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Korovkin second theorem via statistical summability $(C, 1)$

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Abstract

Korovkin type approximation theorems are useful tools to check whether a given sequence $(L_n)_{n \geq 1}$ of positive linear operators on $C[0, 1]$ of all continuous functions on the real interval $[0, 1]$ is an approximation process. That is, these theorems exhibit a variety of test functions, which assure that the approximation property holds on the whole space if it holds for them. Such a property was discovered by Korovkin in 1953 for the functions $1, x$ and x^2 in the space $C[0, 1]$ as well as for the functions $1, \cos$ and $\sin$ in the space of all continuous 2π -periodic functions on the real line. In this paper, we use the notion of statistical summability $(C, 1)$ to prove the Korovkin approximation theorem for the functions $1, \cos$ and $\sin$ in the space of all continuous 2π -periodic functions on the real line and show that our result is stronger. We also study the rate of weighted statistical convergence.

MSC: 41A10; 41A25; 41A36; 40A30; 40G15

Keywords: density; statistical convergence; statistical summability $(C, 1)$; positive linear operator; Korovkin type approximation theorem

1 Introduction and preliminaries

We shall denote by $\mathbb{N}$ be the set of all natural numbers. Let $K \subseteq \mathbb{N}$ and $K_n = \{k \leq n : k \in K\}$. Then the *natural density* of K is defined by $\delta(K) = \lim_n n^{-1}|K_n|$ if the limit exists, where the vertical bars indicate the number of elements in the enclosed set. The sequence $x = (x_k)$ is said to be *statistically convergent* to L if for every $\varepsilon > 0$, the set $K_\varepsilon := \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ has natural density zero (cf. [1, 2]), i.e. for each $\varepsilon > 0$,

$$\lim_n \frac{1}{n} |\{j \leq n : |x_j - L| \geq \varepsilon\}| = 0.$$

In this case, we write $L = st\text{-}\lim x$. Note that every convergent sequence is statistically convergent but not conversely.

Recently, Móricz [3] has defined the concept of statistical summability $(C, 1)$ as follows:

For a sequence $x = (x_k)$, let us write $t_n = \frac{1}{n+1} \sum_{k=0}^n x_k$. We say that a sequence $x = (x_k)$ is *statistically summable* $(C, 1)$ if $st\text{-}\lim_{n \rightarrow \infty} t_n = L$. In this case we write $L = C_1(st)\text{-}\lim x$.

Note that every $(C, 1)$ -summable sequence is also statistically summable $(C, 1)$ to the same limit but not conversely.

Let $x = (x_k)$ be a sequence defined by

$$x_k = \begin{cases} 1 & \text{if } k \text{ odd,} \\ -1 & \text{if } k \text{ even.} \end{cases} \quad (1.1)$$

Then this sequence is neither convergent nor statistically convergent. But x is $(C, 1)$ -summable to 0, and hence statistically summable $(C, 1)$ to 0.

For more details, related concepts and applications, we refer to [4–17] and references therein.

Let $F(\mathbb{R})$ denote the linear space of all real-valued functions defined on $\mathbb{R}$. Let $C(\mathbb{R})$ be the space of all bounded and continuous functions f defined on $\mathbb{R}$. We know that $C(\mathbb{R})$ is a Banach space with norm

$$\|f\|_{\infty} := \sup_{x \in \mathbb{R}} |f(x)|, \quad f \in C(\mathbb{R}).$$

We denote by $C_{2\pi}(\mathbb{R})$ the space of all 2π -periodic functions $f \in C(\mathbb{R})$, which is a Banach spaces with

$$\|f\|_{2\pi} = \sup_{t \in \mathbb{R}} |f(t)|.$$

The classical Korovkin first and second theorems state as follows [18, 19].

Theorem I *Let (T_n) be a sequence of positive linear operators from $C[0, 1]$ into $F[0, 1]$. Then $\lim_n \|T_n(f, x) - f(x)\|_{\infty} = 0$, for all $f \in C[0, 1]$ if and only if $\lim_n \|T_n(f_i, x) - f_i(x)\|_{\infty} = 0$, for $i = 0, 1, 2$, where $f_0(x) = 1$, $f_1(x) = x$ and $f_2(x) = x^2$.*

Theorem II *Let (T_n) be a sequence of positive linear operators from $C_{2\pi}(\mathbb{R})$ into $F(\mathbb{R})$. Then $\lim_n \|T_n(f, x) - f(x)\|_{\infty} = 0$, for all $f \in C_{2\pi}(\mathbb{R})$ if and only if $\lim_n \|T_n(f_i, x) - f_i(x)\|_{\infty} = 0$, for $i = 0, 1, 2$, where $f_0(x) = 1$, $f_1(x) = \cos x$ and $f_2(x) = \sin x$.*

Several mathematicians have worked on extending or generalizing the Korovkin's theorems in many ways and to several settings, including function spaces, abstract Banach lattices, Banach algebras, Banach spaces and so on. This theory is very useful in real analysis, functional analysis, harmonic analysis, measure theory, probability theory, summability theory and partial differential equations. But the foremost applications are concerned with constructive approximation theory, which uses it as a valuable tool. Even today, the development of Korovkin-type approximation theory is far from complete. Note that the first and the second theorems of Korovkin are actually equivalent to the algebraic and the trigonometric version, respectively, of the classical Weierstrass approximation theorem [20]. Recently, the Korovkin second theorem is proved in [21] by using the concept of statistical convergence. Quite recently, Korovkin second theorem is proved by Demirci and Dirik [22] for statistical σ -convergence [23]. For some recent work on this topic, we refer to [24–33]. In this work, we prove Korovkin second theorem by applying the notion of statistical summability $(C, 1)$. We also give an example to justify that our result is stronger than Theorem II.

2 Main result

We write $L_n(f; x)$ for $L_n(f(s); x)$; and we say that L is a positive operator if $L(f; x) \geq 0$ for all $f(x) \geq 0$.

Theorem 2.1 Let (T_k) be a sequence of positive linear operators from $C_{2\pi}(\mathbb{R})$ into $C_{2\pi}(\mathbb{R})$. Then for all $f \in C_{2\pi}(\mathbb{R})$

$$C_1(st)\text{-}\lim_{k \rightarrow \infty} \|T_k(f; x) - f(x)\|_{2\pi} = 0. \quad (2.1)$$

if and only if

$$C_1(st)\text{-}\lim_{k \rightarrow \infty} \|T_k(1; x) - 1\|_{2\pi} = 0, \quad (2.2)$$

$$C_1(st)\text{-}\lim_{k \rightarrow \infty} \|T_k(\cos t; x) - \cos x\|_{2\pi} = 0, \quad (2.3)$$

$$C_1(st)\text{-}\lim_{k \rightarrow \infty} \|T_k(\sin t; x) - \sin x\|_{2\pi} = 0. \quad (2.4)$$

Proof Since each of $1, \cos x, \sin x$ belongs to $C_{2\pi}(\mathbb{R})$, conditions (2.2)-(2.4) follow immediately from (2.1). Let the conditions (2.2)-(2.4) hold and $f \in C_{2\pi}(\mathbb{R})$. Let I be a closed subinterval of length 2π of $\mathbb{R}$. Fix $x \in I$. By the continuity of f at x , it follows that for given $\varepsilon > 0$ there is a number $\delta > 0$ such that for all t

$$|f(t) - f(x)| < \varepsilon, \quad (2.5)$$

whenever $|t - x| < \delta$. Since f is bounded, it follows that

$$|f(t) - f(x)| \leq \|f\|_{2\pi}, \quad (2.6)$$

for all $t \in \mathbb{R}$. For all $t \in (x - \delta, 2\pi + x - \delta]$, it is well known that

$$|f(t) - f(x)| < \varepsilon + \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \psi(t), \quad (2.7)$$

where $\psi(t) = \sin^2(\frac{t-x}{2})$. Since the function $f \in C_{2\pi}(\mathbb{R})$ is 2π -periodic, the inequality (2.6) holds for $t \in \mathbb{R}$. We can also write (2.7) as

$$-\varepsilon - \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \psi(t) < f(t) - f(x) < \varepsilon + \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \psi(t).$$

Now applying the operator $T_k(1; x)$ to this inequality, we obtain

$$\begin{aligned} T_k(1; x) \left(-\varepsilon - \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \psi(t) \right) &< T_k(1; x) (f(t) - f(x)) \\ &< T_k(1; x) \left(\varepsilon + \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \psi(t) \right). \end{aligned}$$

As we know that x is fixed and so $f(x)$ is a constant number. Therefore,

$$\begin{aligned} -\varepsilon T_k(1; x) - \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} T_k(\psi(t); x) &< T_k(f; x) - f(x) T_k(1; x) \\ &< \varepsilon T_k(1; x) + \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} T_k(\psi(t); x). \end{aligned} \quad (2.8)$$

Also,

$$\begin{aligned} T_k(f; x) - f(x) &= T_k(f; x) - f(x)T_k(1; x) + f(x)T_k(1; x) - f(x) \\ &= T_k(f; x) - f(x)T_k(1; x) + f(x)[T_k(1; x) - 1]. \end{aligned} \quad (2.9)$$

From (2.8) and (2.9), we have

$$T_k(f; x) - f(x) < \varepsilon T_k(1; x) + \frac{2\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} T_k(\psi(t); x) + f(x)[T_k(1; x) - 1]. \quad (2.10)$$

Now

$$\begin{aligned} T_k(\psi(t); x) &= T_k\left(\sin^2\left(\frac{t-x}{2}\right); x\right) \\ &= T_k\left(\frac{1}{2}(1 - \cos t \cos x - \sin t \sin x); x\right) \\ &= \frac{1}{2}\{T_k(1; x) - \cos x T_k(\cos t; x) - \sin x T_k(\sin t; x)\} \\ &= \frac{1}{2}\{[T_k(1; x) - 1] - \cos x [T_k(\cos t; x) - \cos x] - \sin x [T_k(\sin t; x) - \sin x]\}. \end{aligned}$$

Substituting the value of $T_k(\psi(t); x)$ in (2.10), we get

$$\begin{aligned} T_k(f; x) - f(x) &< \varepsilon T_k(1; x) + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \{[T_k(1; x) - 1] - \cos x [T_k(\cos t; x) - \cos x] \\ &\quad - \sin x [T_k(\sin t; x) - \sin x]\} + f(x)[T_k(1; x) - 1] \\ &= \varepsilon + \varepsilon [T_k(1; x) - 1] + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \{[T_k(1; x) - 1] - \cos x [T_k(\cos t; x) - \cos x] \\ &\quad - \sin x [T_k(\sin t; x) - \sin x]\} + f(x)[T_k(1; x) - 1]. \end{aligned}$$

Therefore,

$$\begin{aligned} |T_k(f; x) - f(x)| &\leq \varepsilon + \left(\varepsilon + |f(x)| + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}}\right) |T_k(1; x) - 1| + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \{|\cos x| |T_k(\cos t; x) - \cos x| \\ &\quad + |\sin x| |T_k(\sin t; x) - \sin x|\} \\ &\leq \varepsilon + \left(\varepsilon + |f(x)| + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}}\right) |T_k(1; x) - 1| + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \{|T_k(\cos t; x) - \cos x| \\ &\quad + |T_k(\sin t; x) - \sin x|\}, \end{aligned}$$

since $|\cos x| \leq 1$ and $|\sin x| \leq 1$ for all $x \in I$. Now taking $\sup_{x \in I}$, we get

$$\begin{aligned} \|T_k(f; x) - f(x)\|_{2\pi} &\leq \varepsilon + K(\|T_k(1; x) - 1\|_{2\pi} + \|T_k(\cos t; x) - \cos x\|_{2\pi} \\ &\quad + \|T_k(\sin t; x) - \sin x\|_{2\pi}), \end{aligned}$$

where

$$K = \max \left\{ \varepsilon + \|f\|_{2\pi} + \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}}, \frac{\|f\|_{2\pi}}{\sin^2 \frac{\delta}{2}} \right\}.$$

Now replacing $T_k(\cdot; x)$ by $\frac{1}{n+1} \sum_{k=0}^n T_k(\cdot; x) = L_n(\cdot; x)$

$$\begin{aligned} \|L_n(f; x) - f(x)\|_{\infty} &\leq \varepsilon + K \left(\|L_n(1; x) - 1\|_{2\pi} + \|L_n(\cos t; x) - \cos x\|_{2\pi} \right. \\ &\quad \left. + \|L_n(\sin t; x) - \sin x\|_{2\pi} \right). \end{aligned} \quad (2.11)$$

For a given $r > 0$, choose $\varepsilon' > 0$ such that $\varepsilon' < r$. Define the following sets

$$\begin{aligned} D &= \{k \leq n : \|L_n(f; x) - f(x)\|_{2\pi} \geq r\}, \\ D_1 &= \left\{ k \leq n : \|L_n(1; x) - 1\|_{2\pi} \geq \frac{r - \varepsilon'}{3K} \right\}, \\ D_2 &= \left\{ k \leq n : \|L_n(\cos t; x) - \cos x\|_{2\pi} \geq \frac{r - \varepsilon'}{3K} \right\}, \\ D_3 &= \left\{ k \leq n : \|L_n(\sin t; x) - \sin x\|_{2\pi} \geq \frac{r - \varepsilon'}{3K} \right\}. \end{aligned}$$

Then

$$D \subset D_1 \cup D_2 \cup D_3,$$

and so

$$\delta(D) \leq \delta(D_1) + \delta(D_2) + \delta(D_3).$$

Therefore, using conditions (2.1), (2.2) and (2.3), we get

$$C_1(st)\text{-}\lim_n \|T_n(f, x) - f(x)\|_{2\pi} = 0.$$

This completes the proof of the theorem. $\square$

3 Rate of weighted statistical convergence

In this section, we study the rate of weighted statistical convergence of a sequence of positive linear operators defined from $C_{2\pi}(\mathbb{R})$ into $C_{2\pi}(\mathbb{R})$.

Definition 3.1 Let (a_n) be a positive nonincreasing sequence. We say that the sequence $x = (x_k)$ is statistically summable $(C, 1)$ to the number L with the rate $o(a_n)$ if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{a_n} \left| \{m \leq n : |t_m - L| \geq \varepsilon\} \right| = 0.$$

In this case, we write $x_k - L = C_1(st)\text{-}o(a_n)$.

As usual we have the following auxiliary result whose proof is standard.

Lemma 3.1 *Let (a_n) and (b_n) be two positive nonincreasing sequences. Let $x = (x_k)$ and $y = (y_k)$ be two sequences such that $x_k - L_1 = C_1(st)\text{-}o(a_n)$ and $y_k - L_2 = C_1(st)\text{-}o(b_n)$. Then*

- (i) $\alpha(x_k - L_1) = C_1(st)\text{-}o(a_n)$, for any scalar α ,
- (ii) $(x_k - L_1) \pm (y_k - L_2) = C_1(st)\text{-}o(c_n)$,
- (iii) $(x_k - L_1)(y_k - L_2) = C_1(st)\text{-}o(a_n b_n)$,

where $c_n = \max\{a_n, b_n\}$.

Now, we recall the notion of modulus of continuity. The modulus of continuity of $f \in C_{2\pi}(\mathbb{R})$, denoted by $\omega(f, \delta)$ is defined by

$$\omega(f, \delta) = \sup_{|x-y|<\delta} |f(x) - f(y)|.$$

It is well known that

$$|f(x) - f(y)| \leq \omega(f, \delta) \left(\frac{|x-y|}{\delta} + 1 \right). \quad (3.1)$$

Then we have the following result.

Theorem 3.2 *Let (T_k) be a sequence of positive linear operators from $C_{2\pi}(\mathbb{R})$ into $C_{2\pi}(\mathbb{R})$. Suppose that*

- (i) $\|T_k(1; x) - 1\|_{2\pi} = C_1(st)\text{-}o(a_n)$,
- (ii) $\omega(f, \lambda_k) = C_1(st)\text{-}o(b_n)$, where $\lambda_k = \sqrt{T_k(\varphi_x; x)}$ and $\varphi_x(y) = \sin^2(\frac{y-x}{2})$.

Then for all $f \in C_{2\pi}(\mathbb{R})$, we have

$$\|T_k(f; x) - f(x)\|_{2\pi} = C_1(st)\text{-}o(c_n),$$

where $c_n = \max\{a_n, b_n\}$.

Proof Let $f \in C_{2\pi}(\mathbb{R})$ and $x \in [-\pi, \pi]$. Using (2.9) and (3.1), we obtain

$$\begin{aligned} |T_k(f; x) - f(x)| &\leq T_k(|f(y) - f(x)|; x) + |f(x)| |T_k(1; x) - 1| \\ &\leq T_k\left(\frac{|x-y|}{\delta} + 1; x\right) \omega(f, \delta) + |f(x)| |T_k(1; x) - 1| \\ &\leq T_k\left(1 + \frac{\pi^2}{\delta^2} \sin^2\left(\frac{y-x}{2}\right); x\right) \omega(f, \delta) + |f(x)| |T_k(1; x) - 1| \\ &\leq \left(T_k(1; x) + \frac{\pi^2}{\delta^2} T_k(\varphi_x; x)\right) \omega(f, \delta) + |f(x)| |T_k(1; x) - 1|. \end{aligned}$$

Put $\delta = \lambda_k = \sqrt{T_k(\varphi_x; x)}$. Hence, we get

$$\begin{aligned} \|T_k(f; x) - f(x)\|_{2\pi} &\leq \|f\|_{2\pi} \|T_k(1; x) - 1\|_{2\pi} \\ &\quad + (1 + \pi^2) \omega(f, \lambda_k) + \omega(f, \lambda_k) \|T_k(1; x) - 1\|_{2\pi} \\ &\leq K \{ \|T_k(1; x) - 1\|_{2\pi} + \omega(f, \lambda_k) + \omega(f, \lambda_k) \|T_k(1; x) - 1\|_{2\pi} \}, \end{aligned}$$

where $K = \max\{\|f\|_{2\pi}, 1 + \pi^2\}$. Now replacing $T_k(\cdot; x)$ by $\frac{1}{n+1} \sum_{k=0}^n T_k(\cdot; x) = L_n(\cdot; x)$

$$\|L_n(f; x) - f(x)\|_{2\pi} \leq K \left\{ \|L_n(1; x) - 1\|_{2\pi} + \omega(f, \lambda_k) + \omega(f, \lambda_k) \|L_n(1; x) - 1\|_{2\pi} \right\}.$$

Now, using Definition 3.1 and Conditions (i) and (ii), we get the desired result.

This completes the proof of the theorem. $\square$

4 Example and the concluding remark

In the following, we construct an example of a sequence of positive linear operators satisfying the conditions of Theorem 2.1, but does not satisfy the conditions of Theorem II.

For any $n \in \mathbb{N}$, denote by $S_n(f)$ the n th partial sum of the Fourier series of f , i.e.

$$S_n(f)(x) = \frac{1}{2}a_0(f) + \sum_{k=1}^n a_k(f) \cos kx + b_k(f) \sin kx.$$

For any $n \in \mathbb{N}$, write

$$F_n(f) := \frac{1}{n+1} \sum_{k=0}^n S_k(f).$$

A standard calculation gives that for every $t \in \mathbb{R}$

$$\begin{aligned} F_n(f; x) &:= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \frac{1}{n+1} \sum_{k=0}^n \frac{\sin((2k+1)(x-t)/2)}{\sin((x-t)/2)} dt \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \frac{1}{n+1} \sum_{k=0}^n \frac{\sin^2((n+1)(x-t)/2)}{\sin^2((x-t)/2)} dt \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \varphi_n(x-t) dt, \end{aligned}$$

where

$$\varphi_n(x) := \begin{cases} \frac{\sin^2((n+1)x/2)}{\sin^2(x/2)} & \text{if } x \text{ is not a multiple of } 2\pi, \\ n+1 & \text{if } x \text{ is a multiple of } 2\pi. \end{cases}$$

The sequence $(\varphi_n)_{n \in \mathbb{N}}$ is a positive kernel which is called the *Fejér kernel*, and the corresponding operators F_n , $n \geq 1$, are called the *Fejér convolution operators*.

Note that the Theorem II is satisfied for the sequence (F_n) . In fact, we have

For every $f \in C_{2\pi}(\mathbb{R})$,

$$\lim_{n \rightarrow \infty} F_n(f) = f.$$

Let $L_n : C_{2\pi}(\mathbb{R}) \rightarrow C_{2\pi}(\mathbb{R})$ be defined by

$$L_n(f; x) = (1 + x_n)F_n(f; x), \tag{4.1}$$

where the sequence $x = (x_k)$ is defined by (1.1). Now

$$\begin{aligned}F_n(1; x) &= 1, \\F_n(\cos t; x) &= \frac{n}{n+1} \cos x, \\F_n(\sin t; x) &= \frac{n}{n+1} \sin x.\end{aligned}$$

We observe that the sequence (L_n) satisfies the conditions (2.2), (2.3) and (2.4). Hence, by Theorem 2.1, we have

$$C_1(st)\text{-}\lim_{n \rightarrow \infty} \|L_n(f) - f\|_{2\pi} = 0,$$

i.e. our theorem holds. But on the other hand, Theorem II does not hold for our operator defined by (4.1), since the sequence (L_n) is not convergent.

Hence, our Theorem 2.1 is stronger than that of Theorem II.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors contributed equally and significantly in writing this paper. Both the authors read and approved the final manuscript.

Acknowledgements

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant no. (410/130/1432). The authors, therefore, acknowledge with thanks DSR technical and financial support.

Received: 25 July 2012 Accepted: 18 March 2013 Published: 3 April 2013

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doi:10.1186/1029-242X-2013-149

Cite this article as: Mohiuddine and Alotaibi: Korovkin second theorem via statistical summability $(C, 1)$. *Journal of Inequalities and Applications* 2013 **2013**:149.

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