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Extremum of Mahler volume for generalized cylinder in $\mathbb{R}^3$

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Abstract

A special case of Mahler volume for the class of symmetric convex bodies in $\mathbb{R}^3$ is treated here. It is shown that a cube has the minimal Mahler volume and a cylinder has the maximal Mahler volume for all generalized cylinders. Further, the Mahler volume of bodies of revolution obtained by rotating the unit disk in $\mathbb{R}^2$ is presented.
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Keywords: Mahler volume, convex body, polar body, body of revolution

1 Introduction

Throughout this article a convex body K in Euclidean n-space $\mathbb{R}^n$ is a compact convex set that contains the origin in its interior. Its polar body K^* is defined by

$$K^* = \{x \in \mathbb{R}^n : x \cdot y \leq 1, \text{for all } y \in K\},$$

where $x{\cdot}y$ denotes the standard inner product of x and y in $\mathbb{R}^n$.

If K is an origin symmetric convex body, then the product

$$V(K)V(K^*)$$

is called the volume product of K, where $V(K)$ denotes n-dimensional volume of K, which is known as the *Mahler volume* of K, and it is invariant under linear transformation.

One of the main questions still open in convex geometric analysis is the problem of finding a sharp lower estimate for the Mahler volume of a convex body K (see the survey article [1]).

A sharp upper estimate of the volume product is provided by the Blaschke-Santaló inequality: For every centered convex body K in $\mathbb{R}^n$

$$V(K)V(K^*) \leq \omega_n^2,$$

with equality if and only if K is an ellipsoid centered at the origin, where ω_n is the volume of the unit ball in $\mathbb{R}^n$ (see, e.g., [2-5]).

The Mahler conjecture for the class of origin-symmetric bodies is that:

$$V(K)V(K^*) \geq \frac{4^n}{n!} \tag{1.1}$$

with equality holding for parallelepipeds and their polars. For $n = 2$, the inequality is proved by Mahler himself [6], and in 1986, Reisner [7] showed that parallelograms are the only minimizers. Reisner [8] established inequality (1.1) for a class of bodies that have a high degree of symmetry, known as zonoids, which are limits of finite Minkowski sums of line segments. Lopez and Reisner [9] proved the inequality (1.1) for $n \leq 8$ and the minimal bodies are characterized. Recently, Nazaeov et al. [10] proved that the cube is a strict local minimizer for the Mahler volume in the class of origin-symmetric convex bodies endowed with the Banach-Mazur distance.

Bourgain and Milman [11] have proved that there exists a constant $c > 0$ independent of the dimension n, such that for all origin-symmetric bodies K,

$$V(K)V(K^*) \geq c^n \omega_n^2,$$

which is now known as the reverse Santaló inequality. Recently, Kuperberg [12] found a beautiful new approach to the reverse Santaló inequality. What's especially remarkable about Kuperberg's inequality is that it provides an explicit value for c. However, the Mahler conjecture is still open even in the three-dimensional case, Tao [13] made an excellent remark about the open question.

In the present article, we treat a special case of Mahler volume in $\mathbb{R}^3$. We now introduce some notations: A real-valued function $f(x)$ is called *concave*, if for any $x, y \in [a, b]$ and any $\lambda \in [0, 1]$, they have

$$f(\lambda x + (1 - \lambda)y) \geq \lambda f(x) + (1 - \lambda)f(y).$$

Definition 1 *In three-dimensional Cartesian coordinate system OXYZ, if C' is an origin-symmetric convex body in coordinate plane YOZ, then the set:*

$$C = \{(x, y, z)| - 1 \leq x \leq 1, (0, y, z) \in C'\} \tag{1.2}$$

is defined as a generalized cylinder in $\mathbb{R}^3$.

Definition 2 *In the coordinate plane XOY, let*

$$D = \{(x, y)| - a \leq x \leq a, |y| \leq f(x)\}, \tag{1.3}$$

where $f(x)$ *([-a, a], a >0), is a nonnegative concave and even function. Rotating D about the X-axis in* $\mathbb{R}^3$*, we can get a geometric object*

$$R = \{(x, y, z)| - a \leq x \leq a, (y^2 + z^2)^{\frac{1}{2}} \leq f(x)\}. \tag{1.4}$$

We define the geometric object R as a body of revolution generated by the function f(x) (or by the domain D), and call the function f(x) as the generated function of R and D as the generated domain of R.

If the generated domain of R is a rectangle and a diamond, R is called a *cylinder* and a *bicone*, respectively.

Let C denotes the set of all generalized cylinders. In this article, we proved that among the generalized cylinders, a cube has the minimal Mahler volume and a cylinder has the maximal Mahler volume, theorem as following:

Theorem 1 *For* $C \in \mathcal{C}$*, we have*

$$V(C_0)V(C_0^*) \leq V(C)V(C^*) \leq V(C_1)V(C_1^*), \tag{1.5}$$

where $C_0 = [-1, 1] \times [-1, 1] \times [-1, 1]$ *is a cube and* $C_1 = [-1, 1] \times B^2$ *is cylinder.*

Further, we get the following theorem:

Theorem 2 *For a class of bodies of revolution obtained by rotating the "unit disk" in planar XOY, where the "unit disk" is the following set:*

$$U = \{(x, y) | |x|^p + |y|^p \leq 1\}, p \geq 1, \tag{1.6}$$

the Mahler volume is increasing for $1 \le p \le 2$ *and decreasing for* $2 \le p \le +\infty$.

More interrelated notations, definitions, and their background materials are exhibited in the following section.

2 Definition and notation

The setting for this article is n-dimensional Euclidean space $\mathbb{R}^n$. Let $\mathcal{K}^n$ denotes the set of convex bodies (compact, convex subsets with non-empty interiors), $\mathcal{K}_o^n$ denotes the subset of $\mathcal{K}^n$ that contains the origin in their interiors. As usual, B^n denotes the unit ball centered at the origin, S^{n-1} the unit sphere, o the origin, and $||\cdot||$ the norm in $\mathbb{R}^n$.

If $u \in S^{n-1}$ is a direction, $u^\perp$ is the $(n - 1)$-dimensional subspace orthogonal to u. For $x, y \in \mathbb{R}^n$, $x \cdot y$ is the inner product of x and y, and $[x, y]$ denotes the line segment with endpoints x and y.

If K is a set, ∂K is its boundary, *int* K is its interior, and *conv* K denotes its convex hull. $V(K)$ denotes n-dimensional volume of K. Let $K|S$ be the orthogonal projection of K into a subspace S.

Let $K \in \mathcal{K}^n$ and $H = \{x \in \mathbb{R}^n | x \cdot v = d\}$ denotes a hyperplane, H^+ and H^- denote the two closed halfspaces bounded by H.

Associated with each convex body K in $\mathbb{R}^n$, its *support function* $h_K : \mathbb{R}^n$ - $[0, \infty)$, is defined for $x \in \mathbb{R}^n$, by

$$h_K(x) = \max\{x \cdot y : \quad y \in K\},$$

and its *radial function* $\rho_K : \mathbb{R}^n \backslash \{0\} \to (0, \infty)$, is defined for $x \neq 0$, by

$$\rho_K(x) = \max\{\lambda \geq 0 : \lambda x \in K\}.$$

From the definitions of the support and radial functions and the definition of the polar bodies, it follows that (see [4])

$$h_{K^*}(u) = \frac{1}{\rho_K(u)} \quad \text{and} \quad \rho_{K^*}(u) = \frac{1}{h_K(u)}, \quad u \in S^{n-1},$$
$$K^* = \{x \in \mathbb{R}^n : h_K(x) \leq 1\},$$
$$K^{**} = K.$$

If P is a polytope, i.e., $P = \text{conv}\{p_1, ..., p_m\}$, where p_i $(i = 1, ..., m)$ are vertices of polytope P. By the definition of polar body, we have

$$\begin{aligned} P^* &= \{x \in \mathbb{R}^n : x \cdot p_1 \leq 1, \ldots, x \cdot p_m \leq 1\} \\ &= \bigcap_{i=1}^m \{x \in \mathbb{R}^n : x \cdot p_i \leq 1\}, \end{aligned}$$

which implies that P^* is the intersection of m closed halfspace with exterior normal vector p_i and the distance of hyperplane $\{x \in \mathbb{R}^n : x \cdot p_i = 1\}$ from the origin is $1/||p_i||$.

For $K \in \mathcal{K}_o^n$, if $x = (x_1, x_2, ..., x_n) \in K$, $x' = (\varepsilon_1 x_1, ..., \varepsilon_n x_n) \in K$ for any signs $\varepsilon_i = \pm 1$ ($i = 1, ..., n$), then K is a *1-unconditional convex body*. In fact, K is symmetric around all coordinate hyperplanes.

To proof the inequality, we give the following definitions.

Definition 3 *In Definition 2, if the function*

$$f(x) = kx + b, \quad x \in [-a, 0],$$

where k and b are real constants, and f(-a) = 0, then the body of revolution is defined as a bicone. In three-dimensional Cartesian coordinate system OXYZ, if C′ is an origin-symmetric convex body in coordinate plane YOZ and points A = (-a, 0, 0) and A′ = (a, 0, 0), then the set.

$$B = conv\{C', A, A'\} \tag{2.1}$$

is defined as a generalized bicone in $\mathbb{R}^3$.

3 Proof of the main results

In this section, we only consider convex bodies in three-dimensional Cartesian coordinate system with origin *O*, and its three coordinate axes are denoted by *X*-, *Y* -, and *Z*-axis.

Let *C* be a generalized cylinder as following:

$$C = \{(x, y, z) | -1 \leq x \leq 1, (0, y, z) \in C'\},$$

where C' is an origin-symmetric convex body in coordinate plane *Y OZ*.

We require the following lemmas to prove our main result.

Lemma 1 *If* $K \in \mathcal{K}_o^3$, *for any* $u \in S^2$, *then*

$$K^* \cap u^\perp = (K|u^\perp)^*. \tag{3.1}$$

On the other hand, if $K' \in \mathcal{K}_o^3$ *satisfies*

$$K' \cap u^\perp = (K|u^\perp)^*,$$

for any $u \in S^2 \cap v_0^\perp$ (v_0 *is a fixed vector*), *then*

$$K' = K^*. \tag{3.2}$$

Proof First, we prove (3.1).

Let $x \in u^\perp$, $y \in K$ and $y' = y|u^\perp$, since the hyperplane $u^\perp$ is orthogonal to the vector $y - y'$, then

$$y \cdot x = (y' + y - y') \cdot x = y' \cdot x + (y - y'') \cdot x = y' \cdot x.$$

If $x \in K^* \mid u^\perp$, for any $y' \in K \mid u^\perp$, there exists $y \in K$ such that $y' = y|u^\perp$, then $x \cdot y' = x \cdot y \leq 1$, and $x \in (K|u^\perp)^*$. Hence,

$$K^* \cap u^\perp \subseteq (K|u^\perp)^*.$$

If $x \in (K|u^\perp)^*$, then for any $y \in K$ and $y' = y|u^\perp$, $x \cdot y = x \cdot y' \leq 1$, thus $x \in K^*$, and since $x \in u^\perp$, thus $x \in K^* \mid u^\perp$. Then,

$$(K|u^\perp)^* \subseteq K^* \cap u^\perp.$$

Next, we prove (3.2).

Let $S^1 = S^2 \cap v_0^{\perp}$. For any direction vector $v \in S^2$, there always exists a $u \in S^1$ satisfying $v \in u^{\perp}$. Since

$$K' \cap u^{\perp} = (K|u^{\perp})^*,$$

and by (3.1)

$$K^* \cap u^{\perp} = (K|u^{\perp})^*,$$

thus

$$K' \cap u^{\perp} = K^* \cap u^{\perp}.$$

Then, we get

$$\rho_{K'}(v) = \rho_{K^*}(v).$$

By the arbitrary of direction v, we obtain the desired result. ■

For any $C \in \mathcal{C}$ and any $u \in B^2 \mid v^{\perp}$ ($v = (1, 0, 0)$), $C|u^{\perp}$ is a rectangle by the above definition. We study the polar body of a rectangle in the planar. From Figure 1, if $C|u^{\perp} = [-1, 1]\times[-a, a]$, its polar body in planar XOY is a diamond (vertices are (-1, 0), (1, 0), (0, -1/a), (0, 1/a)), thus we can get the following Lemma 2.

Lemma 2 *For any* $C \in \mathcal{C}$, *if*

$$C = \{(x, y, z)| - 1 \leq x \leq 1, (0, y, z) \in C'\},$$

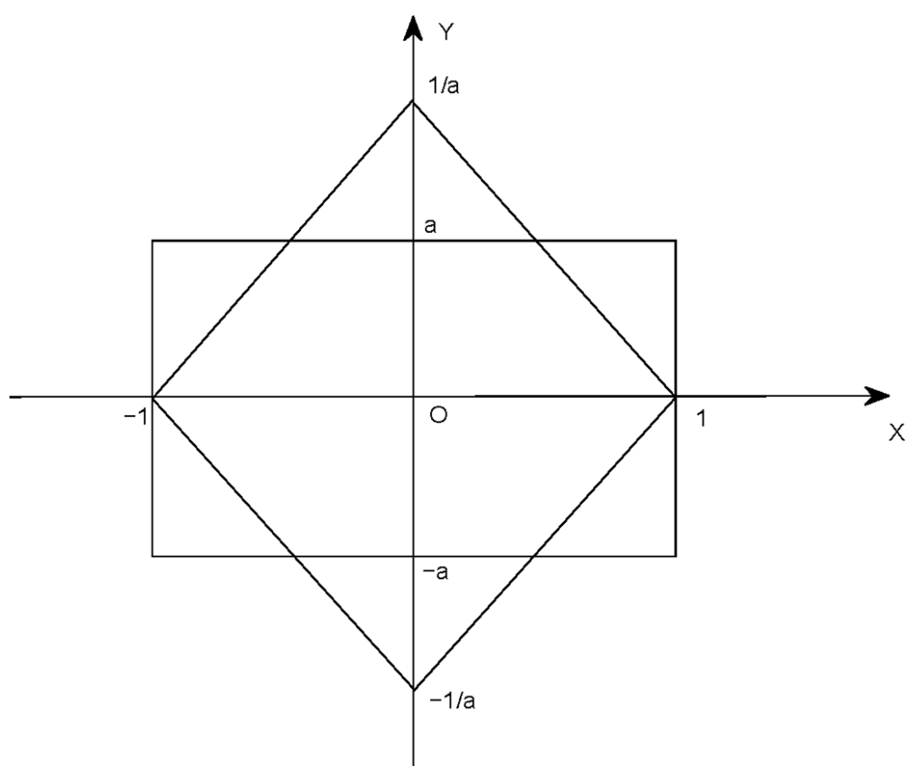

Figure 1 Rectangle and its polar body in planar XOY.

where C' is an origin-symmetric convex body in coordinate plane YOZ, then C^* is a generalized bicone with vertices (-1, 0, 0) *and* (1, 0, 0) *and the base* $(C')^*$.

Proof Let v_0 = (1, 0, 0), $S^1 = S^2 \cap v_0^\perp$. By Lemma 1, we have

$$C^* \cap u^\perp = (C|u^\perp)^*$$

for any $u \in S^1$. Because that $(C|u^\perp)^*$ is a diamond with vertices (-1, 0, 0) and (1, 0, 0), $C^* \mid u^\perp$ is a diamond with vertices (-1, 0, 0) and (1, 0, 0) for any $u \in S^1$, which implies that C^* is a bicone with vertices (-1, 0, 0) and (1, 0, 0).

In view of

$$C^* \cap v_0^\perp = (C|v_0^\perp)^*$$

and $C|v_0^\perp = C'$, then, the base of C^* is $(C')^*$. ■

In the following, we will restate and prove Theorem 1.

Theorem 1 *For $C \in \mathcal{C}$, we have*

$$V(C_0)V(C_0^*) \le V(C)V(C^*) \le V(C_1)V(C_1^*), \tag{3.3}$$

where C_0 = [-1, 1] × [-1, 1] × [-1, 1] is a cube and C_1 = [-1, 1] × B^2 is cylinder.

Proof Let v = (1, 0, 0), and $V(C) = V(C_0)$ by linear transformation, thus $V(C \cap v\perp) = V(C_0 \cap v^\perp)$.

In planar $v^\perp$, since the square has the minimal Mahler volume in $\mathbb{R}^2$, thus

$$V(C_0 \cap v^\perp)V((C_0 \cap v^\perp)^*) \le V(C \cap v^\perp)V((C \cap v^\perp)^*),$$

we get

$$V((C_0 \cap v^\perp)^*) \le V((C \cap v^\perp)^*),$$

then

$$\begin{aligned} V(C_0^*) &= \frac{1}{3}V((C_0 \cap v^\perp)^*) \times 2 \\ &\le \frac{1}{3}V((C \cap v^\perp)^*) \times 2 \\ &= V(C^*), \end{aligned}$$

where the equality holds if and only if $C \cap v^\perp$ is a square. Hence,

$$V(C_0)V(C_0^*) \le V(C)V(C^*).$$

Similarly, let $V(C) = V(C_1)$ for any $C \in \mathcal{C}$ by linear transformation, then $V(C \cap v^\perp) = V(C_1 \cap v^\perp)$.

Since $C_1 \cap v^\perp$ is a disk, which has the maximal Mahler volume in $\mathbb{R}^2$, thus

$$V(C_1 \cap v^\perp)V((C_1 \cap v^\perp)^*) \ge V(C \cap v^\perp)V((C \cap v^\perp)^*),$$

we get

$$V((C_1 \cap v^\perp)^*) \ge V((C \cap v^\perp)^*).$$

Hence, $V(C_1^*) \ge V(C^*)$, which implies

$$V(C_1)V(C_1^*) \ge V(C)V(C^*).$$

■

Theorem 1 implies that among the generalized cylinders, a cube has the minimal Mahler volume and a cylinder has the maximal Mahler volume.

4 Mahler volume of a special class of bodies of revolution

In this section, we study a special case in the coordinate plane XOY, and define the "unit disk" in planar XOY as following set:

$$U = \{(x, y) \| x|^p + |y|^p \le 1\}, \quad p \ge 1. \tag{4.1}$$

We need the following lemmas to prove our result.

Lemma 3 *Let P is a 1-unconditional convex body and P^* is its polar body in the coordinate plane XOY. Let R and R' are two bodies of revolution obtained by rotating P and P^*, respectively. Then $R' = R^*$.*

Proof Let $v_0 = \{1, 0, 0\}$ and $S^1 = S^2 \cap v_0^{\perp}$, for any $u \in S^1$, we have

$$R|u^{\perp} = R \cap u^{\perp}.$$

Since $R' \cap u\perp = (R \cap u^{\perp})^*$ for any $u \in S^1$, we get

$$R' \cap u^{\perp} = (R|u^{\perp})^*,$$

for any $u \in S^1$. By Lemma 1, we have $R' = R^*$. ■

Lemma 4 *If*

$$\frac{1}{p} + \frac{1}{q} = 1,$$

then the polar body of

$$U = \{(x, y) \| x|^p + |y|^p \le 1\}, \quad p \ge 1$$

is the following set:

$$U' = \{(x, y) \| x|^q + |y|^q \le 1\}, \quad q \ge 1. \tag{4.2}$$

Proof For any $(x, y) \in U$ and $(x', y') \in U'$, we have

$$xx' + yy' \le |xx'| + |yy'| \le \left(|x|^p + |y|^p\right)^{\frac{1}{p}} \left(|x'|^q + |y'|^q\right)^{\frac{1}{q}} \le 1,$$

which implies $U' \subset U^*$.

If a point $A' = (x', y') \notin U'$, then

$$|x'|^q + |y'|^q > 1.$$

Let $A_0' = (|x'|, |y'|)$, then $A_0' \notin U'$. There exists a real $r > 1$ and a point $A^0 \in \partial U'$ satisfying $A_0' = rA^0$. If $A^0 = (x_0, y_0)$, then $x_0 > 0$ and $y_0 > 0$. Let $x = x_0^{\frac{q}{p}}$ and $y = y_0^{\frac{q}{p}}$, then

$$x^p + y^p = x_0^q + y_0^q = 1$$

and

$$xx_0 + yy_0 = x_0^{1+q/p} + y_0^{1+q/p} = x_0^q + y_0^q = 1,$$

which implies $(x, y) \in U$ and $\langle (x, y), (|x'|, |y'|)\rangle = r > 1$, thus $A_0' \notin U^*$. Because that U^* is a 1-unconditional convex body, we have $A' \notin U^*$. Then, $U^* \subset U'$. ■

Rotating U and U', we can get two bodies of revolution R and R'. By Lemma 3, we have $R' = R^*$. Let $F(p) = V(R)V(R^*)$.

In the following, we restate and prove Theorem 2.

Theorem 2 *For a class of bodies of revolution obtained by rotating the "unit disk" in planar XOY, where the "unit disk" is the following set:*

$$U = \{(x, y) \,|\, |x|^p + |y|^p \le 1\}, \quad p \ge 1, \tag{4.3}$$

the Mahler volume is increasing for $1 \le p \le 2$ *and decreasing for* $2 \le p \le +\infty$.

Proof By integration, we get $V_R(p)$ and $V_{R^*}(q)$, which are volume functions of R and R^* about p and q as following:

$$V_R(p) = 2\pi \int_0^1 (1 - x^p)^{\frac{2}{p}} dx, \quad p \ge 1,$$

and

$$V_{R^*}(q) = 2\pi \int_0^1 (1 - x^q)^{\frac{2}{q}} dx, \quad \frac{1}{p} + \frac{1}{q} = 1.$$

Thus, we have the Mahler volume $V(R)V(R^*)$, which is a function about p as following:

$$F(p) = V_R(p)V_{R^*}(q) = 4\pi^2 \int_0^1 (1 - x^p)^{\frac{2}{p}} dx \int_0^1 (1 - x^q)^{\frac{2}{q}} dx, \tag{4.4}$$

where $p \ge 1$, and $\frac{1}{p} + \frac{1}{q} = 1$.

Let $1 - x^p = y$, we have

$$\int_0^1 (1 - x^p)^{\frac{2}{p}} dx = \frac{1}{p} \int_0^1 y^{\frac{2}{p}} (1 - y)^{\frac{1}{p} - 1} dy = \frac{1}{p} B\left(\frac{2}{p} + 1, \frac{1}{p}\right),$$

where $B(\cdot, \cdot)$ is Beta function. Thus we have

$$F(p) = \frac{4\pi^2}{pq} B\left(\frac{2}{p} + 1, \frac{1}{p}\right) B\left(\frac{2}{q} + 1, \frac{1}{q}\right),$$

where $p \ge 1$, and $\frac{1}{p} + \frac{1}{q} = 1$.

By the relationship between Gamma function and Beta function:

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)},$$

we have

$$F(p) = \frac{4\pi^2}{pq} \cdot \frac{\Gamma(\frac{2}{p} + 1)\Gamma(\frac{1}{p})}{\Gamma(\frac{3}{p} + 1)} \cdot \frac{\Gamma(\frac{2}{q} + 1)\Gamma(\frac{1}{q})}{\Gamma(\frac{3}{p} + 1)}.$$

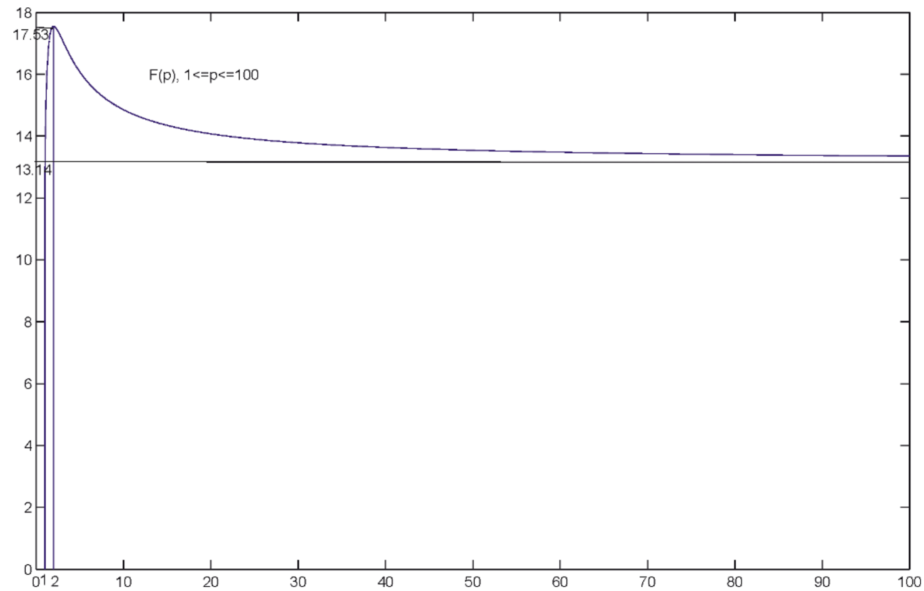

Figure 2 The figure of the function *F*(*p*).

And by the following properties of Gamma function:

$$\Gamma(z+1) = z\Gamma(z) \text{ and } \Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)},$$

we have

$$F(p) = \frac{16\pi^3}{9} \cdot \frac{(p-1)(p-2)}{p(2p-3)(p-3)} \cdot \frac{\sin(\frac{3\pi}{p})}{\sin(\frac{2\pi}{p})\sin(\frac{\pi}{p})}, \quad p \geq 1.$$

We can easily prove

$$\lim_{p\to 1} F(p) = \lim_{p\to +\infty} F(p) = \frac{4\pi^2}{3}, \tag{4.5}$$

then R and R^* are bicone and cylinder, or cylinder and bicone, and

$$\lim_{p\to 2} F(p) = \frac{16\pi^2}{9}, \tag{4.6}$$

then R and R^* are the same unit ball, which have the maximal Mahler volume.

In fact, $F(p) = F(q)$ holds when $\frac{1}{p} + \frac{1}{q} = 1$, so we just need to prove $F(p)$ is increasing when $1 \leq p \leq 2$, which can be easily proved by $F'(p) \geq 0$ when $1 \leq p \leq 2$. Based on the above conclusions, we have that a cylinder has the minimal Mahler volume and a ball has the maximal Mahler volume in this special class of bodies of revolution. ∎

We can draw the figure of the function $F(p)$ by using MATLAB (see Figure 2). From the figure, we see that function $F(p)$ is increasing when $1 \leq p \leq 2$ and decreasing when $2 \leq p \leq +\infty$, so $F(2)$ is a maximum and $F(1) = F(+\infty)$ is a minimum.

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Competing interests
The author declares that they have no competing interests.

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