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Strong convergence results for arrays of rowwise pairwise NQD random variables

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Abstract

Let $\{X_{ni}, 1 \le i \le n, n \ge 1\}$ be an array of rowwise pairwise NQD random variables. Some sufficient conditions of complete convergence for weighted sums of arrays of rowwise pairwise NQD random variables are presented without assumption of identical distribution. Our results partially extend the corresponding ones for independent random variables and negatively associated random variables.

MSC: 60F15

Keywords: arrays of rowwise pairwise NQD random variables; weighted sums; complete convergence

1 Introduction

Throughout the paper, let $I(A)$ be the indicator function of the set A. C denotes a positive constant which may be different in various places and $a_n = O(b_n)$ stands for $a_n \le Cb_n$. Denote $\log x = \ln\max(x, e)$, where $\ln x$ is the natural logarithm.

The concept of complete convergence was introduced by Hsu and Robbins [1] as follows. A sequence of random variables $\{U_n, n \ge 1\}$ is said to *converge completely* to a constant C if $\sum_{n=1}^{\infty} P(|U_n - C| > \varepsilon) < \infty$ for all $\varepsilon > 0$. In view of the Borel-Cantelli lemma, this implies that $U_n \to C$ almost surely (a.s.). The converse is true if the $\{U_n, n \ge 1\}$ are independent. Hsu and Robbins [1] proved that the sequence of arithmetic means of independent and identically distributed (i.i.d.) random variables converges completely to the expected value if the variance of the summands is finite. Since then many authors, such as Spitzer [2], Baum and Katz [3], Gut [4] and so forth, have studied the complete convergence for partial sums and weighted sums of random variables. The main purpose of the present investigation is to provide the complete convergence results for weighted sums of arrays of rowwise pairwise negatively quadrant dependent random variables.

Firstly, let us recall the definition of pairwise negatively quadrant dependent random variables.

Definition 1.1 Two random variables X and Y are said to be negatively quadrant dependent (NQD in short) if for any $x, y \in \mathbb{R}$,

$$P(X \le x, Y \le y) \le P(X \le x)P(Y \le y). \tag{1.1}$$

A sequence of random variables $\{X_n, n \ge 1\}$ is said to be pairwise NQD if X_i and X_j are NQD for all $i, j \in \mathbb{N}$ and $i \ne j$.

An array of random variables $\{X_{ni}, 1 \le i \le n, n \ge 1\}$ is called rowwise pairwise NQD random variables if for every $n \ge 1$, $\{X_{ni}, 1 \le i \le n\}$ are pairwise NQD random variables.

The concept of pairwise NQD was introduced by Lehmann [5]. Obviously, a sequence of pairwise NQD random variables is a family of very wide scope, which contains a pairwise independent random variable sequence and a negatively orthant dependent (NOD) random variable sequence as special cases. For more details about NOD random variables, one can refer to Joag-Dev and Proschan [6], Wang *et al.* [7, 8] and so forth. Many known types of negative dependence such as negative upper (lower) orthant dependence and negative association (see Joag-Dev and Proschan [6]) have been developed on the basis of this notion. Among them, the negatively associated class is the most important and special case of a pairwise NQD sequence. So, it is very significant to study probabilistic properties of this wider pairwise NQD class. Since Lehmann's article appeared, a number of limit theorems for pairwise NQD random variables have been established. For example, Matula [9] obtained the Kolmogorov strong law of large numbers for a pairwise NQD random variable sequence with identical distribution, Wang *et al.* [10] and Wu [11] investigated some limit properties for such a sequence, Li and Wang [12] obtained the central limit theorem, Gan and Chen [13] studied further some limit properties for the pairwise NQD sequence without limitation of an identically distributed condition and obtained Baum-Katz (Baum and Katz [3]) type complete convergence and the strong stability of Jamison (Jamison *et al.* [14]) type weighted sums for the pairwise NQD sequence which may have different distributions, Huang *et al.* [15] studied the complete convergence for sequences of pairwise NQD random variables and so forth.

Our goal in this paper is to further study the complete convergence for weighted sums of arrays of rowwise pairwise NQD random variables under some moment conditions. We will give some sufficient conditions for complete convergence for an array of rowwise pairwise NQD random variables without assumption of identical distribution. The results presented in this paper are obtained by using the truncated method and the generalized Kolmogorov type inequality of pairwise NQD random variables.

Definition 1.2 An array of random variables $\{X_{ni}, i \ge 1, n \ge 1\}$ is said to be stochastically dominated by a random variable X if there exists a positive constant C such that

$$P\big(|X_{ni}| > x\big) \le CP\big(|X| > x\big) \tag{1.2}$$

for all $x \ge 0$, $i \ge 1$ and $n \ge 1$.

The following lemmas are useful for the proof of the main results.

Lemma 1.1 (*cf.* Lehmann [5]) *Let X and Y be NQD, then*

(i) $EXY \le EXEY$;
(ii) $P(X > x, Y > y) \le P(X > x)P(Y > y)$, *for any $x, y \in R$;*
(iii) *If f and g are both nondecreasing (or nonincreasing) functions, then $f(X)$ and $g(Y)$ are NQD.*

Lemma 1.2 (cf. Wu [11]) *Let $\{X_n, n \ge 1\}$ be a sequence of pairwise NQD random variables with $EX_n = 0$ and $EX_n^2 < \infty$ for every $n \ge 1$. Denote $T_j(k) = \sum_{i=j+1}^{j+k} X_i$, $j \ge 0$, $k \ge 1$. Then, for* *every* $n \geq 1$,

$$ET_j^2(k) \leq \sum_{i=j+1}^{j+k} EX_i^2, \tag{1.3}$$

$$E\Bigl(\max_{1\leq k\leq n} T_j^2(k)\Bigr) \leq 4(\log_2 2n)^2 \sum_{i=j+1}^{j+n} EX_i^2. \tag{1.4}$$

Lemma 1.3 *Let* $\{X_n, n \geq 1\}$ *be a sequence of random variables which is stochastically dominated by a random variable* X. *For any* $\alpha > 0$ *and* $b > 0$, *the following two statements hold*:

$$E|X_n|^\alpha I\bigl(|X_n| \leq b\bigr) \leq C_1\bigl[E|X|^\alpha I\bigl(|X| \leq b\bigr) + b^\alpha P\bigl(|X| > b\bigr)\bigr], \tag{1.5}$$

$$E|X_n|^\alpha I\bigl(|X_n| > b\bigr) \leq C_2 E|X|^\alpha I\bigl(|X| > b\bigr), \tag{1.6}$$

where C_1 *and* C_2 *are positive constants.*

Proof The proof can be found in Wu [16]. So, we omit the details. □

The following two lemmas are from Sung [17].

Lemma 1.4 *Let* X *be a random variable and* $\{a_{ni}, 1 \leq i \leq n, n \geq 1\}$ *be an array of constants satisfying*

$$\sum_{i=1}^n |a_{ni}|^\alpha = O(n) \tag{1.7}$$

for some $\alpha > 0$. *Let* $b_n = n^{1/\alpha}(\log n)^{1/\gamma}$ *for some* $\gamma > 0$. *Then*

$$\sum_{n=1}^\infty n^{-1} \sum_{i=1}^n P\bigl(|a_{ni}X| > b_n\bigr) \leq \begin{cases} CE|X|^\alpha, & \text{for } \alpha > \gamma, \\ CE|X|^\alpha \log(1+|X|), & \text{for } \alpha = \gamma, \\ CE|X|^\gamma, & \text{for } \alpha < \gamma. \end{cases}$$

Lemma 1.5 *Let* X *be a random variable and* $\{a_{ni}, 1 \leq i \leq n, n \geq 1\}$ *be an array of constants satisfying* $a_{ni} = 0$ *or* $a_{ni} > 1$, *and* $\sum_{i=1}^n |a_{ni}|^\alpha \leq n$ *some* $\alpha > 0$. *Let* $b_n = n^{1/\alpha}(\log n)^{1/\alpha}$. *If* $q > \alpha$, *then*

$$\sum_{n=1}^\infty n^{-1} b_n^{-q} \sum_{i=1}^n E|a_{ni}X|^q I\bigl(|a_{ni}X| \leq b_n\bigr) \leq CE|X|^\alpha \log\bigl(1+|X|\bigr).$$

2 Main results

Theorem 2.1 *Let* $\{X_{ni}, 1 \leq i \leq n, n \geq 1\}$ *be an array of rowwise pairwise NQD random variables which is stochastically dominated by a random variable* X *and* $\{a_{ni}, 1 \leq i \leq n, n \geq 1\}$ *be an array of constants satisfying* (1.7) *for some* $0 < \alpha < 2$. *Let* $b_n \doteq n^{1/\alpha}(\log n)^{1/\alpha}$. *If* $EX_{ni} = 0$ *for* $1 < \alpha < 2$ *and* $E|X|^\alpha \log(1+|X|) < \infty$, *then for any* $\varepsilon > 0$,

$$\sum_{n=1}^\infty n^{-1} P\left(\left|\sum_{i=1}^n a_{ni}X_{ni}\right| > \varepsilon b_n\right) < \infty. \tag{2.1}$$

Proof Without loss of generality, we may assume that $\sum_{i=1}^{n}|a_{ni}|^{\alpha} \leq n$ and $a_{ni} \geq 0$ for all $1 \leq i \leq n$ and $n \geq 1$. For fixed $n \geq 1$, define for $1 \leq i \leq n$ that

$$X_i^{(n)} = -b_n I(a_{ni}X_{ni} < -b_n) + a_{ni}X_{ni} I\big(|a_{ni}X_{ni}| \leq b_n\big) + b_n I(a_{ni}X_{ni} > b_n),$$
$$T^{(n)} = \sum_{i=1}^{n}\big(X_i^{(n)} - EX_i^{(n)}\big).$$

It is easy to check that for any $\varepsilon > 0$,

$$\left(\left|\sum_{i=1}^{n} a_{ni}X_{ni}\right| > \varepsilon b_n\right) \subset \Big(\max_{1\leq i\leq n}|a_{ni}X_{ni}| > b_n\Big) \cup \left(\left|\sum_{i=1}^{n} X_i^{(n)}\right| > \varepsilon b_n\right),$$

which implies that

$$\begin{aligned}
P\left(\left|\sum_{i=1}^{n} a_{ni}X_{ni}\right| > \varepsilon b_n\right) &\leq P\Big(\max_{1\leq i\leq n}|a_{ni}X_{ni}| > b_n\Big) + P\left(\left|\sum_{i=1}^{n} X_i^{(n)}\right| > \varepsilon b_n\right) \\
&\leq \sum_{i=1}^{n} P\big(|a_{ni}X_{ni}| > b_n\big) + P\left(\left|T^{(n)}\right| > \varepsilon b_n - \left|\sum_{i=1}^{n} EX_i^{(n)}\right|\right) \\
&\leq C\sum_{i=1}^{n} P\big(|a_{ni}X| > b_n\big) + P\left(\left|T^{(n)}\right| > \varepsilon b_n - \left|\sum_{i=1}^{n} EX_i^{(n)}\right|\right).
\end{aligned}$$

Firstly, we will show that

$$b_n^{-1}\left|\sum_{i=1}^{n} EX_i^{(n)}\right| \to 0, \quad \text{as } n \to \infty. \tag{2.2}$$

When $1 < \alpha < 2$, we have by $EX_{ni} = 0$, (1.6) of Lemma 1.3 and Markov's inequality that

$$\begin{aligned}
b_n^{-1}\left|\sum_{i=1}^{n} EX_i^{(n)}\right| &\leq \sum_{i=1}^{n} P\big(|a_{ni}X_{ni}| > b_n\big) + b_n^{-1}\left|\sum_{i=1}^{n} a_{ni}EX_{ni}I\big(|a_{ni}X_{ni}| > b_n\big)\right| \\
&\leq C\sum_{i=1}^{n} P\big(|a_{ni}X| > b_n\big) + b_n^{-1}\sum_{i=1}^{n} E|a_{ni}X_{ni}|I\big(|a_{ni}X_{ni}| > b_n\big) \\
&\leq Cb_n^{-\alpha}\sum_{i=1}^{n}|a_{ni}|^{\alpha}E|X|^{\alpha} + Cb_n^{-\alpha}\sum_{i=1}^{n}|a_{ni}|^{\alpha}E|X|^{\alpha} \\
&\leq CE|X|^{\alpha}(\log n)^{-1} \to 0 \quad \text{as } n \to \infty.
\end{aligned} \tag{2.3}$$

When $0 < \alpha \leq 1$, we have by (1.5) of Lemma 1.3 and Markov's inequality that

$$\begin{aligned}
b_n^{-1}\left|\sum_{i=1}^{n} EX_i^{(n)}\right| &\leq \sum_{i=1}^{n} P\big(|a_{ni}X_{ni}| > b_n\big) + b_n^{-1}\sum_{i=1}^{n} E|a_{ni}X_{ni}|I\big(|a_{ni}X_{ni}| \leq b_n\big) \\
&\leq C\sum_{i=1}^{n} P\big(|a_{ni}X| > b_n\big)
\end{aligned}$$

$$
\begin{aligned}
&+ Cb_n^{-1}\sum_{i=1}^{n}\bigl[E|a_{ni}X|I\bigl(|a_{ni}X|\le b_n\bigr)+b_nP\bigl(|a_{ni}X|>b_n\bigr)\bigr]\\
&\le C\sum_{i=1}^{n}P\bigl(|a_{ni}X|>b_n\bigr)+Cb_n^{-1}\sum_{i=1}^{n}E|a_{ni}X|I\bigl(|a_{ni}X|\le b_n\bigr)\\
&\le Cb_n^{-\alpha}\sum_{i=1}^{n}|a_{ni}|^{\alpha}E|X|^{\alpha}+Cb_n^{-\alpha}\sum_{i=1}^{n}E|a_{ni}X|^{\alpha}I\bigl(|a_{ni}X|\le b_n\bigr)\\
&\le Cb_n^{-\alpha}nE|X|^{\alpha}+Cb_n^{-\alpha}nE|X|^{\alpha}\\
&= 2CE|X|^{\alpha}(\log n)^{-1}\to 0 \quad \text{as } n\to\infty. \qquad (2.4)
\end{aligned}
$$

By (2.3) and (2.4), we can get (2.2) immediately. Hence, for n large enough,

$$P\left(\left|\sum_{i=1}^{n}a_{ni}X_{ni}\right|>\varepsilon b_n\right)\le C\sum_{i=1}^{n}P\bigl(|a_{ni}X|>b_n\bigr)+P\left(\bigl|T^{(n)}\bigr|>\frac{\varepsilon}{2}b_n\right).$$

To prove (2.1), we only need to show that

$$I \doteq \sum_{n=1}^{\infty}n^{-1}\sum_{i=1}^{n}P\bigl(|a_{ni}X|>b_n\bigr)<\infty, \qquad (2.5)$$

$$J \doteq \sum_{n=1}^{\infty}n^{-1}P\left(\bigl|T^{(n)}\bigr|>\frac{\varepsilon}{2}b_n\right)<\infty. \qquad (2.6)$$

By Lemma 1.4 and the condition $E|X|^{\alpha}\log(1+|X|)<\infty$, we can see that

$$I\le CE|X|^{\alpha}\log\bigl(1+|X|\bigr)<\infty,$$

which implies (2.5).

For fixed $n\ge 1$, it is easily seen that $\{X_i^{(n)}-EX_i^{(n)}, 1\le i\le n\}$ are still pairwise NQD with mean zero by (iii) of Lemma 1.1. Hence, it follows from Markov's inequality, (1.3) of Lemma 1.2 and (1.5) of Lemma 1.3 that

$$
\begin{aligned}
J &\le C\sum_{n=1}^{\infty}n^{-1}b_n^{-2}E\bigl|T^{(n)}\bigr|^2\le C\sum_{n=1}^{\infty}n^{-1}b_n^{-2}\sum_{i=1}^{n}E\bigl|X_i^{(n)}\bigr|^2\\
&\le C\sum_{n=1}^{\infty}n^{-1}b_n^{-2}\sum_{i=1}^{n}\bigl[E|a_{ni}X_{ni}|^2I\bigl(|a_{ni}X_{ni}|\le b_n\bigr)+b_n^2P\bigl(|a_{ni}X_{ni}|>b_n\bigr)\bigr]\\
&\le C\sum_{n=1}^{\infty}n^{-1}b_n^{-2}\sum_{i=1}^{n}\bigl[E|a_{ni}X|^2I\bigl(|a_{ni}X|\le b_n\bigr)+b_n^2P\bigl(|a_{ni}X|>b_n\bigr)\bigr]\\
&= C\sum_{n=1}^{\infty}n^{-1}b_n^{-2}\sum_{i=1}^{n}E|a_{ni}X|^2I\bigl(|a_{ni}X|\le b_n\bigr)+C\sum_{n=1}^{\infty}n^{-1}\sum_{i=1}^{n}P\bigl(|a_{ni}X|>b_n\bigr)\\
&\doteq J_1+J_2. \qquad (2.7)
\end{aligned}
$$

By Lemma 1.4 and the condition $E|X|^{\alpha}\log(1+|X|)<\infty$ again, we can see that

$$J_2\le CE|X|^{\alpha}\log\bigl(1+|X|\bigr)<\infty.$$

To prove $J_1 < \infty$, we divide $\{a_{ni}, 1 \le i \le n\}$ into three subsets $\{a_{ni} : |a_{ni}| \le 1/(\log n)^s\}$, $\{a_{ni} : 1/(\log n)^s < |a_{ni}| \le 1\}$, $\{a_{ni} : |a_{ni}| > 1\}$, where $s = \frac{1}{2-\alpha}$. Hence

$$\begin{aligned} J_1 &= C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:|a_{ni}|\le 1/(\log n)^s} E|a_{ni}X|^2 I\big(|a_{ni}X| \le b_n\big) \\ &\quad + C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:1/(\log n)^s<|a_{ni}|\le 1} E|a_{ni}X|^2 I\big(|a_{ni}X| \le b_n\big) \\ &\quad + C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:|a_{ni}|>1} E|a_{ni}X|^2 I\big(|a_{ni}X| \le b_n\big) \\ &\doteq J_{11} + J_{12} + J_{13}. \end{aligned}$$

Note that

$$\sum_{i:|a_{ni}|\le 1/(\log n)^s} |a_{ni}|^\alpha \le n(\log n)^{-s\alpha},$$

we have by $E|X|^\alpha < \infty$ that

$$\begin{aligned} J_{11} &\doteq C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:|a_{ni}|\le 1/(\log n)^s} E|a_{ni}X|^2 I\big(|a_{ni}X| \le b_n\big) \\ &\le C\sum_{n=1}^{\infty} n^{-1} b_n^{-\alpha} \sum_{i:|a_{ni}|\le 1/(\log n)^s} E|a_{ni}X|^\alpha I\big(|a_{ni}X| \le b_n\big) \quad (\text{since } 0 < \alpha < 2) \\ &\le C\sum_{n=1}^{\infty} n^{-1} b_n^{-\alpha} \sum_{i:|a_{ni}|\le 1/(\log n)^s} |a_{ni}|^\alpha \\ &\le C\sum_{n=1}^{\infty} n^{-1} (\log n)^{-1-s\alpha} < \infty. \end{aligned}$$

Note that $s = \frac{1}{2-\alpha}$ and

$$\sum_{i:1/(\log n)^s<|a_{ni}|\le 1} |a_{ni}|^2 \le n,$$

we have by $E|X|^\alpha < \infty$ again that

$$\begin{aligned} J_{12} &\doteq C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:1/(\log n)^s<|a_{ni}|\le 1} E|a_{ni}X|^2 I\big(|a_{ni}X| \le b_n\big) \\ &\le C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:1/(\log n)^s<|a_{ni}|\le 1} |a_{ni}|^2 EX^2 I\big(|X| \le b_n(\log n)^s\big) \\ &\le C\sum_{n=1}^{\infty} b_n^{-2} EX^2 I\big(|X| \le n^{1/\alpha}(\log n)^{s+1/\alpha}\big) \\ &= C\sum_{n=1}^{\infty} b_n^{-2} \sum_{i=1}^{n} EX^2 I\Big((i-1)^{1/\alpha}\big(\log(i-1)\big)^{s+1/\alpha} < |X| \le i^{1/\alpha}(\log i)^{s+1/\alpha}\Big) \end{aligned}$$

$$
\begin{aligned}
&= C\sum_{i=1}^{\infty} EX^2 I\big((i-1)^{1/\alpha}\big(\log(i-1)\big)^{s+1/\alpha} < |X| \le i^{1/\alpha}(\log i)^{s+1/\alpha}\big)\sum_{n=i}^{\infty} n^{-2/\alpha}(\log n)^{-2/\alpha} \\
&\le C\sum_{i=1}^{\infty} EX^2 I\big((i-1)^{1/\alpha}\big(\log(i-1)\big)^{s+1/\alpha} < |X| \le i^{1/\alpha}(\log i)^{s+1/\alpha}\big)(\log i)^{-2/\alpha}\sum_{n=i}^{\infty} n^{-2/\alpha} \\
&\le C\sum_{i=1}^{\infty} EX^2 I\big((i-1)^{1/\alpha}\big(\log(i-1)\big)^{s+1/\alpha} < |X| \le i^{1/\alpha}(\log i)^{s+1/\alpha}\big)(\log i)^{-2/\alpha} i^{1-2/\alpha} \\
&\le C\sum_{i=1}^{\infty} E|X|^{\alpha} I\big((i-1)^{1/\alpha}\big(\log(i-1)\big)^{s+1/\alpha} < |X| \le i^{1/\alpha}(\log i)^{s+1/\alpha}\big) \quad \left(\text{since } s = \frac{1}{2-\alpha}\right) \\
&\le CE|X|^{\alpha} < \infty.
\end{aligned}
$$

By Lemma 1.5 and the condition $E|X|^{\alpha}\log(1+|X|) < \infty$, we can see that

$$
J_{13} \doteq C\sum_{n=1}^{\infty} n^{-1} b_n^{-2} \sum_{i:|a_{ni}|>1} E|a_{ni}X|^2 I\big(|a_{ni}X| \le b_n\big) \le CE|X|^{\alpha}\log\big(1+|X|\big) < \infty.
$$

Therefore, $J_1 < \infty$ follows from the statements above. This completes the proof of the theorem. □

Remark 2.1 The key to the proof of Theorem 2.1 is the Kolmogorov type inequality for pairwise NQD random variables. For many sequences of random variables, such as independent sequence, negatively associated sequence (see Shao [18]), negatively dependent sequence (see Asadian *et al.* [19]), ρ^*-mixing sequence (see Utev and Peligrad [20]), φ-mixing sequence (see Wang *et al.* [21]) and so forth, the Kolmogorov type inequality also holds. So, Theorem 2.1 also holds for these sequences.

Remark 2.2 Theorem 2.1 only holds for $0 < \alpha < 2$. It is still an open question whether Theorem 2.1 holds for $\alpha = 2$. In addition, it is still an open question whether

$$
\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon b_n\right) < \infty
$$

holds for any $\varepsilon > 0$ under the conditions of Theorem 2.1. The authors suggest that a solution can be obtained if a better moment inequality than the one presented above in Lemma 1.2 could be established.

Competing interests
The author declares that they have no competing interests.

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