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The riesz convergence and riesz core of double sequences

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Abstract

In this article, we have introduced the Riesz convergence and Riesz core of double sequences and determined the necessary and sufficient conditions on a four-dimensional matrix A to yield P_R - core$\{Ax\} \subseteq P$ - core$\{x\}$ and P_R - core$\{Ax\} \subseteq st_2$ - core $\{x\}$ for all $x \in \ell_\infty^2$.

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1. Introduction

A double sequence $x = [x_{jk}]_{j,k=0}^{\infty}$ is said to be convergent in the Pringsheim sense or P-convergent if for every ε >0 there exists an $N \in \mathbb{N}$ such that $| x_{jk} - \ell | < \varepsilon$ whenever $j, k > N$, [1]. In this case, we write P-lim $x = \ell$. By c_2, we mean the space of all P-convergent sequences.

A double sequence x is bounded if

$$\|x\| = \sup_{j,k \geq 0} |x_{jk}| < \infty.$$

By ℓ_∞^2 we denote the space of all bounded double sequences.

Note that, in contrast to the case for single sequences, a convergent double sequence need not be bounded. So, we denote by c_2^∞ the space of double sequences which are bounded and convergent.

Let $E \subseteq \mathbb{N} \times \mathbb{N}$ and $E(m, n) = \{(j, k): j \leq m, k \leq n\}$. Then, the double natural density of E is defined by

$$\delta_2(E) = P - \lim_{m,n} \frac{|E(m,n)|}{mn}$$

if the limit on the right hand side exists; where the vertical bars denotes the cardinality of the set $E(m,n)$.

A real double sequence $x = [x_{jk}]$ is said to be statistical (or briefly st-) convergent [2] to the number L if for every ε >0, the set $\{(j,k): |x_{jk} - L| > \varepsilon\}$ has double natural density zero. In this case, we write st_2 - lim $x = L$. Let st_2 be the space of all st-convergent double sequences. Clearly, a convergent double sequence is also st-convergent but the converse it is not true, in general. Also, note that a st-convergent double sequence need not be bounded. For example, consider the sequence $x = [x_{jk}]$ defined by

$$x_{jk} = \begin{cases} jk, & \text{if } j \text{ and } k \text{ are squares,} \\ 1, & \text{otherwise.} \end{cases} \tag{1.1}$$

Then, clearly st_2-lim $x = 1$. Nevertheless x neither convergent nor bounded. The st_2-lim sup and st_2 - lim inf of a double sequence were introduced in [3] and also the statistical core of a double sequence was defined by the closed interval [st_2 - lim sup, st_2 - lim inf].

Let $A = [a_{jk}^{mn}]_{j,k=0}^{\infty}$ be a four-dimensional infinite matrix of real numbers for all m,n = 0,1,.... The sums

$$y_{mn} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} a_{jk}^{mn} x_{jk}$$

are called the A- transforms of the double sequence $x = [x_{jk}]$. We say that a sequence $x = [x_{jk}]$ is A-summable to the limit ℓ if the A- transform of $x = [x_{jk}]$ exists for all m, n = 0,1,... and is convergent to ℓ in the Pringsheim sense, i.e.,

$$\lim_{p,q\to\infty} \sum_{j=0}^{p}\sum_{k=0}^{q} a_{jk}^{mn} x_{jk} = y_{mn}$$

and

$$\lim_{m,n\to\infty} y_{mn} = \ell.$$

We say that a matrix A is bounded-regular if every bounded-convergent sequence x is A-summable to the same limit and the A-means are also bounded. The necessary and sufficient conditions for A to be bounded-regular or RH- regular are known (see [4,5]).

A double sequence $x = [x_{jk}]$ of real numbers is said to be Cesáro convergent to a number L if and only if there exists an $L \in \mathbb{R}$ such that

$$\lim_{p,q\to\infty} \frac{1}{(p+1)(q+1)} \sum_{j=1}^{p}\sum_{k=1}^{q} x_{jk}^{mn} = L,$$

and is denoted by C_1 - lim $x = L$. We denote the space of all Cesáro convergent double sequences by C_1. That is,

$$C_1 = \{x \in \ell_\infty^2 : \exists L \in \mathbb{R} \ni \quad C_1 - \lim\ x\ = L\}.$$

The concept core of single sequences (see [6]) was extended by Patterson [7] to the double sequences by defining the Pringsheim core (or P-core) of a real bounded double sequence $x = [x_{jk}]$ as the closed interval [P - lim inf x,P - lim sup x]. Later this concept has been studied by many authors. For example we refer [2,8-10].

Let

$$C_1^*(x) = \limsup_{p,q\to\infty} \frac{1}{(p+1)(q+1)} \sum_{j=0}^{p}\sum_{k=0}^{q} x_{jk}.$$

The Cesáro core (or P_C-core) of a real-valued bounded double sequence $x = [x_{jk}]$ has been defined by the closed interval $[-C_1^*(-x), C_1^*(x)]$ in [11]. Also; where an inequality related to the P_C and P-cores has been investigated.

In this article we have introduced the Riesz convergence and Riesz core of a double sequence and also we have investigated some inequalities related to the P-, statistical and Riesz cores.

2. Main results

Definition 2.1. *Let (q_i), (p_j) be sequences of non-negative numbers which are not all zero and $Q_m = q_1 + q_2 + \cdots + q_m$, $q_1 > 0$, $P_n = p_1 + p_2 + \cdots + p_n$, $p_1 > 0$. Then, the transformation given by*

$$t_{mn}^{qp}(x) = \frac{1}{Q_m}\frac{1}{P_n}\sum_{i=1}^{m}\sum_{j=1}^{n} q_i p_j x_{ij}$$

is called the Riesz mean of double sequence $x = [x_{jk}]$.

Definition 2.2. *If $P - \lim t_{mn}^{qp}(x) = s$, $s \in \mathbb{R}$, then the sequence $x = [x_{jk}]$ is said to be Riesz convergent to s.*

If $x = [x_{jk}]$ is Riesz convergent to s, then we write P_R- lim $x = s$. In what follows c_R^2 will denote the set of all Riesz convergent sequences. Since a Riesz convergent double sequence need not be bounded, by $c_R^{2,\infty}$ we will denote the set of all bounded and Riesz convergent double sequences. $c_{0,R}^{2,\infty}$ will denote the set of all double sequences which bounded and Riesz convergent to zero.

Note that in the case $q_i= 1$ for all i and $p_j = 1$ for all j, the Riesz mean reduced to the Cesáro mean and the Riesz convergence is said to be Cesáro convergence.

Now, we will give some lemmas characterized some classes of matrices related to the $c_R^{2,\infty}$.

Lemma 2.3. *A matrix $A = (a_{jk}^{mn}) \in (\ell_\infty^2, c_{0,R}^{2,\infty})$if and only if*

$$\|A\| = \sup_{mn} \sum_{jk} \left|a_{jk}^{mn}\right| < \infty, \tag{2.1}$$

$$P - \lim_{mn} \alpha(m,n,r,s,q,p) = 0 (r, s \in \mathbb{N}), \tag{2.2}$$

$$P - \lim_{mn} \sum_{r} \left|\alpha(m,n,r,s,q,p)\right| = 0 (s \in \mathbb{N}), \tag{2.3}$$

$$P - \lim_{mn} \sum_{s} \left|\alpha(m,n,r,s,q,p)\right| = 0 (r \in \mathbb{N}), \tag{2.4}$$

(2.5) $$P - \lim_{mn} \sum_{rs} \left|\alpha(m,n,r,s,q,p)\right| = 0,$$

where

$$\alpha(m,n,r,s,q,p) = \frac{1}{Q_r}\frac{1}{P_s}\sum_{j=1}^{r}\sum_{k=1}^{s} q_j p_k a_{jk}^{mn}.$$

Proof. Let $A = (a_{jk}^{mn}) \in (\ell_\infty^2, c_{0,R}^{2,\infty})$. This means that Ax exists for all $x = [x_{jk}] \in \ell_\infty^2$ and $Ax \in c_{0,R}^{2,\infty}$ which implies (2.1). Let us define a sequence $y = [y_{rs}]$ by

$$y_{rs} = \begin{cases} \operatorname{sgn} \alpha(m_i, n_j, r, s, q, p), & r_{i-1} < r < r_{i,} \quad s_{j-1} < s < s_j \\ 0 & \text{otherwise.} \end{cases}$$

Then, the necessity of (2.5) follows from $P - \lim t_{rs}^{qp}(Ax)$.

It is known by the assumption that

$$P - \lim \sum_{r,s} \alpha(m,n,r,s,q,p)x_{jk} = 0.$$

So; if we define the sequences e_{ij}^{rs}, e^r, e^s as follows

$$e_{ij}^{rs} = \begin{cases} 1, & (j,k) = (r,s) \\ 0 & \text{otherwise,} \end{cases}$$

$e^r = \Sigma_s\, e^{rs}$ $(s \in \mathbb{N})$ and $e^s = \Sigma_r\, e^{rs}$ $(r \in \mathbb{N})$, then the necessity of (2.2), (2.3), and (2.4) follows from $P - \lim t_{rs}^{qp}(Ae^{rs})$, $P - \lim t_{rs}^{qp}(Ae^{r})$ and $P - \lim t_{rs}^{qp}(Ae^{s})$, respectively.

Since the proof of the sufficiency part is routine, we omit the details.

Lemma 2.4. *A matrix* $A = (a_{jk}^{mn}) \in (c_2^\infty, c_R^{2,\infty})_{\text{reg}}$*if and only if* (2.1)-(2.4) *hold and*

$$P - \lim_{mn} \sum_{rs} \left|\alpha(m,n,r,s,q,p)\right| = 1. \tag{2.6}$$

Proof. The necessity of the conditions can be shown by the same way used in the proof of Lemma 2.3.

For the sufficiency let the conditions hold and $x = [x_{jk}] \in c_2^\infty$ with P - lim $x_{jk} = L$, (say). Then, there exists an $N > 0$ such that $|x_{jk}| < |L| + \varepsilon$ for every whenever $j, k > N$. Now; let us write

$$\begin{aligned}\sum_{rs} \alpha(m,n,r,s,q,p)x_{rs} = \sum_{r=0}^{N}\sum_{s=0}^{N} \alpha(m,n,r,s,q,p)x_{rs} &+ \sum_{r=N+1}^{\infty}\sum_{s=0}^{N-1} \alpha(m,n,r,s,q,p)x_{rs} \\ &+ \sum_{r=0}^{N-1}\sum_{s=N+1}^{\infty} \alpha(m,n,r,s,q,p)x_{rs} \\ &+ \sum_{r=N+1}^{\infty}\sum_{s=N+1}^{\infty} \alpha(m,n,r,s,q,p)x_{rs}\end{aligned}$$

which implies that

$$\left|\sum_{rs}\alpha(m,n,r,s,q,p)x_{rs}\right| = \|x\|\sum_{r=0}^{N}\sum_{s=0}^{N}\left|\alpha(m,n,r,s,q,p)\right| + \|x\|\sum_{r=N+1}^{\infty}\sum_{s=0}^{N-1}\left|\alpha(m,n,r,s,q,p)\right|$$
$$+ \|x\|\sum_{r=0}^{N-1}\sum_{s=N+1}^{\infty}\left|\alpha(m,n,r,s,q,p)\right|$$
$$+ (|L|+\varepsilon)\left|\sum_{r=N+1}^{\infty}\sum_{s=N+1}^{\infty}\alpha(m,n,r,s,q,p)\right|.$$

So, by letting $m,n \to \infty$ under the light of the assumption, we get that $P-\lim t_{rs}^{qp}(Ax) = L$.

This completes the proof.

Lemma 2.5. *A matrix* $A = (a_{jk}^{mn}) \in (st_2 \cap \ell_\infty^2, c_R^{2,\infty})_{\text{reg}}$*if and only if*

$$A = (a_{jk}^{mn}) \in (c_2^{\infty}, c_R^{2,8})_{\text{reg}} \tag{2.7}$$

$$P - \lim_{mn}\sum_{r,s\in E}\left|\alpha(m,n,r,s,q,p)\right| = 0 \tag{2.8}$$

for every $E \subset \mathbb{N}\times\mathbb{N}$ *with* $\delta_2(E) = 0$.

Proof. If $A = (a_{jk}^{mn}) \in (st_2 \cap \ell_\infty^2, c_R^{2,\infty})_{\text{reg}}$, the necessity of (2.7) follows from the fact that $c_2^{\infty} \subset st_2 \cap \ell_\infty^2$. For the necessity of the condition (2.8), let us choose a sequence $z = [z_{rs}]$ by

$$z_{rs} = \begin{cases} x_{rs}, & r,s \in E \\ 0, & \text{otherwise,} \end{cases}$$

where $x = [x_{rs}] \in \ell_\infty^2$ and $E \subset \mathbb{N}\times\mathbb{N}$ with $\delta_2\ (E) = 0$. Then; it is easy to see that st_2 - lim $z = 0$ and

$$t_{rs}^{qp}(Az) = \sum_{r,s\in E}\alpha(m,n,r,s,q,p)x_{rs}.$$

So; a matrix $B = [b_{rs}^{mn}]$ defined by

$$b_{rs}^{mn} = \begin{cases} \alpha(m,n,r,s,q,p), & r,s \in E \\ 0, & \text{otherwise,} \end{cases}$$

for every q,p is in the class $(\ell_\infty^2, c_{0,R}^{2,\infty})$. Therefore, the necessity of (2.8) follows from the condition (2.5) of Lemma 2.3.

For the converse take a sequence $x = [x_{rs}] \in st_2 \cap \ell_\infty^2$ with st_2 - lim $x = l$. Then; it is known that $\delta_2 = \delta_2(\{(r,\ s): |x_{rs} - l| \geq \varepsilon\}) = 0$ and $|x_{rs} - l| < \varepsilon$ whenever $r,s \notin E$. Now, write

$$\sum_{r,s}\alpha(m,n,r,s,q,p)x_{rs} = \sum_{r,s}\alpha(m,n,r,s,q,p)(x_{rs}-1) + l\sum_{r,s}\alpha(m,n,r,s,q,p). \tag{2.9}$$

The inequality

$$\left|\sum_{r,s}\alpha(m,n,r,s,q,p)(x_{rs}-l)\right| = \left|\sum_{r,s\in E}\alpha(m,n,r,s,q,p)(x_{rs}-1)+\sum_{r,s\notin E}\alpha(m,n,r,s,q,p)(x_{rs}-1)\right|$$
$$\leq \|x_{rs}-l\|\sum_{r,s\in E}\left|\alpha(m,n,r,s,q,p)\right| + \varepsilon\,\|A\|$$

and condition (2.8) implies that

$$P-\lim_{mn}\sum_{r,s}\alpha(m,n,r,s,q,p)(x_{rs}-l)=0.$$

So; by letting $m,\ n\to\infty$ in (2.9) we have $P-\lim t_{rs}^{qp}(Ax)=l$ and this completes the proof.

Definition 2.6. *The Riesz core* (*or P_R-core*) *of a double sequence $x = [x_{jk}]$ is the closed interval* $[P-\lim\inf_{m,n}t_{mn}^{qp}(x), P-\lim\sup_{m,n}t_{mn}^{qp}(x)]$..

Note that in the case $q_i = 1$ for all i and $p_j = 1$ for all j, Riesz core is reduced to the Cesáro core, [11].

Now; we are ready to give some inequalities related to the P-, P_R- and st_2-core of double sequences.

Theorem 2.7. *Let* $\|A\| < \infty$. *Then,*

$$P-\lim\sup t_{rs}^{qp}(Ax)\leq P-\lim\sup(x), \tag{2.10}$$

for all $x\in\ell_\infty^2$ *if and only if* $A\in(c_2^\infty, c_R^{2,\infty})_{\text{reg}}$ *and*

$$P-\lim_{m,n}\sum_{r,s}\left|\alpha(m,n,r,s,q,p)\right|=1. \tag{2.11}$$

Proof. Let (2.10) holds for all $x\in\ell_\infty^2$. Then, it is easy to get that

$$-P-\lim\sup(-x)\leq -P-\lim\ \sup\ t_{rs}^{qp}(-Ax)\leq P-\lim\sup\ \ t_{rs}^{qp}(Ax)\leq P-\lim\sup(x).$$

Since - P - lim sup(-x) = P - lim inf(x) and $-P-\lim\sup t_{rs}^{qp}(-Ax)=P-\lim\inf t_{rs}^{qp}(Ax)$, by choosing $x\in c_2^\infty$, we reach that $P-\lim t_{rs}^{qp}(Ax)=P-\lim(x)$. Since x is arbitrary, this means that $A\in(c_2^\infty, c_R^{2,\infty})_{\text{reg}}$.

By Lemma 3.1 of Patterson [7], there exists a $y\in\ell_\infty^2$ with $\|\ y\ \|\leq 1$ such that

$$P-\lim\sup\ \ t_{rs}^{qp}(Ay)=P-\lim\sup\sum_{r,s}\left|\alpha(m,n,r,s,q,p)\right|.$$

So; we have from assumption that

$$P-\lim\sup\sum_{r,s}\left|\alpha(m,n,r,s,q,p)\right|=p-\lim\sup t_{rs}^{qp}(Ay)\leq P-\lim\sup(y)\leq\|y\|\leq 1. \tag{2.12}$$

By the same way, one can see that

$$P-\lim\inf\sum_{r,s}\left|\alpha(m,n,r,s,q,p)\right|\geq 1. \tag{2.13}$$

Therefore, by combining the inequalities (2.12) and (2.13), we obtain the necessity of (2.11).

Conversely; suppose that $A \in (c_2^{\infty}, c_R^{2,\infty})_{\text{reg}}$ and (2.11) holds. For any arbitrary bounded sequence $x = [x_{rs}]$, there exists M, $N > 0$ such that $x_{rs} \leq P$ - lim sup $x + \varepsilon$ whenever $r > M$, $s > N$. Now, we can write the following inequality,

$$\begin{aligned}
\left|\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\alpha(m,n,r,s,q,p)x_{rs}\right| &= \left|\sum_{r=0}^{\infty}\sum_{s=o}^{\infty}\left(\frac{\left|\alpha(m,n,r,s,q,p)\right| + \alpha(m,n,r,s,q,p)}{2}\right.\right. \\
&\quad \left.\left. - \frac{\left|\alpha(m,n,r,s,q,p)\right| - \alpha(m,n,r,s,q,p)}{2}\right)x_{rs}\right| \\
&\leq \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\left|\alpha(m,n,r,s,q,p)\right| |x_{rs}| \\
&\quad + \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\left|(\left|\alpha(m,n,r,s,q,p)\right| - \alpha(m,n,r,s,q,p))x_{rs}\right| \\
&\leq \|x\|\sum_{r=0}^{M}\sum_{s=0}^{N}\left|\alpha(m,n,r,s,q,p)\right| \\
&\quad + \|x\|\sum_{r=M+1}^{\infty}\sum_{s=0}^{N}\left|\alpha(m,n,r,s,q,p)\right| \\
&\quad + \|x\|\sum_{r=0}^{\infty}\sum_{s=N+1}^{N}\left|\alpha(m,n,r,s,q,p)\right| \\
&\quad + (P - \limsup x + \varepsilon)\sum_{r=M+1}^{\infty}\sum_{s=N+1}^{\infty}\left|\alpha(m,n,r,s,q,p)\right| \\
&\quad + \|x\|\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(\left|\alpha(m,n,r,s,q,p)\right| - \alpha(m,n,r,sq,p)).
\end{aligned}$$

Using the conditions characterized the class $(c_2^{\infty}, c_R^{2,\infty})_{\text{reg}}$ and (2.11), we reach that $P - \limsup t_{rs}^{qp}(Ax) \leq P - \limsup(x)$ and this completes the proof of the theorem.

Theorem 2.8. *Let* $\|A\| < \infty$. *Then,*

$$P - \limsup t_{rs}^{qp}(Ax) \leq st_2 - \limsup(x), \tag{2.14}$$

for all $x \in$ $x \in \ell_{\infty}^2$*if and only if* $A \in (st_2 \cap \ell_{\infty}^2, c_R^{2,\infty})_{\text{reg}}$*and* (2.11) *holds.*

Proof. Let (2.14) holds for all $x \in \ell_{\infty}^2$. Then, by the same argument used in Theorem 2.7, one can see that $A \in (st_2 \cap \ell_{\infty}^2, c_R^{2,\infty})_{\text{reg}}$. On the other since st_2 - lim sup$(x) \leq P$ - lim sup(x) for all $x \in \ell_{\infty}^2$, the necessity of (2.11) follows from Theorem 2.7.

For the converse suppose that $A \in (st_2 \cap \ell_{\infty}^2, c_R^{2,\infty})_{\text{reg}}$ and (2.11) holds. If $x = [x_{rs}] \in \ell_{\infty}^2$,, it is known that for every $\varepsilon > 0$,

$$\delta_2(E) = \delta_2(\{(r,s) : x_{rs} > st_2 - \limsup(x) + \varepsilon\}) = 0$$

and $x_{rs} \leq st_2$ - lim sup$(x) + \varepsilon$ whenever $r, s \notin E$. Taking this knowledge in the mind, let us write

$$\begin{aligned}
\sum_{r,s}\alpha(m,n,r,s,q,p)x_{rs} &\le \left|\sum_{r,s}\frac{|\alpha(m,n,r,s,q,p)x_{rs}|+\alpha(m,n,r,s,q,p)x_{rs}}{2}\right.\\
&\quad\left.+\sum_{r,s}\frac{|\alpha(m,n,r,s,q,p)x_{rs}|-\alpha(m,n,r,s,q,p)x_{rs}}{2}\right|\\
&\le \left|\sum_{r,s\in E}\alpha(m,n,r,s,q,p)x_{rs}+\sum_{r,s\notin E}\alpha(m,n,r,s,q,p)x_{rs}\right|\\
&\quad+\|x\|\sum_{r,s}\left(|\alpha(m,n,r,s,q,p)|-\alpha(m,n,r,s,q,p)\right)\\
&\le \|x\|\sum_{r,s\in E}|\alpha(m,n,r,s,q,p)|\\
&\quad+(st_2-\limsup(x)+\varepsilon)\sum_{r,s\notin E}|\alpha(m,n,r,s,q,p)\\
&\quad+\|x\|\sum_{r,s}\left(|\alpha(m,n,r,s,q,p)|-\alpha(m,n,r,s,q,p)\right).
\end{aligned}$$

So, the conditions characterized the class $(st_2\cap\ell_\infty^2, c_R^{2,\infty})_{\text{reg}}$ and (2.11) imply that $P-\limsup t_{rs}^{qp}(Ax)\le st_2-\limsup(x)+\varepsilon$. Since ε was arbitrary, this completes the proof.

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Authors' contributions
AMA designed the problems and carried out the proof of the Lemmas. CÇ defined the Riesz core and gave the proof of the theorems. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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