha)]}$, and $\arg(a_{k+p}) - \arg(b_{k+p}) = \alpha - \varphi$ ($n, p \in \mathbb{N} = \{1, 2, \dots\}$), then $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$.

Proof By Theorem 2.1, we see the inequality (2.1) which implies that $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$.

Since $\arg(a_{k+p}) - \arg(b_{k+p}) = \alpha - \varphi$, if $\arg(a_{k+p}) = \varphi_{k+p}$, we see $\arg(b_{k+p}) = \varphi_{k+p} - \alpha + \varphi$. Therefore,

$$e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p} = e^{i\varphi} |a_{k+p}| e^{i\varphi_{k+p}} - e^{i\alpha} |b_{k+p}| e^{i(\varphi_{k+p} - \alpha + \varphi)} = (|a_{k+p}| - |b_{k+p}|) e^{i(\varphi_{k+p} + \varphi)}$$

implies that

$$|e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| = ||a_{k+p}| - |b_{k+p}||. \quad (2.4)$$

Using (2.4) in (2.1), the proof of the corollary is complete. $\square$

Theorem 2.4 If $f \in \mathcal{P}(\Omega, \lambda)$ satisfies

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| \leq \delta - \sqrt{2(1 - \cos(\alpha - \varphi))}$$

for some $-\pi \leq \varphi - \alpha \leq \pi$ and $\delta > \sqrt{2[1 - \cos(\varphi - \alpha)]}$, then $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{M}(g)$.

The proof of this theorem is similar with Theorem 2.1.

Corollary 2.5 If $f \in \mathcal{P}(\Omega, \lambda)$ satisfies

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (1 + \lambda k p^{\Omega-1}) ||a_{k+p}| - |b_{k+p}|| \leq \delta - \sqrt{2(1 - \cos(\varphi - \alpha))}$$

for some $-\pi \leq \varphi - \alpha \leq \pi$, $\delta > \sqrt{2[1 - \cos(\varphi - \alpha)]}$, and $\arg(a_{k+p}) - \arg(b_{k+p}) = \alpha - \varphi$, then $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{M}(g)$.

Next, we derive the following theorem.

Theorem 2.6 If $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$, $0 \leq \varphi < \alpha \leq \pi$ and $\arg(e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}) = k\varphi$, then

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| \leq \delta - p\{\cos \varphi - \cos \alpha\}.$$

Proof For $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{N}(g)$, we have

$$\begin{aligned} & \left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| \\ &= \left| p(e^{i\varphi} - e^{i\alpha}) + \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) (e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}) z^k \right| \end{aligned}$$

$$= \left| p(e^{i\varphi} - e^{i\alpha}) + \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| e^{ik\varphi} z^k \right|$$

$$< \delta.$$

Let us consider z such that $\arg z = -\varphi$. Then $z^k = |z|^k e^{-ik\varphi}$. For such a point $z \in U$, we see that

$$\left| e^{i\varphi} \frac{\delta \mathcal{O}'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\delta \mathcal{O}'(g(z))}{z^{p-1}} \right|$$

$$= \left| p(e^{i\varphi} - e^{i\alpha}) + \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| |z|^k \right|$$

$$= \left\{ \left[\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| |z|^k + p(\cos \varphi - \cos \alpha) \right]^2 \right. \\ \left. + p^2 (\sin \varphi - \sin \alpha)^2 \right\}^{\frac{1}{2}}$$

$$< \delta.$$

This implies that

$$\left\{ \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| |z|^k + p(\cos \varphi - \cos \alpha) \right\}^2 < \delta^2$$

or

$$p(\cos \varphi - \cos \alpha) + \sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| |z|^k < \delta$$

for $z \in U$. Letting $|z| \rightarrow 1^-$, we have that

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (k+p)(1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| \leq \delta - p(\cos \varphi - \cos \alpha). \quad \square$$

Theorem 2.7 $f \in (\varphi, \alpha, \delta, \lambda, \Omega)_p - \mathcal{M}(g)$, $0 \leq \varphi < \alpha \leq \pi$ and $\arg(e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}) = k\varphi$, then

$$\sum_{k=n}^{\infty} \left(\frac{k+p}{p} \right)^{\Omega} (1 + \lambda k p^{\Omega-1}) |e^{i\varphi} a_{k+p} - e^{i\alpha} b_{k+p}| \leq \delta + \cos \alpha - \cos \varphi.$$

The proof of this theorem is similar with Theorem 2.6.

Remark 2.8 Taking $\varphi = 0$, $\Omega = 0$, $\lambda = 0$ and $p = 1$ in Theorem 2.6, we obtain the following theorem due to Orhan *et al.* [6].

Theorem 2.9 If $f \in (\alpha, \delta) - \mathcal{N}(g)$ and $\arg(a_n - e^{i\alpha} b_n) = (n-1)\varphi$ ($n = 2, 3, 4, \dots$), then

$$\sum_{n=2}^{\infty} n |a_n - e^{i\alpha} b_n| \leq \delta + \cos \alpha - 1.$$

Remark 2.10 Taking $\varphi = 0$, $\Omega = 0$ and $\lambda = 0$ in Theorem 2.6, we obtain the following theorem due to Altuntaş *et al.* [7].

Theorem 2.11 If $f \in (\alpha, \delta)_p - \mathcal{N}(g)$ and $\arg(a_{k+p} - e^{i\alpha} b_{k+p}) = k\varphi$, then

$$\sum_{k=n}^{\infty} (k+p) |a_{k+p} - e^{i\alpha} b_{k+p}| \leq \delta - p(1 - \cos \alpha).$$

We give an application of following lemma due to Miller and Mocanu [10].

Lemma 2.12 Let the function

$$w(z) = b_n z^n + b_{n+1} z^{n+1} + b_{n+2} z^{n+2} + \dots \quad (n \in \mathbb{N})$$

be regular in the unit disk U with $w(z) \neq 0$ ($z \in U$). If $z_0 = r_0 e^{i\theta_0}$ ($r_0 < 1$) and $|w(z_0)| = \max_{|z| \leq r_0} |w(z)|$, then $z_0 w'(z_0) = m w(z_0)$ where m is real and $m \geq n \geq 1$.

Theorem 2.13 If $f \in \wp(\Omega, \lambda)$ satisfies

$$\left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| < \delta(p+n) - p\sqrt{2(1 - \cos(\varphi - \alpha))}$$

for some $-\pi \leq \varphi - \alpha \leq \pi$ and $\delta > (\frac{p}{p+n})\sqrt{2(1 - \cos(\varphi - \alpha))}$, then

$$\left| e^{i\varphi} \frac{\wp(f(z))}{z^p} - e^{i\alpha} \frac{\wp(g(z))}{z^p} \right| < \delta + \sqrt{2(1 - \cos(\varphi - \alpha))} \quad (z \in U).$$

Proof Let us define $w(z)$ by

$$e^{i\varphi} \frac{\wp(f(z))}{z^p} - e^{i\alpha} \frac{\wp(g(z))}{z^p} = e^{i\varphi} - e^{i\alpha} + \delta w(z). \quad (2.5)$$

Then $w(z)$ is analytic in U and $w(0) = 0$. By logarithmic differentiation, we obtain from (2.5) that

$$\frac{e^{i\varphi} \wp'(f(z)) - e^{i\alpha} \wp'(g(z))}{e^{i\varphi} \wp(f(z)) - e^{i\alpha} \wp(g(z))} - \frac{p}{z} = \frac{\delta w'(z)}{e^{i\varphi} - e^{i\alpha} + \delta w(z)}.$$

Since

$$\frac{e^{i\varphi} \wp'(f(z)) - e^{i\alpha} \wp'(g(z))}{z^p (e^{i\varphi} - e^{i\alpha} + \delta w(z))} = \frac{p}{z} + \frac{\delta w'(z)}{e^{i\varphi} - e^{i\alpha} + \delta w(z)},$$

we see that

$$e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} = p(e^{i\varphi} - e^{i\alpha}) + \delta w(z) \left(p + \frac{z w'(z)}{w(z)} \right).$$

This implies that

$$\left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| = \left| p(e^{i\varphi} - e^{i\alpha}) + \delta w(z) \left(p + \frac{z w'(z)}{w(z)} \right) \right|.$$

We claim that

$$\left| e^{i\varphi} \frac{\wp'(f(z))}{z^{p-1}} - e^{i\alpha} \frac{\wp'(g(z))}{z^{p-1}} \right| < \delta(p+n) - p\sqrt{2(1-\cos(\varphi-\alpha))}$$

in U .

Otherwise, there exists a point $z_0 \in U$ such that $z_0 w'(z_0) = mw(z_0)$ (by Miller and Mocanu's lemma) where $w(z_0) = e^{i\theta}$ and $m \geq n \geq 1$.

Therefore, we obtain that

$$\begin{aligned} \left| e^{i\varphi} \frac{\wp'(f(z_0))}{z_0^{p-1}} - e^{i\alpha} \frac{\wp'(g(z_0))}{z_0^{p-1}} \right| &= |p(e^{i\varphi} - e^{i\alpha}) + \delta e^{i\theta}(p+m)| \\ &\geq \delta(p+m) - |p(e^{i\varphi} - e^{i\alpha})| \\ &\geq \delta(p+n) - p\sqrt{2(1-\cos(\varphi-\alpha))}. \end{aligned}$$

This contradicts our condition in Theorem 2.13. $\square$

Hence, there is no $z_0 \in U$ such that $|w(z_0)| = 1$. This implies that $|w(z)| < 1$ for all $z \in U$. Thus, we have that

$$\begin{aligned} \left| e^{i\varphi} \frac{\wp(f(z))}{z^p} - e^{i\alpha} \frac{\wp(g(z))}{z^p} \right| &= |(e^{i\varphi} - e^{i\alpha}) + \delta w(z)| \\ &\leq |e^{i\varphi} - e^{i\alpha}| + \delta |w(z)| \\ &< \delta + \sqrt{2(1-\cos(\varphi-\alpha))}. \end{aligned}$$

Letting $\varphi = 0$, $\Omega = 0$, $\lambda = 0$ and $\alpha = \frac{\pi}{2}$ in Theorem 2.13, we can obtain the following corollary.

Corollary 2.14 *If $f \in A(p, n)$ satisfies*

$$\left| \frac{f'(z)}{z^{p-1}} - i \frac{g'(z)}{z^{p-1}} \right| < \delta(p+n) - p\sqrt{2} \quad (z \in U)$$

for some $\delta > \sqrt{2}(\frac{p}{p+n})$, then

$$\left| \frac{f(z)}{z^p} - i \frac{g(z)}{z^p} \right| < \delta + \sqrt{2} \quad (z \in U).$$

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the final manuscript.

Author details

¹Department of Mathematics, Faculty of Science, Atatürk University, Erzurum, 25240, Turkey. ²Department of Mathematics, Faculty of Science and Arts, Avrasya University, Trabzon, Turkey.

Acknowledgements

Dedicated to Professor Hari M Srivastava.

Received: 13 December 2012 Accepted: 21 March 2013 Published: 3 April 2013

References

1. Goodman, AW: Univalent functions and nonanalytic curves. *Proc. Am. Math. Soc.* **8**, 598-601 (1957)
2. Ruscheweyh, S: Neighborhoods of univalent functions. *Proc. Am. Math. Soc.* **81**, 521-527 (1981)
3. Walker, JB: A note on neighborhoods of analytic functions having positive real part. *Int. J. Math. Math. Sci.* **13**, 425-430 (1990)
4. Owa, S, Saitoh, H, Nunokawa, M: Neighborhoods of certain analytic functions. *Appl. Math. Lett.* **6**, 73-77 (1993)
5. Altıntaş, O, Owa, S: Neighborhoods of certain analytic functions with negative coefficients. *Int. J. Math. Math. Sci.* **19**, 797-800 (1996)
6. Orhan, H, Kadioğlu, E, Owa, S: (α, δ) -Neighborhood for certain analytic functions. In: *International Symposium on Geometric Function Theory and Applications*, August 20-24, pp. 207-213 (2007)
7. Altıntaş, F, Owa, S, Kamali, M: $(\alpha, \delta)_p$ -Neighborhood for certain class of multivalent functions. *Panam. Math. J.* **19**(2), 35-46 (2009)
8. Frasin, BA: (α, β, δ) -Neighborhood for certain analytic functions with negative coefficients. *Eur. J. Pure Appl. Math.* **4**(1), 14-19 (2011)
9. Salagean, G: Subclasses of univalent functions. In: *Complex Analysis - Fifth Romanian-Finnish Seminar, Part 1* (Bucharest, 1981), *Lecture Notes in Math.*, vol. 1013, pp. 362-372. Springer, Berlin (1983)
10. Miller, SS, Mocanu, PT: Second order differential inequalities in the complex plane. *J. Math. Anal. Appl.* **65**, 289-305 (1978)

doi:10.1186/1029-242X-2013-152

Cite this article as: Sağsöz and Kamali: $(\varphi, \alpha, \delta, \lambda, \Omega)_p$ -Neighborhood for some classes of multivalent functions. *Journal of Inequalities and Applications* 2013 **2013**:152.

Submit your manuscript to a SpringerOpen[®] journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at ► springeropen.com