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On some A_I-convergent difference sequence spaces of fuzzy numbers defined by the sequence of Orlicz functions

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Abstract
In this paper, using the difference operator of order *m*, the sequences of Orlicz functions, and an infinite matrix, we introduce and examine some classes of sequences of fuzzy numbers defined by *I*-convergence. We study some basic topological and algebraic properties of these spaces. In addition, we shall establish inclusion theorems between these sequence spaces.

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Keywords: ideal; *I*-convergent; infinite matrix; Orlicz function; fuzzy number; difference space

1 Introduction

The notion of ideal convergence was introduced first by Kostyrko *et al.* [1] as a generalization of statistical convergence [2, 3], which was further studied in topological spaces [4]. More applications of ideals can be seen in [5–9].

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [10]. Subsequently, several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy ordering, fuzzy measures of fuzzy events, and fuzzy mathematical programming. In particular, the concept of fuzzy topology has very important applications in quantum particle physics, especially in connection with both string and ε^∞ theory, which were given and studied by El Naschie [11]. The theory of sequences of fuzzy numbers was first introduced by Matloka [12]. Matloka introduced bounded and convergent sequences of fuzzy numbers, studied some of their properties, and showed that every convergent sequence of fuzzy numbers is bounded. In [13], Nanda studied sequences of fuzzy numbers and showed that the set of all convergent sequences of fuzzy numbers forms a complete metric space. Different classes of sequences of fuzzy real numbers have been discussed by Nuray and Savas [14], Altinok, Colak, and Et [15], Savas [16–20], Savas and Mursaleen [21], and many others.

The study of Orlicz sequence spaces was initiated with a certain specific purpose in Banach space theory. Lindenstrauss and Tzafriri [22] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space l_M contains a subspace isomorphic to l_p $(1 \leq p < \infty)$. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in [23]. Orlicz spaces find a number of useful applications in the theory of nonlinear integral equations. Although the Orlicz sequence spaces are the generalization of l_p spaces, the l_p-spaces find themselves enveloped in Orlicz spaces [24]. Recently, Savas [19] generalized $c(\Delta)$ and $l_\infty(\Delta)$ for a single sequence of fuzzy numbers by using the Orlicz function and also established some inclusion theorems.

In the later stage, different classes of Orlicz sequence spaces were introduced and studied by Parashar and Choudhary [25], Savas [26–29], and many others.

Throughout the article, w^F denotes the class of all fuzzy real-valued sequence spaces. Also, $\mathbf{N}$ and $\mathbf{R}$ denote the set of positive integers and the set of real numbers, respectively.

The operator $\Delta^n : w_F \to w_F$ is defined by $(\Delta^0 X)_k = X_k$; $(\Delta^1 X)_k = \Delta X_k = X_k - X_{k+1}$; $(\Delta^n X)_k = \Delta^n X_k = \Delta^{n-1} X_k - \Delta^{n-1} X_{k+1}$ $(n \geq 2)$ for all $n \in \mathbf{N}$. The generalized difference has the following binomial expression for $n \geq 0$:

$$\Delta^n x_k = \sum_{\nu=0}^{n} \binom{n}{\nu} (-1)^\nu x_{k+\nu}. \tag{1}$$

In this paper, we study some new sequence spaces of fuzzy numbers using I-convergence, the sequence of Orlicz functions, an infinite matrix, and the difference operator. We establish the inclusion relation between the sequence spaces $w^{I(F)}[A, \mathbf{M}, \Delta^m, p]$, $w^{I(F)}[A, \mathbf{M}, \Delta^m, p]_0$, $w^F[A, \mathbf{M}, \Delta^m, p]_\infty$, and $w^{I(F)}[A, \mathbf{M}, \Delta^m, p]_\infty$, where $p = (p_k)$ denotes the sequence of positive real numbers for all $k \in \mathbf{N}$ and $\mathbf{M} = (M_k)$ is a sequence of Orlicz functions. In addition, we study some algebraic and topological properties of these new spaces.

2 Definitions and notations

Before continuing with this paper, we present some definitions and preliminaries which we shall use throughout this paper.

Let X and Y be two nonempty subsets of the space w of complex sequences. Let $A = (a_{nk})$ $(n, k = 1, 2, \ldots)$ be an infinite matrix of complex numbers. We write $Ax = (A_n(x))$ if $A_n(x) = \sum_k a_{nk} x_k$ converges for each n. (Throughout, $\sum_k$ denotes summation over k from $k = 1$ to $k = \infty$). If $x = (x_k) \in X \Rightarrow Ax = (A_n(x)) \in Y$, we say that A defines a (matrix) transformation from X to Y and we denote it by $A : X \to Y$.

Let X be a nonempty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an *ideal* if and only if for each $A, B \in I$, we have $A \cup B \in I$, and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A nonempty family of sets $F \subset 2^X$ is a *filter* on X if and only if $\Phi \notin F$, for each $A, B \in F$, we have $A \cap B \in F$, and for each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called *non-trivial* ideal if $I \neq \Phi$ and $X \notin I$. Clearly, $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X. A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $\{\{x\} : x \in X\} \subset I$. A non-trivial ideal I is maximal if there cannot exist any non-trivial ideal $J \neq I$ containing I as a subset. Further details on ideals of 2^X can be found in Kostyrko *et al.* [1].

Let D denote the set of all closed and bounded intervals $X = [x_1, x_2]$ on the real line $\mathbf{R}$. For $X, Y \in D$, we define $X \leq Y$ if and only if $x_1 \leq y_1$ and $x_2 \leq y_2$,

$$d(X, Y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}, \quad \text{where } X = [x_1, x_2] \text{ and } Y = [y_1, y_2].$$

Then it can be easily seen that d defines a metric on D and (D, d) is a complete metric space (see [30]). Also, the relation '$\leq$' is a partial order on D. A fuzzy number X is a fuzzy subset of the real line $\mathbf{R}$, *i.e.*, a mapping $X : \mathbf{R} \to J$ $(= [0, 1])$ associating each real number

t with its grade of membership $X(t)$. A fuzzy number X is *convex* if $X(t) \geq X(s) \wedge X(r) = \min\{X(s), X(r)\}$, where $s < t < r$. If there exists $t_0 \in \mathbf{R}$ such that $X(t_0) = 1$, then the fuzzy number X is called *normal*. A fuzzy number X is said to be upper semicontinuous if for each $\epsilon > 0$, $X^{-1}([0, a + \epsilon))$ for all $a \in [0,1]$ is open in the usual topology in $\mathbf{R}$. Let $\mathbf{R}(J)$ denote the set of all fuzzy numbers which are upper semicontinuous and have compact support, *i.e.*, if $X \in \mathbf{R}(J)$, then for any $\alpha \in [0,1]$, $[X]^\alpha$ is compact, where

$$[X]^\alpha = \{t \in \mathbf{R} : X(t) \geq \alpha, \text{if } \alpha \in [0,1]\},$$
$$[X]^0 = \text{closure of } (\{t \in \mathbf{R} : X(t) > \alpha, \text{if } \alpha = 0\}).$$

The set $\mathbf{R}$ of real numbers can be embedded in $\mathbf{R}(J)$ if we define $\overline{r} \in \mathbf{R}(J)$ by

$$\overline{r}(t) = \begin{cases} 1, & \text{if } t = r; \\ 0, & \text{if } t \neq r. \end{cases}$$

The additive identity and multiplicative identity of $\mathbf{R}(J)$ are defined by $\overline{0}$ and $\overline{1}$ respectively.

The arithmetic operations on $\mathbf{R}(J)$ are defined as follows:

$$(X \oplus Y)(t) = \sup\{X(s) \wedge Y(t-s)\}, \quad t \in \mathbf{R},$$
$$(X \ominus Y)(t) = \sup\{X(s) \wedge Y(s-t)\}, \quad t \in \mathbf{R},$$
$$(X \otimes Y)(t) = \sup\left\{X(s) \wedge Y\left(\frac{t}{s}\right)\right\}, \quad t \in \mathbf{R},$$
$$\left(\frac{X}{Y}\right)(t) = \sup\{X(st) \wedge Y(s)\}, \quad t \in \mathbf{R}.$$

Let $X, Y \in \mathbf{R}(J)$ and the α-level sets be $[X]^\alpha = [x_1^\alpha, x_2^\alpha]$, $[Y]^\alpha = [y_1^\alpha, y_2^\alpha]$, $\alpha \in [0,1]$. Then the above operations can be defined in terms of α-level sets as follows:

$$[X \oplus Y]^\alpha = [x_1^\alpha + y_1^\alpha, x_2^\alpha + y_2^\alpha],$$
$$[X \ominus Y]^\alpha = [x_1^\alpha - y_1^\alpha, x_2^\alpha - y_2^\alpha],$$
$$[X \otimes Y]^\alpha = \left[\min_{i \in \{1,2\}} x_i^\alpha y_i^\alpha, \max_{i \in \{1,2\}} x_i^\alpha y_i^\alpha\right],$$
$$[X^{-1}]^\alpha = [(x_2^\alpha)^{-1}, (x_1^\alpha)^{-1}], \quad x_i^\alpha > 0, \text{ for each } 0 < \alpha \leq 1.$$

For $r \in \mathbf{R}$ and $X \in \mathbf{R}(J)$, the product rX is defined as follows:

$$rX(t) = \begin{cases} X(r^{-1}t), & \text{if } r \neq 0; \\ 0, & \text{if } r = 0. \end{cases}$$

The absolute value $|X|$ of $X \in \mathbf{R}(J)$ is defined by

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if } t \geq 0; \\ 0, & \text{if } t < 0. \end{cases}$$

Define a mapping $\bar{d} : \mathbf{R}(J) \times \mathbf{R}(J) \to \mathbf{R}^+ \cup \{0\}$ by

$$\bar{d}(X, Y) = \sup_{0 \le \alpha \le 1} d\big([X]^\alpha, [Y]^\alpha\big).$$

A metric $\bar{d}$ on $\mathbf{R}(J)$ is said to be a translation invariant if $\bar{d}(X + Z, Y + Z) = \bar{d}(X, Y)$ for $X, Y, Z \in \mathbf{R}(J)$.

Proposition 2.1 *If $\bar{d}$ is a translation invariant metric on $\mathbf{R}(J)$, then*
(i) $\bar{d}(X + Z, 0) \le \bar{d}(X, 0) + \bar{d}(Y, 0)$,
(ii) $\bar{d}(\lambda X, 0) \le |\lambda| \bar{d}(X, 0)$, $|\lambda| > 1$.

The proof is easy and so it is omitted.

A sequence $X = (X_k)$ of fuzzy numbers is said to *converge* to a fuzzy number X_0 if for every $\epsilon > 0$, there exists a positive integer n_0 such that $\bar{d}(X_k, X_0) < \epsilon$ for all $n \ge n_0$.

A sequence $X = (X_k)$ of fuzzy numbers is said to be *bounded* if the set $\{X_k : k \in \mathbf{N}\}$ of fuzzy numbers is bounded. A sequence $X = (X_k)$ of fuzzy numbers is said to be I-convergent to a fuzzy number X_0 if for each $\epsilon > 0$ such that

$$A = \big\{k \in \mathbf{N} : \bar{d}(X_k, X_0) \ge \epsilon\big\} \in I.$$

The fuzzy number X_0 is called I-limit of the sequence (X_k) of fuzzy numbers, and we write $I\text{-}\lim X_k = X_0$.

A sequence $X = (X_k)$ of fuzzy numbers is said to be I-bounded if there exists $M > 0$ such that

$$\big\{k \in \mathbf{N} : \bar{d}(X_k, \bar{0}) > M\big\} \in I.$$

Example 2.1 If we take $I = I_f = \{A \subseteq \mathbf{N} : A \text{ is a finite subset}\}$, then I_f is a non-trivial admissible ideal of $\mathbf{N}$, and the corresponding convergence coincides with the usual convergence.

Example 2.2 If we take $I = I_\delta = \{A \subseteq \mathbf{N} : \delta(A) = 0\}$, where $\delta(A)$ denotes the asymptotic density of the set A, then I_δ is a non-trivial admissible ideal of $\mathbf{N}$, and the corresponding convergence coincides with the statistical convergence.

Lemma 2.1 (Kostyrko, Salat, and Wilczynski [1], Lemma 5.1) *If $I \subset 2^{\mathbf{N}}$ is a maximal ideal, then for each $A \subset \mathbf{N}$, we have either $A \in I$ or $\mathbf{N} - A \in I$.*

Recall in [23] that the Orlicz function $M : [0, \infty) \to [0, \infty)$ is a continuous, convex, nondecreasing function such that $M(0) = 0$ and $M(x) > 0$ for $x > 0$, and $M(x) \to \infty$ as $x \to \infty$. If convexity of the Orlicz function is replaced by $M(x+y) \le M(x)+M(y)$, then this function is called the modulus function and characterized by Ruckle [31]. An Orlicz function M is said to satisfy the Δ_2-condition for all values of u, if there exists $K > 0$ such that $M(2u) \le KM(u)$, $u \ge 0$.

The following well-known inequality will be used throughout the article. Let $p = (p_k)$ be any sequence of positive real numbers with $0 \le p_k \le \sup_k p_k = G$, $H = \max\{1, 2^{G-1}\}$, then

$$|a_k + b_k|^{p_k} \le H\big(|a_k|^{p_k} + |b_k|^{p_k}\big)$$

for all $k \in \mathbf{N}$ and $a_k, b_k \in \mathbf{C}$. Also, $|a_k|^{p_k} \le \max\{1, |a|^G\}$ for all $a \in \mathbf{C}$.

3 Some new sequence spaces of fuzzy numbers

In this section, using the sequence of Orlicz functions, an infinite matrix, the difference operator Δ^m, and I-convergence, we introduce the following new sequence spaces and examine some properties of the resulting sequence spaces. Let I be an admissible ideal of $\mathbf{N}$, and let $p = (p_k)$ be a sequence of positive real numbers for all $k \in \mathbf{N}$ and $A = (a_{nk})$ be an infinite matrix. Let $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions and $X = (X_k)$ be a sequence of fuzzy numbers. We define the following new sequence spaces:

$$\begin{aligned} &w^{I(F)}\bigl[A, \mathbf{M}, \Delta^m, p\bigr] \\ &\quad = \Biggl\{ (X_k) \in w^F : \forall \varepsilon > 0, \Biggl\{ n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk} \biggl[M_k \biggl(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho} \biggr) \biggr]^{p_k} \geq \varepsilon \Biggr\} \in I, \\ &\qquad \text{for some } \rho > 0 \text{ and } X_0 \in \mathbf{R}(J) \Biggr\}, \\ &w^{I(F)}\bigl[A, \mathbf{M}, \Delta^m, p\bigr]_0 \\ &\quad = \Biggl\{ (X_k) \in w^F : \forall \varepsilon > 0, \Biggl\{ n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk} \biggl[M_k \biggl(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho} \biggr) \biggr]^{p_k} \geq \varepsilon \Biggr\} \in I, \\ &\qquad \text{for some } \rho > 0 \Biggr\}, \\ &w^{F}\bigl[A, \mathbf{M}, \Delta^m, p\bigr]_\infty \\ &\quad = \Biggl\{ (X_k) \in w^F : \sup_n \sum_{k=1}^{\infty} a_{nk} \biggl[M_k \biggl(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho} \biggr) \biggr]^{p_k} < \infty, \text{for some } \rho > 0 \Biggr\}, \end{aligned}$$

and

$$\begin{aligned} &w^{I(F)}\bigl[A, \mathbf{M}, \Delta^m, p\bigr]_\infty \\ &\quad = \Biggl\{ (X_k) \in w^F : \exists K > 0 \text{ s.t. } \Biggl\{ n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk} \biggl[M_k \biggl(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho} \biggr) \biggr]^{p_k} \geq K \Biggr\} \in I, \\ &\qquad \text{for some } \rho > 0 \Biggr\}. \end{aligned}$$

Let us consider a few special cases of the above sets.

(i) If $m = 0$, then the above classes of sequences are denoted by $w^{I(F)}[A, \mathbf{M}, p]$, $w^{I(F)}[A, \mathbf{M}, p]_0$, $w^F[A, \mathbf{M}, p]_\infty$, and $w^{I(F)}[A, \mathbf{M}, p]_\infty$, respectively.

(ii) If $M_k(x) = x$ for all $k \in \mathbf{N}$, then the above classes of sequences are denoted by $w^{I(F)}[A, \Delta^m, p]$, $w^{I(F)}[A, \Delta^m, p]_0$, $w^F[A, \Delta^m, p]_\infty$, and $w^{I(F)}[A, \Delta^m, p]_\infty$, respectively.

(iii) If $p = (p_k) = (1, 1, 1, \ldots)$, then we denote the above spaces by $w^{I(F)}[A, \mathbf{M}, \Delta^m]$, $w^{I(F)}[A, \mathbf{M}, \Delta^m]_0$, $w^F[A, \mathbf{M}, \Delta^m]_\infty$, and $w^{I(F)}[A, \mathbf{M}, \Delta^m]_\infty$.

(iv) If we take $A = (C, 1)$, *i.e.*, the Cesàro matrix, then the above classes of sequences are denoted by $w^{I(F)}[\mathbf{M}, \Delta^m, p]$, $w^{I(F)}[\mathbf{M}, \Delta^m, p]_0$, $w^F[\mathbf{M}, \Delta^m, p]_\infty$, and $w^{I(F)}[\mathbf{M}, \Delta^m, p]_\infty$, respectively.

(v) If we take $A = (a_{nk})$ is a de la Valée Poussin mean, *i.e.*,

$$a_{nk} = \begin{cases} \frac{1}{\lambda_n}, & \text{if } k \in I_n = [n - \lambda_n + 1, n], \\ 0, & \text{otherwise,} \end{cases}$$

where (λ_n) is a non-decreasing sequence of positive numbers tending to ∞ and $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$, then the above classes of sequences are denoted by $w_\lambda^{I(F)}[A, \mathbf{M}, \Delta^m, p]$, $w_\lambda^{I(F)}[\mathbf{M}, \Delta^m, p]_0$, $w_\lambda^F[\mathbf{M}, \Delta^m, p]_\infty$, and $w_\lambda^{I(F)}[\mathbf{M}, \Delta^m, p]_\infty$, respectively (see [32]).

(vi) By a lacunary $\theta = (k_r)$, $r = 0, 1, 2, \dots$ where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1}$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and $h_r = k_r - k_{r-1}$. As a final illustration, let

$$a_{nk} = \begin{cases} \frac{1}{h_r}, & \text{if } k_{r-1} < k \leq k_r, \\ 0, & \text{otherwise.} \end{cases}$$

Then we denote the above classes of sequences by $w_\theta^{I(F)}[\mathbf{M}, \Delta^m, p]$, $w_\theta^{I(F)}[\mathbf{M}, \Delta^m, p]_0$, $w_\theta^F[\mathbf{M}, \Delta^m, p]_\infty$, and $w_\theta^{I(F)}[\mathbf{M}, \Delta^m, p]_\infty$, respectively.

(vii) If $I = I_f$, then we obtain

$$\begin{aligned} & w^F\big[A, \mathbf{M}, \Delta^m, p\big] \\ & \quad = \left\{ (X_k) \in w^F : \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho} \right) \right]^{p_k} = 0, \right. \\ & \qquad \left. \text{for some } \rho > 0 \text{ and } X_0 \in \mathbf{R}(J) \right\}, \\ & w^F\big[A, \mathbf{M}, \Delta^m, p\big]_0 \\ & \quad = \left\{ (X_k) \in w^F : \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho} \right) \right]^{p_k} = 0, \text{for some } \rho > 0 \right\}, \\ & w^F\big[A, \mathbf{M}, \Delta^m, p\big]_\infty \\ & \quad = \left\{ (X_k) \in w^F : \sup_n \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho} \right) \right]^{p_k} < \infty, \text{for some } \rho > 0 \right\}. \end{aligned}$$

If $X = (X_k) \in w^F[A, \mathbf{M}, \Delta^m, p]$, then we say that $X = (X_k)$ is strongly A-convergent with respect to the sequence of Orlicz functions $\mathbf{M}$.

(viii) If $I = I_\delta$ is an admissible ideal of $\mathbf{N}$, then we obtain

$$\begin{aligned} & w^{I(F)}\big[A, \mathbf{M}, \Delta^m, p\big] \\ & \quad = \left\{ (X_k) \in w^F : \forall \varepsilon > 0, \left\{ n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\} \in I_\delta, \right. \\ & \qquad \left. \text{for some } \rho > 0 \text{ and } X_0 \in \mathbf{R}(J) \right\}, \end{aligned}$$

$$w^{I(F)}\big[A,\mathbf{M},\Delta^m,p\big]_0 = \left\{(X_k)\in w^F:\forall\varepsilon>0, \left\{n\in\mathbf{N}:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}\in I_\delta, \text{ for some } \rho>0\right\},$$

and

$$w^{I(F)}\big[A,\mathbf{M},\Delta^m,p\big]_\infty = \left\{(X_k)\in w^F:\exists K>0 \text{ s.t. } \left\{n\in\mathbf{N}:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]^{p_k}\geq K\right\}\in I_\delta, \text{for some } \rho>0\right\}.$$

4 Main results

In this section, we examine the basic topological and algebraic properties of the new sequence spaces and obtain the inclusion relation related to these spaces.

Theorem 4.1 *Let (p_k) be a bounded sequence. Then the sequence spaces $w^{I(F)}[A,\mathbf{M},\Delta^m,p]$, $w^{I(F)}[A,\mathbf{M},\Delta^m,p]_0$, and $w^{I(F)}[A,\mathbf{M},\Delta^m,p]_\infty$ are linear spaces.*

Proof We will prove the result for the space $w_\theta^{I(F)}[\mathbf{M},\Delta^m,p]_0$ only and others can be proved in a similar way.

Let $X=(X_k)$ and $Y=(Y_k)$ be two elements in $w_\theta^{I(F)}[\mathbf{M},\Delta^m,p]_0$. Then there exist $\rho_1>0$ and $\rho_2>0$ such that

$$A_{\frac{\varepsilon}{2}} = \left\{r\in\mathbf{N}:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho_1}\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}\in I$$

and

$$B_{\frac{\varepsilon}{2}} = \left\{r\in\mathbf{N}:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m Y_k,\bar{0})}{\rho_2}\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}\in I.$$

Let α, β be two scalars. By the continuity of the function $\mathbf{M}=(M_k)$, the following inequality holds:

$$\begin{aligned}&\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m(\alpha X_k+\beta Y_k,\bar{0}))}{|\alpha|\rho_1+|\beta|\rho_2}\right)\right]^{p_k}\\ &\quad\leq D\sum_{k=1}^{\infty}a_{nk}\left[\frac{|\alpha|}{|\alpha|\rho_1+|\beta|\rho_2}M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho_1}\right)\right]^{p_k}\\ &\qquad+D\sum_{k=1}^{\infty}a_{nk}\left[\frac{|\beta|}{|\alpha|\rho_1+|\beta|\rho_2}M_k\left(\frac{\bar{d}(\Delta^m Y_k,\bar{0})}{\rho_2}\right)\right]^{p_k}\end{aligned}$$

$$\begin{aligned}&\leq DK\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho_1}\right)\right]^{p_k}\\&\quad+DK\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m Y_k,\bar{0})}{\rho_2}\right)\right]^{p_k},\end{aligned}$$

where $K=\max\{1,(\frac{|\alpha|}{|\alpha|\rho_1+|\beta|\rho_2})^G,(\frac{|\alpha|}{|\beta|\rho_1+|\beta|\rho_2})^G\}$.

From the above relation, we obtain the following:

$$\begin{aligned}&\left\{n\in\mathbf{N}:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m(\alpha X_k+\beta Y_k,\bar{0}))}{|\alpha|\rho_1+|\beta|\rho_2}\right)\right]^{p_k}\geq\varepsilon\right\}\\&\quad\subseteq\left\{n\in\mathbf{N}:DK\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho_1}\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}\\&\quad\cup\left\{n\in\mathbf{N}:DK\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m Y_k,\bar{0})}{\rho_2}\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}\in I.\end{aligned}$$

This completes the proof. □

Theorem 4.2 *$w^{I(F)}[A,\mathbf{M},\Delta^m,p]$, $w^{I(F)}[A,\mathbf{M},\Delta^m,p]_0$, and $w^{I(F)}[A,\mathbf{M},\Delta^m,p]_\infty$ are linear topological spaces with the paranorm g_Δ defined by*

$$\begin{aligned}g_\Delta(X)=\inf\Bigg\{&\rho^{\frac{p_n}{H}}:\left(\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]^{p_k}\right)^{1/H}\leq 1,\\&\textit{for some }\rho>0,n=1,2,3,\ldots\Bigg\},\end{aligned}$$

where $H=\max\{1,\sup_k p_k\}$.

Proof Clearly, $g_\Delta(-X)=g_\Delta(X)$ and $g_\Delta(\theta)=0$. Let $X=(X_k)$ and $Y=(Y_k)$ be two elements in $w^{I(F)}[A,\mathbf{M},\Delta^m,p]_\infty$. Then for every $\rho>0$, we write

$$A_1=\left\{\rho>0:\left(\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]^{p_k}\right)^{\frac{1}{H}}\leq 1\right\}$$

and

$$A_2=\left\{\rho>0:\left(\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]^{p_k}\right)^{\frac{1}{H}}\leq 1\right\}.$$

Let $\rho_1\in A_1$ and $\rho_2\in A_2$. If $\rho=\rho_1+\rho_2$, then we get the following:

$$\begin{aligned}\left(\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]\right)&\leq\frac{\rho_2}{\rho_1+\rho_2}\left(\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]\right)\\&\quad+\frac{\rho_2}{\rho_1+\rho_2}\left(\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k,\bar{0})}{\rho}\right)\right]\right).\end{aligned}$$

Hence, we obtain

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho}\right)\right]^{p_k} \leq 1$$

and

$$\begin{aligned} g(x+y) &= \inf\left\{(\rho_1+\rho_2)^{\frac{p_n}{H}} : \rho_1 \in A_1, \rho_2 \in A_2\right\} \\ &\leq \inf\left\{(\rho_1)^{\frac{p_n}{H}} : \rho_1 \in A_1\right\} + \inf\left\{(\rho_2)^{\frac{p_n}{H}} : \rho_2 \in A_2\right\} = g(x) + g(y). \end{aligned}$$ □

Let $u_k^m \to t$, where $u_k^m, u \in \mathbb{C}$, and let $g(X_k^m - X_k) \to 0$ as $m \to \infty$. To prove that $g(u_k^m X_k^m - uX_k) \to 0$ as $m \to \infty$, let $u_k \to u$, where $u_k, u \in \mathbb{C}$ and $g(X_k^m - X_k) \to 0$ as $m \to \infty$.

We have

$$A_3 = \left\{\rho_k > 0 : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho_k}\right)\right]^{p_k} \leq 1\right\}$$

and

$$A_4 = \left\{\rho'_k > 0 : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, \bar{0})}{\rho'_k}\right)\right]^{p_k} \leq 1\right\}.$$

If $\rho_k \in A_3$ and $\rho'_k \in A_4$ and by continuity of the function $\mathbf{M} = M_k$, we have that

$$\begin{aligned} &M_k\left(\frac{\bar{d}(\Delta^m(u^m X_k^m - uX), \bar{0})}{|u^m - u|\rho_k + |u|\rho'_k}\right) \\ &\quad\leq M_k\left(\frac{\bar{d}(\Delta^m(u^m X_k^m - uX_k), \bar{0})}{|u^m - u|\rho_k + |u|\rho'_k}\right) + M_k\left(\frac{\bar{d}(\Delta^m(uX_k - uX), \bar{0})}{|u^m - u|\rho_k + |u|\rho'_k}\right) \\ &\quad\leq \frac{|u^m - u|\rho_k}{|u^m - u|\rho_k + |u|\rho'_k} M_k\left(\frac{\bar{d}(\Delta^m X_k^m, \bar{0})}{\rho_k}\right) \\ &\qquad + \frac{|u|\rho'_k}{|u^m - u|\rho_k + |u|\rho'_k} M_k\left(\frac{\bar{d}(\Delta^m(X_k^m - X_k), \bar{0})}{\rho'_k}\right). \end{aligned}$$

From the above inequality, it follows that

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m(u^m X_k^m - uX), \bar{0})}{|u^m - u|\rho_k + |u|\rho'_k}\right)\right]^{p_k} \leq 1,$$

and consequently

$$\begin{aligned} g\left(u_k^m x_k - u x_k\right) &= \inf\left\{\left(\left|u_k^m - u\right|\rho_k + |u|\rho'_k\right)^{\frac{p_n}{G}} : \rho_k \in A_3, \rho'_k \in A_4\right\} \\ &\leq \left|u_k^m - u\right|^{\frac{p_n}{G}} \inf\left\{(\rho_k)^{\frac{p_n}{G}} : \rho_k \in A_3\right\} + |u|^{\frac{p_n}{G}} \inf\left\{\left(\rho'_k\right)^{\frac{p_n}{G}} : \rho'_k \in A_4\right\} \\ &\leq \max\left\{|u|, |u|^{\frac{p_n}{G}}\right\} g\left(x_k^m - x_k\right). \end{aligned}$$

Note that $g(x^m) \leq g(x) + g(x^m - x)$ for all $m \in \mathbb{N}$.

Hence, by our assumption, the right-hand side tends to 0 as $m \to \infty$. This completes the proof of the theorem.

Theorem 4.3 *Let I be an admissible ideal and $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions. Then the following hold*:

$w^{I(F)}[A,\mathbf{M},\Delta^{m-1},p]_0 \subset w^{I(F)}[A,\mathbf{M},\Delta^m,p]_0$; $w^{I(F)}[A,\mathbf{M},\Delta^{m-1},p] \subset w^{I(F)}[A,\mathbf{M},\Delta^m,p]$;
$w^{I(F)}[A,\mathbf{M},\Delta^{m-1},p]_\infty \subset w^{I(F)}[A,\mathbf{M},\Delta^m,p]_\infty$ *for* $m \geq 1$ *and the inclusions are strict.*

In general, for all $i = 1,2,3,\dots,m-1$, the following hold:

$w^{I(F)}[A,\mathbf{M},\Delta^{i},p]_0 \subset w^{I(F)}[A,\mathbf{M},\Delta^m,p]_0$; $w^{I(F)}[A,\mathbf{M},\Delta^{i},p] \subset w^{I(F)}[A,\mathbf{M},\Delta^m,p]$;
$w^{I(F)}[A,\mathbf{M},\Delta^{i},p]_\infty \subset w^{I(F)}[A,\mathbf{M},\Delta^m,p]_\infty$ *and the inclusions are strict.*

Proof Let $X = (X_k)$ be an element in $w^{I(F)}[A,\mathbf{M},\Delta^{m-1},p]_\infty$. Then there exists $K > 0$, and for given $\varepsilon > 0$, $\rho > 0$, we have

$$\left\{n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m-1}X_k,\bar{0})}{\rho}\right)\right]^{p_k} \geq K\right\} \in I.$$

Since $\mathbf{M} = (M_k)$ is non-decreasing and convex, it follows that

$$\begin{aligned}
&\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m}X_k,\bar{0})}{2\rho}\right)\right]^{p_k} \\
&\quad\leq \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k+1}-\Delta^{m-1}X_k,\bar{0})}{2\rho}\right)\right]^{p_k} \\
&\quad\leq D\sum_{k=1}^{\infty} a_{nk}\left[\frac{1}{2}M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k+1},\bar{0})}{\rho}\right)\right]^{p_k} \\
&\qquad + D\sum_{k=1}^{\infty} a_{nk}\left[\frac{1}{2}M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k},\bar{0})}{\rho}\right)\right]^{p_k} \\
&\quad\leq D\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k+1},\bar{0})}{\rho}\right)\right]^{p_k} + D\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k},\bar{0})}{\rho}\right)\right]^{p_k}.
\end{aligned}$$

Hence, we have

$$\begin{aligned}
&\left\{n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m}X_k,\bar{0})}{2\rho}\right)\right]^{p_k} \geq K\right\} \\
&\quad\subseteq \left\{n \in \mathbf{N} : D\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k+1},\bar{0})}{\rho}\right)\right]^{p_k} \geq \frac{K}{2}\right\} \\
&\qquad\cup \left\{n \in \mathbf{N} : D\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^{m-1}X_{k},\bar{0})}{\rho}\right)\right]^{p_k} \geq \frac{K}{2}\right\} \in I.
\end{aligned}$$

□

The inclusion is strict, it follows from the following example.

Example 4.1 Let $M_k(x) = x$, $p_k = 1$ for all $k \in \mathbf{N}$ and $A = (C, 1)$, i.e., the Cesàro matrix, $m = 3$. Consider the sequence $X = (X_k)$ of fuzzy numbers as follows:

$$X_k(t) = \begin{cases} -\frac{t}{k^3-1} - 1, & \text{for } k^3 - 1 \le t \le 0; \\ -\frac{t}{k^3+1} - 1, & \text{for } 0 < t \le k^3 + 1; \\ 0, & \text{otherwise.} \end{cases}$$

For $\alpha \in (0, 1]$, α-level sets of X_k, ΔX_k, $\Delta^2 X_k$, and $\Delta^3 X_k$ are

$$\begin{aligned} [X_k]^\alpha &= \big[(1-\alpha)\big(k^3 - 1\big), (1-\alpha)\big(k^3 + 1\big)\big], \\ [\Delta X_k]^\alpha &= \big[(1-\alpha)\big(-3k^2 - 3k - 3\big), (1-\alpha)\big(-3k^2 - 3k + 1\big)\big], \\ \big[\Delta^2 X_k\big]^\alpha &= \big[(1-\alpha)(6k+2), (1-\alpha)(6k+10)\big], \\ \big[\Delta^3 X_k\big]^\alpha &= \big[-14(1-\alpha), 2(1-\alpha)\big], \end{aligned}$$

respectively. It is easy to see that the sequence $[\Delta^2 X_k]^\alpha$ is not I-bounded although $[\Delta^3 X_k]^\alpha$ is I-bounded.

Theorem 4.4 *(a) Let* $0 < \inf p_k \le p_k \le 1$. *Then*
$w^{I(F)}[A, \mathbf{M}, \Delta^m, p] \subseteq w^{I(F)}[A, \mathbf{M}, \Delta^m]$; $w^{I(F)}[A, \mathbf{M}, \Delta^m, p]_0 \subseteq w^{I(F)}[A, \mathbf{M}, \Delta^m]_0$.
(b) Let $1 \le p_k \le \sup p_k < \infty$. *Then*
$w^{I(F)}[A, \mathbf{M}, \Delta^m] \subseteq w^{I(F)}[A, \mathbf{M}, \Delta^m, p]$; $w^{I(F)}[A, \mathbf{M}, \Delta^m]_0 \subseteq w^{I(F)}[A, \mathbf{M}, \Delta^m, p]_0$.

Proof (a) Let $X = (X_k)$ be an element in $w^{I(F)}[A, \mathbf{M}, \Delta^m, p]$. Since $0 < \inf p_k \le p_k \le 1$, we have

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right] \le \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right]^{p_k}.$$

Therefore,

$$\begin{aligned} &\left\{n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right] \ge \varepsilon\right\} \\ &\quad \subseteq \left\{n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right]^{p_k} \ge \varepsilon\right\} \in I. \end{aligned}$$

The other part can be proved in a similar way.

(b) Let $X = (X_k)$ be an element in $w^{I(F)}[A, \mathbf{M}, \Delta^m, p]$. Since $1 \le p_k \le \sup p_k < \infty$, then for each $0 < \varepsilon < 1$, there exists a positive integer n_0 such that

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right] \le \varepsilon < 1 \quad \text{for all } n \ge n_0.$$

This implies that

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right]^{p_k} \le \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right].$$

Therefore, we have

$$\left\{n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\}$$
$$\subseteq \left\{n \in \mathbf{N} : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(\Delta^m X_k, X_0)}{\rho}\right)\right] \geq \varepsilon\right\} \in I. \qquad \square$$

The other part can be proved in a similar way.

The following corollary follows immediately from the above theorem.

Corollary 4.5 *Let $A = (C, 1)$, i.e., the Cesàro matrix, and $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions.*

(*a*) *Let $0 < \inf p_k \leq p_k \leq 1$. Then*
$w^{I(F)}[\mathbf{M}, \Delta^m, p] \subseteq w^{I(F)}[\mathbf{M}, \Delta^m]$; $w^{I(F)}[\mathbf{M}, \Delta^m, p]_0 \subseteq w^{I(F)}[\mathbf{M}, \Delta^m]_0$.

(*b*) *Let $1 \leq p_k \leq \sup p_k < \infty$. Then*
$w^{I(F)}[\mathbf{M}, \Delta^m] \subseteq w^{I(F)}[\mathbf{M}, \Delta^m, p]$; $w^{I(F)}[\mathbf{M}, \Delta^m]_0 \subseteq w^{I(F)}[\mathbf{M}, \Delta^m, p]_0$.

Competing interests

The author declares that they have no competing interests.

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