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Intuitionistic (λ, μ)-fuzzy sets in Γ-semigroups

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Abstract

We first introduce (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of an ordered Γ-semigroup. Then we prove that in regular and in intra-regular ordered semigroups the (λ, μ)-fuzzy ideals and the (λ, μ)-fuzzy interior ideals coincide. Lastly, we introduce (λ, μ)-fuzzy simple ordered Γ-semigroup and characterize the simple ordered Γ-semigroups in terms of (λ, μ)-fuzzy interior ideals.

Keywords: Γ-semigroup; (λ, μ)-fuzzy interior ideal; (λ, μ)-fuzzy simple; regular ordered Γ-semigroup; intra-regular ordered Γ-semigroup

1 Introduction and preliminaries

The formal study of semigroups began in the early twentieth century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis.

Γ-semigroups were first defined by Sen and Saha [1] as a generalization of semigroups and studied by many researchers [2–13].

The concept of fuzzy sets was first introduced by Zadeh [14] in 1965, and then the fuzzy sets have been used in the reconsideration of classical mathematics. Recently, Yuan [15] introduced the concept of a fuzzy subfield with thresholds. A fuzzy subfield with thresholds λ and μ is also called a (λ, μ)-fuzzy subfield. Yao continued to research (λ, μ)-fuzzy normal subfields, (λ, μ)-fuzzy quotient subfields, (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals in [16–19].

In this paper, we study (λ, μ)-fuzzy ideals in ordered Γ-semigroups. This can be seen as an application of [19] and as a generalization of [20, 21].

Let $S = \{x, y, z, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. An ordered Γ-semigroup $S_\Gamma = (S, \Gamma, \leq)$ is a poset $(S, \leq)$, and there is a mapping $S \times \Gamma \times S \to S$ (images to be denoted by $a\alpha b$) such that, for all $x, y, z \in S$, $\alpha, \beta, \gamma \in \Gamma$, we have

(1) $(x\beta y)\gamma z = x\beta(y\gamma z)$;

(2) $x \leq y \Rightarrow \begin{cases} x\alpha z \leq y\alpha z, \\ z\alpha x \leq z\alpha y. \end{cases}$

If $(S, \Gamma, \leq)$ is an ordered Γ-semigroup and A is a subset of S, we denote by $(A]$ the subset of S defined as follows:

$$(A] = \{t \in S | t \leq a \text{ for some } a \in A\}.$$

Given an ordered Γ-semigroup S, a fuzzy subset of S (or a fuzzy set in S) is an arbitrary mapping $f : S \to [0,1]$, where $[0,1]$ is the usual closed interval of real numbers. For any $t \in [0,1]$, f_t is defined by $f_t = \{x \in S | f(x) \geq t\}$.

For each subset A of S, the characteristic function f_A is a fuzzy subset of S defined by

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

In the following, we will use S, S_Γ or $(S, \Gamma, \leq)$ to denote an ordered Γ-semigroup. In the rest of this paper, we will always assume that $0 \leq \lambda < \mu \leq 1$.

2 Intuitionistic (λ, μ)-fuzzy Γ-ideals

In what follows, we will use S to denote a Γ-semigroup unless otherwise specified.

Definition 1 For an IFS $A = (f_A, g_A)$ in S, consider the following axioms:

(ΓS_1) $f_A(x\gamma y) \vee \lambda \geq f_A(x) \wedge f_A(y) \wedge \mu$,
(ΓS_2) $g_A(x\gamma y) \wedge \mu \leq f_A(x) \vee f_A(y) \vee \lambda$

for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (f_A, g_A)$ is called a first (resp. second) intuitionistic (λ, μ)-fuzzy Γ-subsemigroup (briefly (λ, μ)-*IF*ΓS_1 (resp. (λ, μ)-*IF*ΓS_2)) of S if it satisfies (ΓS_1) (resp. ΓS_2).

$A = (f_A, g_A)$ is called an intuitionistic (λ, μ)-fuzzy Γ-subsemigroup (briefly (λ, μ)-*IF*ΓS) of S if it is both a first and a second intuitionistic fuzzy Γ-subsemigroup.

Theorem 1 *If U is a Γ-subsemigroup of S, then $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ, μ)-IFΓS of S.*

Proof Let $x, y \in S$ and $\gamma \in \Gamma$.

(1) If $x, y \in U$, then $x\gamma y \in U$ from the hypothesis. Thus

$$\chi_U(x\gamma y) \vee \lambda = 1 \vee \lambda = 1 \geq \chi_U(x) \wedge \chi_U(y) \wedge \mu$$

and

$$\widetilde{\chi_U}(x\gamma y) \wedge \mu = \big(1 - \chi_U(x\gamma y)\big) \wedge \mu = (1 - 1) \wedge \mu = 0 \leq \widetilde{\chi_U}(x) \vee \widetilde{\chi_U}(y) \vee \lambda.$$

(2) If $x \notin U$ or $y \notin U$, then $\chi_U(x) = 0$ or $\chi_U(y) = 0$. Thus

$$\chi_U(x\gamma y) \vee \lambda \geq 0 = \chi_U(x) \wedge \chi_U(y) \wedge \mu$$

and

$$\widetilde{\chi_U}(x\gamma y) \wedge \mu \leq 1 = \widetilde{\chi_U}(x) \vee \widetilde{\chi_U}(y) \vee \lambda.$$

And we complete the proof. □

Theorem 2 *Let U be a non-empty subset of S. If $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ, μ)-IFΓS_1 or (λ, μ)-IFΓS_2 of S, then U is a Γ-subsemigroup of S.*

Proof (1) Suppose that $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-*IFΓS*$_1$ of S. For any $u, v \in U$ and $\gamma \in \Gamma$, we need to show that $u\gamma v \in U$. From (ΓS_1), we know that

$$\chi_U(u\gamma v) \vee \lambda \geq \chi_U(u) \wedge \chi_U(v) \wedge \mu = 1 \wedge 1 \wedge \mu = \mu.$$

Notice that $\lambda < \mu$, thus $\chi_U(u\gamma v) \geq \mu > 0$.

And also because U is a crisp set of S, then we conclude that $\chi_U(u\gamma v) = 1$; that is, $u\gamma v \in U$. Thus U is a Γ-subsemigroup of S.

(2) Now assume that $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-*IFΓS*$_2$ of S. For any $u, v \in U$ and $\gamma \in \Gamma$, we also need to show that $u\gamma v \in U$. It follows from (ΓS_2) that

$$\widetilde{\chi_U}(x\gamma y) \wedge \mu \leq \widetilde{\chi_U}(x) \vee \widetilde{\chi_U}(y) \vee \lambda = 0 \vee 0 \vee \lambda = \lambda.$$

Notice that $\lambda < \mu$, thus $\widetilde{\chi_U}(x\gamma y) \leq \lambda$.

And also because U is a crisp set of S, then we conclude that $\widetilde{\chi_U}(x\gamma y) = 0$, *i.e.*, $\chi_U(u\gamma v) = 1$. That is, $u\gamma v \in U$. Thus U is a Γ-subsemigroup of S. □

Definition 2 For an IFS $A = (f_A, g_A)$ in S, consider the following axioms:

$(L\Gamma I_1)$ $f_A(x\gamma y) \vee \lambda \geq f_A(y) \wedge \mu$,
$(L\Gamma I_2)$ $g_A(x\gamma y) \wedge \mu \leq g_A(y) \vee \lambda$

for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (f_A, g_A)$ is called a first (resp. second) intuitionistic (λ,μ)-fuzzy left Γ-ideal (briefly (λ,μ)-*IFLΓI*$_1$ (resp. (λ,μ)-*IFLΓI*$_2$)) of S if it satisfies $(L\Gamma I_1)$ (resp. $(L\Gamma I_2)$).

$A = (f_A, g_A)$ is called an intuitionistic (λ,μ)-fuzzy left Γ-ideal (briefly (λ,μ)-*IFLΓI*) of S if it is both a first and a second intuitionistic (λ,μ)-fuzzy left Γ-ideal.

Definition 3 For an IFS $A = (f_A, g_A)$ in S, consider the following axioms:

$(R\Gamma I_1)$ $f_A(x\gamma y) \vee \lambda \geq f_A(x) \wedge \mu$,
$(R\Gamma I_2)$ $g_A(x\gamma y) \wedge \mu \leq g_A(x) \vee \lambda$

for all $x, y \in S$ and $\gamma \in \Gamma$. Then $A = (f_A, g_A)$ is called a first (resp. second) intuitionistic (λ,μ)-fuzzy right Γ-ideal (briefly (λ,μ)-*IFRΓI*$_1$ (resp. (λ,μ)-*IFRΓI*$_2$)) of S if it satisfies $(R\Gamma I_1)$ (resp. $(R\Gamma I_2)$).

$A = (f_A, g_A)$ is called an intuitionistic (λ,μ)-fuzzy right Γ-ideal (briefly (λ,μ)-*IFRΓI*) of S if it is both a first and a second intuitionistic (λ,μ)-fuzzy right Γ-ideal.

Definition 4 For an IFS $A = (f_A, g_A)$ in S, it is called an intuitionistic (λ,μ)-fuzzy Γ-ideal (briefly (λ,μ)-*IFΓI*) of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right Γ-ideal.

Theorem 3 *If $A = (f_A, g_A)$ is a (λ,μ)-IFLΓI$_1$ of S. U is a left-zero Γ-subsemigroup of S. For any $x, y \in U$, one of the following must hold*:

(1) $f_A(x) = f_A(y)$;
(2) $f_A(x) \neq f_A(y) \Rightarrow (f_A(x) \vee f_A(y) \leq \lambda$, *or* $f_A(x) \wedge f_A(y) \geq \mu)$.

Proof Let $x, y \in U$. Since U is left-zero, we have that $x\gamma y = x$ and $y\gamma x = y$ for all $\gamma \in \Gamma$.

From the hypothesis, we have that

$$f_A(x) \vee \lambda = f_A(x\gamma y) \vee \lambda \geq f_A(y) \wedge \mu$$

and

$$f_A(y) \vee \lambda = f_A(y\gamma x) \vee \lambda \geq f_A(x) \wedge \mu.$$

Obviously, if $f_A(x) = f_A(y)$, then the previous two inequalities hold.

Suppose $f_A(x) < f_A(y)$. If $f_A(x) \vee f_A(y) > \lambda$ and $f_A(x) \wedge f_A(y) < \mu$, four cases are possible:

(1) If $f_A(x) > \lambda$ and $f_A(x) < \mu$, then $f_A(x) \vee \lambda = f_A(x) < f_A(y)$. Note that $f_A(x) < \mu$, we obtain that $f_A(x) < f_A(y) \wedge \mu$; that is, $f_A(x) \vee \lambda < f_A(y) \wedge \mu$. This is a contradiction to the previous proposition.

(2) If $f_A(x) > \lambda$ and $f_A(y) < \mu$, then $f_A(x) \vee \lambda = f_A(x) < f_A(y) = f_A(y) \wedge \mu$. This is a contradiction to the previous proposition.

(3) If $f_A(y) > \lambda$ and $f_A(x) < \mu$, then from $f_A(x) < \mu$ and $f_A(x) < f_A(y)$, we obtain that $f_A(x) < f_A(y) \wedge \mu$. From $\lambda < f_A(y)$ and $\lambda < \mu$, we conclude that $\lambda < f_A(y) \wedge \mu$. So, $f_A(x) \vee \lambda < f_A(y) \wedge \mu$. This is a contradiction to the previous proposition.

(4) If $f_A(y) > \lambda$ and $f_A(y) < \mu$, then from $F_A(X) < f_A(y)$ and $\lambda < f_A(y)$, we obtain $F_A(X) \vee \lambda < f_A(y) = f_A(y) \wedge \mu$. This is a contradiction to the previous proposition.

If $f_A(y) < f_A(x)$, we can prove the results dually.

Thus if $f_A(y) \neq f_A(x)$, then $f_A(x) \vee f_A(y) \leq \lambda$ or $f_A(x) \wedge f_A(y) \geq \mu$. □

Similarly, we can prove the following three theorems.

Theorem 4 *If $A = (f_A, g_A)$ is a (λ, μ)-IFRΓI_1 of S. U is a right-zero Γ-subsemigroup of S. For any $x, y \in U$, one of the following must hold*:

(1) $f_A(x) = f_A(y)$;

(2) $f_A(x) \neq f_A(y) \Rightarrow (f_A(x) \vee f_A(y) \leq \lambda$, *or* $f_A(x) \wedge f_A(y) \geq \mu)$.

Theorem 5 *If $A = (f_A, g_A)$ is a (λ, μ)-IFLΓI_2 of S. U is a left-zero Γ-subsemigroup of S. For any $x, y \in U$, one of the following must hold*:

(1) $g_A(x) = g_A(y)$;

(2) $g_A(x) \neq g_A(y) \Rightarrow (g_A(x) \vee g_A(y) \leq \lambda$, *or* $g_A(x) \wedge g_A(y) \geq \mu)$.

Theorem 6 *If $A = (f_A, g_A)$ is a (λ, μ)-IFRΓI_2 of S. U is a right-zero Γ-subsemigroup of S. For any $x, y \in U$, one of the following must hold*:

(1) $g_A(x) = g_A(y)$;

(2) $g_A(x) \neq g_A(y) \Rightarrow (g_A(x) \vee g_A(y) \leq \lambda$, *or* $g_A(x) \wedge g_A(y) \geq \mu)$.

Lemma 1 *If U is a left Γ-ideal of S, then $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ, μ)-IFLΓI of S.*

Proof Let $x, y \in S$ and $\gamma \in \Gamma$.

(1) If $y \in U$, then $x\gamma y \in U$ since U is a left Γ-ideal of S. It follows that

$$\chi_U(x\gamma y) \vee \lambda = 1 \vee \lambda = 1 \geq \chi_U(y) \wedge \mu$$

and

$$\widetilde{\chi_U}(x\gamma y) \wedge \mu = 0 \wedge \mu = 0 \leq \widetilde{\chi_U}(y) \vee \lambda.$$

(2) If $y \notin U$, then $\chi_U(y) = 0$. It follows that

$$\chi_U(x\gamma y) \vee \lambda \geq 0 = \chi_U(y) \wedge \mu$$

and

$$\widetilde{\chi_U}(x\gamma y) \wedge \mu \leq 1 = \widetilde{\chi_U}(y) \vee \lambda.$$

Consequently, $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ, μ)-*IFLΓI* of S. □

Theorem 7 *Let S be regular. If E_S is a left-zero Γ-subsemigroup of S, then, for all $e, e' \in E_S$, we have*

$$\chi_{L[e]}(e') = \chi_{L[e]}(e)$$

or

$$\chi_{L[e]}(e') \vee \chi_{L[e]}(e) \leq \lambda,$$

or

$$\chi_{L[e]}(e') \wedge \chi_{L[e]}(e) \geq \mu.$$

Proof Since S is regular, E_S is non-empty. Let $e = e\gamma e, e' = e'\gamma' e' \in E_S$, where $\gamma, \gamma' \in \Gamma$. Because S is regular, $L[e] = S\Gamma e$. From the fact that $L[e]$ is a left Γ-ideal of S, we obtain that $\widetilde{L[e]} = (\chi_{L[e]}, \widetilde{\chi_{L[e]}})$ is a (λ, μ)-*IFLΓI*$_1$ of S by the previous lemma.

Applying Theorem 3, we obtain the results. □

The following theorem can be proved in a similar way.

Theorem 8 *Let S be regular. If E_S is a left-zero Γ-subsemigroup of S, then, for all $e, e' \in E_S$, we have*

$$\widetilde{\chi_{L[e]}}(e') = \widetilde{\chi_{L[e]}}(e)$$

or

$$\widetilde{\chi_{L[e]}}(e') \vee \widetilde{\chi_{L[e]}}(e) \leq \lambda,$$

or

$$\widetilde{\chi_{L[e]}}(e') \wedge \widetilde{\chi_{L[e]}}(e) \geq \mu.$$

Theorem 9 *Let S be regular. If for all $e, e' \in E_S$ we have*

$$\chi_{L[e]}(e') = \chi_{L[e]}(e)$$

or

$$\chi_{L[e]}(e') \vee \chi_{L[e]}(e) \leq \lambda,$$

or

$$\chi_{L[e]}(e') \wedge \chi_{L[e]}(e) \geq \mu,$$

then E_S is a left-zero Γ-subsemigroup of S.

Proof Since S is regular, E_S is non-empty. Let $e = e\gamma e, e' = e'\gamma' e' \in E_S$, where $\gamma, \gamma' \in \Gamma$. Because S is regular, $L[e] = S\Gamma e$. From the fact that $L[e]$ is a left Γ-ideal of S, we obtain that $\widetilde{L[e]} = (\chi_{L[e]}, \widetilde{\chi_{L[e]}})$ is a (λ, μ)-*IFL*ΓI_1 of S by the previous lemma.

(1) If $\chi_{L[e]}(e') = \chi_{L[e]}(e) = 1$, then $e' \in L[e] = S\Gamma e$. Thus

$$e' = x\beta e = x\beta(e\gamma e) = (x\beta e)\gamma e = e'\gamma e$$

for some $x \in S$ and $\beta \in \Gamma$.

(2) $\chi_{L[e]}(e') \vee \chi_{L[e]}(e) \leq \lambda$ will never happen since $\chi_{L[e]}(e') \vee \chi_{L[e]}(e) = 1$.

(3) If $\chi_{L[e]}(e') \wedge \chi_{L[e]}(e) \geq \mu$, that is, $\chi_{L[e]}(e') \geq \mu$, then $\chi_{L[e]}(e') = 1$. And so $e' \in L[e] = S\Gamma e$. The following proof will be the same as in case (1).

Consequently, E_S is a left-zero Γ-subsemigroup of S. □

The following theorem can be proved similarly.

Theorem 10 *Let S be regular. If for all $e, e' \in E_S$ we have*

$$\widetilde{\chi_{L[e]}}(e') = \widetilde{\chi_{L[e]}}(e)$$

or

$$\widetilde{\chi_{L[e]}}(e') \vee \widetilde{\chi_{L[e]}}(e) \leq \lambda,$$

or

$$\widetilde{\chi_{L[e]}}(e') \wedge \widetilde{\chi_{L[e]}}(e) \geq \mu,$$

then E_S is a left-zero Γ-subsemigroup of S.

3 Intuitionistic (λ, μ)-fuzzy interior Γ-ideals

Definition 5 For an IFS $A = (f_A, g_A)$ in S, consider the following axioms:

$(I\Gamma I_1)$ $f_A(x\beta s\gamma y) \vee \lambda \geq f_A(s) \wedge \mu$,
$(I\Gamma I_2)$ $g_A(x\beta s\gamma y) \wedge \mu \leq g_A(s) \vee \lambda$

for all $s, x, y \in S$ and $\beta, \gamma \in \Gamma$. Then $A = (f_A, g_A)$ is called a first (resp. second) intuitionistic (λ, μ)-fuzzy interior Γ-ideal (briefly (λ, μ)-*IFI*ΓI_1 (resp. (λ, μ)-*IFI*ΓI_2)) of S if it satisfies $(I\Gamma I_1)$ (resp. $(I\Gamma I_2)$).

$A = (f_A, g_A)$ is called an intuitionistic (λ, μ)-fuzzy interior Γ-ideal (briefly (λ, μ)-*IFI*ΓI) of S if it is both a first and a second intuitionistic (λ, μ)-fuzzy interior Γ-ideal.

Theorem 11 *Every* (λ, μ)-*IF*ΓI *of* S *is a* (λ, μ)-*IFI*ΓI *of* S.

Proof Let $A = (f_A, g_A)$ be a (λ, μ)-*IF*ΓI of S. For all $s, x, y \in S$ and $\beta, \gamma \in \Gamma$, we have

$$\begin{aligned} f_A(x\beta s\gamma y) \vee \lambda &= f_A(x\beta s\gamma y) \vee \lambda \vee \lambda \geq \bigl(f_A(x\beta s) \wedge \mu\bigr) \vee \lambda \\ &= \bigl(f_A(x\beta s) \vee \lambda\bigr) \wedge (\mu \vee \lambda) \geq f(s) \wedge \mu. \end{aligned}$$

Similarly, we have $g_A(x\beta s\gamma y) \wedge \mu \leq g_A(s) \vee \lambda$.

So, $A = (f_A, g_A)$ is a (λ, μ)-*IFI*ΓI of S. □

Theorem 12 *If* S *is regular, then every* (λ, μ)-*IFI*ΓI *of* S *is a* (λ, μ)-*IF*ΓI *of* S.

Proof Let $A = (f_A, g_A)$ be a (λ, μ)-*IFI*ΓI of S and $x, y \in S$. Since S is regular, there exist $s, s' \in S$ and $\beta, \beta', \gamma, \gamma' \in \Gamma$ such that $x = x\beta s\gamma x$ and $y = y\beta' s'\gamma' y$. Thus

$$f_A(x\alpha y) \vee \lambda = f_A\bigl(x\alpha\bigl(y\beta' s'\gamma' y\bigr)\bigr) \vee \lambda = f_A\bigl(x\alpha y\beta'\bigl(s'\gamma' y\bigr)\bigr) \vee \lambda \geq f_A(y) \wedge \mu$$

and

$$g_A(x\alpha y) \wedge \mu = g_A\bigl(x\alpha\bigl(y\beta' s'\gamma' y\bigr)\bigr) \wedge \mu = g_A\bigl(x\alpha y\beta'\bigl(s'\gamma' y\bigr)\bigr) \wedge \mu \leq g_A(y) \vee \lambda$$

for all $\alpha \in \Gamma$. It follows that $A = (f_A, g_A)$ is a (λ, μ)-*IFL*ΓI of S. Similarly, we can prove that A is a (λ, μ)-*IFR*ΓI of S. This completes the proof. □

Theorem 13 *If* U *is an interior* Γ*-ideal of* S, *then* $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ *is a* (λ, μ)-*IFI*ΓI *of* S.

Proof Let $s, x, y \in S$ and $\beta, \gamma \in \Gamma$.

(1) If $s \in U$, then $x\beta s\gamma y \in U$ since U is an interior Γ-ideal of S. So,

$$\chi_U(x\beta s\gamma y) \vee \lambda = 1 \vee \lambda = 1 \geq \chi_U(s) \wedge \mu$$

and

$$\widetilde{\chi_U}(x\beta s\gamma y) \wedge \mu = 0 \wedge \mu = 0 \leq \widetilde{\chi_U}(s) \vee \lambda.$$

(2) If $s \notin U$, then $\chi_U(s) = 0$. Thus

$$\chi_U(x\beta s\gamma y) \vee \lambda \geq 0 = \chi_U(s) \wedge \mu$$

and

$$\widetilde{\chi_U}(x\beta s\gamma y) \wedge \mu \leq 1 = \widetilde{\chi_U}(s) \vee \lambda.$$

Consequently, we obtain that $\widetilde{U}$ is a (λ, μ)-*IFI*ΓI of S. □

Theorem 14 *Let S be regular and U be a non-empty subset of S. If $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-IFIΓI_1 or (λ,μ)-IFIΓI_2 of S, then U is an interior Γ-ideal of S.*

Proof It is obvious that U is a Γ-subsemigroup of S by Theorem 2.

Case 1. Suppose that $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-*IFIΓI_1* of S and $x \in S\Gamma U\Gamma S$. Thus $x = s\beta u\gamma t$ for some $s, t \in S$, $u \in U$ and $\beta, \gamma \in \Gamma$. It follows from $(I\Gamma I_1)$ that

$$\chi_U(x) \vee \lambda = \chi_U(s\beta u\gamma t) \vee \lambda \geq \chi_U(u) \wedge \mu = 1 \wedge \mu = \mu.$$

Notice that $\lambda < \mu$, we obtain that $\chi_U(x) \geq \mu$, that is, $\chi_U(x) = 1$. So, $x \in U$. Thus U is an interior Γ-ideal of S.

Case 2. Suppose that $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-*IFIΓI_2* of S and $x \in S\Gamma U\Gamma S$. Then $x = s\beta u\gamma t$ for some $s, t \in S$, $u \in U$ and $\beta, \gamma \in \Gamma$. Using $(I\Gamma I_2)$, we conclude that

$$\widetilde{\chi_U}(x) \wedge \mu = \widetilde{\chi_U}(s\beta u\gamma t) \wedge \mu \leq \widetilde{\chi_U}(u) \vee \lambda = 0 \vee \lambda = \lambda.$$

Notice that $\lambda < \mu$, we obtain that $\widetilde{\chi_U}(x) \leq \lambda$, that is, $\widetilde{\chi_U}(x) = 0$. So, $x \in U$. Thus U is an interior Γ-ideal of S. □

4 Intuitionistic (λ,μ)-fuzzy simple Γ-semigroups

Definition 6 S is called first (resp. second) intuitionistic (λ,μ)-fuzzy left simple if for any (λ,μ)-*IFLΓI_1* (resp. (λ,μ)-*IFLΓI_2*) $A = (f_A, g_A)$ of S, we have $f_A(a) \vee \lambda \geq f_A(b) \wedge \mu$ (resp. $g_A(a) \vee \lambda \geq g_A(b) \wedge \mu$) for all $a, b \in S$.

S is said to be intuitionistic (λ,μ)-fuzzy left simple if it is both first and second intuitionistic (λ,μ)-fuzzy left simple.

Theorem 15 *If S is left simple, then S is intuitionistic (λ,μ)-fuzzy left simple.*

Proof Let $A = (f_A, g_A)$ be a (λ,μ)-*IFLΓI* of S and $x, x' \in S$. Because S is left simple, there exist $s, s' \in S$ and $\gamma, \gamma' \in \Gamma$ such that $x = s\gamma x'$ and $x' = s'\gamma' x$. Thus, since A is a (λ,μ)-*IFLΓI* of S, we have that

$$f_A(x) \vee \lambda = f_A(s\gamma x') \vee \lambda \geq f_A(x') \wedge \mu,$$
$$f_A(x') \vee \lambda = f_A(s'\gamma' x) \vee \lambda \geq f_A(x) \wedge \mu$$

and

$$g_A(x) \wedge \mu = g_A(s\gamma x') \wedge \mu \leq g_A(x') \vee \lambda,$$
$$g_A(x') \wedge \mu = g_A(s'\gamma' x) \wedge \mu \leq g_A(x) \vee \lambda.$$

Consequently, S is intuitionistic (λ,μ)-fuzzy left simple. □

Theorem 16 *If S is first or second intuitionistic (λ,μ)-fuzzy left simple, then S is left simple.*

Proof Let U be a left Γ-ideal of S. Suppose that S is first (or second) intuitionistic (λ,μ)-fuzzy left simple. Because $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-*IFLΓI* of S, $\widetilde{U} = (\chi_U, \widetilde{\chi_U})$ is a (λ,μ)-*IFLΓI_1* (and (λ,μ)-*IFLΓI_2*) of S. □

5 Conclusion and further research

In this paper, we generalized the results of [20, 21]. We introduced (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of an ordered Γ-semigroup and we got some interesting results. When $\lambda = 0$ and $\mu = 1$, we meet ordinary fuzzy ideals and fuzzy interior ideals. From this point of view, (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals are more general concepts than fuzzy ones.

In [19], Yao gave the definition of (λ, μ)-fuzzy bi-ideals in semigroups. One can study (λ, μ)-fuzzy bi-ideals in ordered Γ-semigroups. We would like to explore this in next papers.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
After a long time discuss between YF and BL, the paper is finally finished. During YF's writing of this paper, BL gave some very good advice and she helped to draft the manuscript. All authors read and approved the final manuscript.

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