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Large deviations for random sums of differences between two sequences of random variables with applications to risk theory

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Abstract
This paper investigates some precise large deviations for the random sums of the differences between two sequences of independent and identically distributed random variables, where the minuend random variables have subexponential tails, and the subtrahend random variables have finite second moments. As applications to risk theory, the customer-arrival-based insurance risk model is considered, and some uniform asymptotics for the ruin probabilities of an insurance company are derived as the number of customers or the time tends to infinity.
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1 Introduction and main result

Throughout, let $\{X_k, k \geq 1\}$ be a sequence of independent and identically distributed (i.i.d.) nonnegative random variables (r.v.s) with a common distribution B, $\{Y_k, k \geq 1\}$ be also a sequence of i.i.d. nonnegative r.v.s. Denote the differences by $Z_k = X_k - Y_k$, $k \geq 1$, with a common distribution F and a finite mean $\mu < 0$. Let $\{N(t), t \geq 0\}$ be a nonnegative integer-valued process. We assume that $\{X_k, k \geq 1\}$, $\{Y_k, k \geq 1\}$ and $\{N(t), t \geq 0\}$ are mutually independent. Define a random walk process $S_n = \sum_{k=1}^{n} Z_k$, $n \geq 1$, by convention, $S_0 = 0$. In this paper, we are interested in the precise large deviations for the randomly index sums (random sums) $S_{N(t)}$ under the assumption that the distribution B is heavy tailed. A well-known notion in extremal value theory, the subexponentiality, describes an important property of the right tail of a distribution. The subexponential class of distributions, denoted by $\mathscr{S}$, is the most important class of heavy-tailed distributions. A distribution V on $[0, \infty)$ is said to belong to the class $\mathscr{S}$ if its tail $\overline{V} = 1 - V$ satisfies

$$\lim_{x \to \infty} \frac{\overline{V^{n*}}(x)}{\overline{V}(x)} = n$$

for some (or, equivalently, for all) $n \geq 2$, where V^{n*} denotes the n-fold convolution of V. A related class is the dominatedly-varying-tailed distribution class denoted by $\mathscr{D}$. A distribution V on $(-\infty,\infty)$ is said to belong to the class $\mathscr{D}$ if for any $0 < y < 1$,

$$\limsup_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)} < \infty.$$

Furthermore, for the distribution V on $(-\infty,\infty)$, denote its upper Matuszewska index by

$$J_V^+ = -\lim_{y\to\infty} \frac{\log \overline{V}_*(y)}{\log y} \quad \text{with } \overline{V}_*(y) := \liminf_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)} \text{ for } y > 1.$$

Precise large deviation probabilities for random sums have been extensively investigated by many researchers who have mainly concentrated on the sequence of nonnegative r.v.s, whose distributions belong to some subclasses of the classes $\mathscr{S}$ and $\mathscr{D}$. Klüppelberg and Mikosch [1] dealt with the case where the distribution B is extended-regularly-varying-tailed, and Tang *et al.* [2] improved this result with some weaker conditions on the process $N(t)$. Later, Ng *et al.* [3] investigated a more general case where B has a consistently varying tail. For some further works, one can refer to Liu [4], Wang *et al.* [5], Yang and Wang [6] among others. We remark that Baltrūnas *et al.* [7] obtained an important equivalently precise large deviations result for the random sums of nonnegative subexponential r.v.s. For the case of real-valued r.v.s, Yang and Wang [8] derived some similar results for the real-valued r.v.s or the differences of two nonnegative r.v.s with dominatedly-varying-tailed distributions; however, their results are weakly equivalent and the process $N(t)$ is restricted to some renewal counting process. Recently, Chen and Zhang [9] considered the case of dependent real-valued r.v.s with consistently varying tails and the process $N(t)$ satisfying the condition

$$\mathbb{E}\big(N(t)\big)^p \mathbf{1}_{\{N(t)>(1+\delta)\lambda(t)\}} = o\big(\lambda(t)\big), \quad t\to\infty \tag{1.1}$$

for some $p > J_F^+$ and any $\delta > 0$, where $\lambda(t) := \mathbb{E}N(t)$ is assumed to tend to ∞ as $t\to\infty$. Motivated by the above contributions, in this paper we aim to establish some precise large deviations results, which are some equivalent relations, for the random sums of the differences between two sequences of nonnegative r.v.s under a mild condition on the process $N(t)$:

$$\frac{N(t)}{\lambda(t)} \xrightarrow{\mathbb{P}} 1, \quad t\to\infty, \tag{1.2}$$

and

$$\lambda(t) \sim \lambda t, \quad t\to\infty, \tag{1.3}$$

for some $\lambda > 0$. Clearly, the condition (1.1) implies (1.2); see, *e.g.*, Tang *et al.* [2].

Hereafter, all limit relationships hold for t tending to ∞ unless stated otherwise. For two positive functions $a(t)$ and $b(t)$, we write $a(t) \sim b(t)$ if $\lim a(t)/b(t) = 1$; write $a(t) = o(b(t))$ if $\lim a(t)/b(t) = 0$; and write $a(t) = O(b(t))$ if $\limsup a(t)/b(t) < \infty$. Furthermore, for two positive bivariate functions $a(t,x)$ and $b(t,x)$, we write $a(t,x) \sim b(t,x)$ uniformly for all x in a nonempty set Δ if

$$\lim_{t\to\infty} \sup_{x\in\Delta} \left| \frac{a(t,x)}{b(t,x)} - 1 \right| = 0.$$

Asymptotic formulae that hold with such a uniformity feature are usually of higher theoretical and practical interests. The indicator function of an event A is denoted by $\mathbf{1}_A$.

To formulate our main results, we firstly introduce some notations and assumptions. Let $Q(u) = -\log \overline{B}(u)$, $u \geq 0$, be the hazard function of the distribution B. We assume that there exists a nonnegative function q such that $Q(u) = Q(0) + \int_0^u q(v)\,\mathrm{d}v$, $u \geq 0$, which is called the hazard rate of B. Denote the hazard ratio index by $r := \limsup tq(t)/Q(t)$. The following condition is essential for our purposes.

Condition A Assume that Y has a finite second moment, the distribution B is absolutely continuous and satisfies

(1) $r < 1/2$;

(2) $\liminf tq(t) \geq \begin{cases} 2 & \text{if } r = 0, \\ c_B/(1-r) & \text{if } 0 < r < 1 \text{ for some } c_B > 2 + \sqrt{2}. \end{cases}$

Condition A is due to Condition B of Baltrūnas *et al.* [10], which plays an important role in proving the precise large deviations result for partial sums; see Yang [11]. By Lemma 3.8(a) of Baltrūnas *et al.* [10], we know that if $r < 1$, then $B \in \mathcal{S}$. (1.2) is a mild restriction on the process $N(t)$. It can be satisfied for many common nonnegative integer-valued processes such as the renewal counting process generated by i.i.d. or some dependent r.v.s (see, *e.g.*, Theorem 2.5.10 of Embrechts *et al.* [12], Theorem 6.1 of Yang and Wang [8], Theorem 1.4 of Wang and Cheng [13] *etc.*), the compound renewal counting process (see Theorems 2.3 and 2.4 of Tang *et al.* [2]) among others. Indeed, some recent works proposed a common used and weaker condition than (1.1):

$$\mathbb{E}\big(N(t)\big)^p \mathbf{1}_{\{N(t)>(1+\delta)\lambda(t)\}} = O\big(\lambda(t)\big)$$

for some $p > J_F^+$ and any $\delta > 0$. Comparing with this condition, (1.2) is weaker due to Lemma 2.5 of Ng *et al.* [3]. The condition (1.3) is also satisfied for, *e.g.*, the renewal counting process generated by independent or some dependent r.v.s according to some elementary renewal theorems (see, *e.g.*, Proposition 2.5.12 of Embrechts *et al.* [12], Theorem 6.1 of Yang and Wang [8], Theorems 1.2 and 1.3 of Wang and Cheng [13] *etc.*).

Throughout the paper, we assume that $\mu = \mathbb{E}(X_1 - Y_1) < 0$. Under Condition A, we state our main results below.

Theorem 1.1 *Assume that Condition* A, (1.2) *and* (1.3) *hold. If $B \in \mathcal{D}$, then for any $\gamma > |\mu|$,*

$$\mathbb{P}\big(S_{N(t)} - \mu\lambda(t) > x\big) \sim \lambda t \overline{B}(x) \tag{1.4}$$

holds uniformly for all $x \geq \gamma\lambda(t)$.

Corollary 1.1 *Assume that Y has a finite second moment*, (1.2) *and* (1.3) *hold. If the hazard function of r.v. X_1 is of the form $Q(u) = Q(0) + \int_0^u q(v)\,\mathrm{d}v$, where $2 < \liminf tq(t) \leq \limsup tq(t) < \infty$, then for any $\gamma > |\mu|$*, (1.4) *holds uniformly for all $x \geq \gamma\lambda(t)$.*

For some applications of these results in insurance, finance and queueing system, one can refer to Klüppelberg and Mikosch [1], Mikosch and Nagaev [14], Baltrūnas *et al.* [10] among others. In Section 2 we consider the customer-arrival-based insurance risk model

(CIRM) and obtain some uniformly asymptotic behavior of the accumulated risks of an insurance company as the number of customers tends to infinity and the time tends to infinity. The proofs of Theorem 1.1 and Corollary 1.1 will be postponed in Section 3.

2 Applications to risk theory

In this section, we apply our main results to the Customer-arrival-based Insurance Risk Model (CIRM). Their proofs are straightforward and we omit them. Such a risk model satisfies the following three requirements:

(1) The customer-arrival process $\{N(t), t \geq 0\}$ is a general counting process, namely a nonnegative, nondecreasing, right continuous and integer-valued random process. Denote the times of successive customer-arrival by τ_n, $n = 1, 2, \ldots$.

(2) At the time τ_n, the nth customer purchases an insurance policy. Assume that an insurance period lasts δ_0. Then in an insurance period δ_0, the insurance company has a potential risk of payment.

(3) The potential claims $\{X_k, k \geq 1\}$, independent of $\{N(t), t \geq 0\}$, are nonnegative i.i.d. r.v.s with a common distribution B and a finite mean μ_B. The price of an insurance policy is $(1 + \rho)\mu_B$, where the positive constant ρ is interpreted as a relative safety loading. The net loss of the nth customer is $X_n - (1 + \rho)\mu_B$.

Denote by $R(x, t) = x - W(t)$ the risk reserve process up to time $t \geq 0$, where x is the initial capital reserve and the claim surplus process $W(t)$ is defined as

$$W(t) = \sum_{k=1}^{N(t)} \bigl(X_k - (1 + \rho)\mu_B\bigr), \quad t \geq 0.$$

In the discrete case, the claim surplus process can be rewritten as

$$W_n = \sum_{k=1}^{n} \bigl(X_k - (1 + \rho)\mu_B\bigr), \quad n \geq 1.$$

This model was introduced by Ng *et al.* [15]. Clearly, Lemma 3.1 and Theorem 1.1 lead to some precise large deviation results for the processes W_n and $W(t)$ in the CIRM.

Theorem 2.1 *In the CIRM,*

(i) *Assume that Condition* A *holds, then for any* $\gamma > 0$

$$\lim_{n\to\infty} \sup_{x \geq \gamma n} \left| \frac{\mathbb{P}(W_n > x)}{n\overline{B}(x + \rho n \mu_B)} - 1 \right| = 0. \tag{2.1}$$

(ii) *Assume that Condition* A, (1.2) *and* (1.3) *hold. If* $B \in \mathscr{D}$, *then for any* $\gamma > |\mu|$

$$\lim_{t\to\infty} \sup_{x \geq \gamma \lambda(t)} \left| \frac{\mathbb{P}(W(t) > x)}{\lambda t \overline{B}(x + \rho \mu_B \lambda(t))} - 1 \right| = 0. \tag{2.2}$$

We address that the large deviation problems for the prospective loss process $W(t)$ describe the uniformly asymptotic behavior of the accumulated risks.

3 Proof of main result

In the sequel, the constant C always represents a positive constant, which may vary from place to place. Before proving Theorem 1.1, we require some lemmas.

We firstly introduce two auxiliary lemmas. The first one is an important precise large deviation for partial sums, which was originally due to Baltrūnas *et al.* [10] and modified by Daley *et al.* [16].

Lemma 3.1 *Assume that Condition* A *holds, then*

$$\lim_{n\to\infty}\sup_{t\geq t_n}\left|\frac{\mathbb{P}(S_n-n\mu>t)}{n\overline{B}(t)}-1\right|=0 \tag{3.1}$$

holds for any sequence $\{t_n, n\geq 1\}$ *satisfying*

$$\lim_{n\to\infty}\sqrt{n}\sup_{u\geq t_n}\frac{Q(u)}{u}=0. \tag{3.2}$$

The second lemma describes the relations among the hazard ratio index, the class $\mathcal{S}$ and the hazard function, which can be found in Baltrūnas *et al.* [10] or Baltrūnas *et al.* [7].

Lemma 3.2 *If* $r<1$, *then*

(1) $B\in\mathcal{S}$;
(2) $Q(u)/u$ *decreases for sufficiently large* u;
(3) *for any* $\epsilon>0$, *there exist positive* u_ϵ *and* c_ϵ *such that* $Q(u)\leq c_\epsilon u^{r+\epsilon}$ *for* $u\geq u_\epsilon$.

In order to prove Theorem 1.1, we rewrite $\mathbb{P}(S_{N(t)}-\mu\lambda(t)>x)$ as the sum $\sum_{n=1}^{\infty}\mathbb{P}(S_n-\mu\lambda(t)>x)\mathbb{P}(N(t)=n)$ and divide the sum into three parts

$$\begin{aligned}
&\mathbb{P}\bigl(S_{N(t)}-\mu\lambda(t)>x\bigr)\\
&\quad=\biggl(\sum_{|n-\lambda(t)|\leq\varepsilon(t)\lambda(t)}+\sum_{n<(1-\varepsilon(t))\lambda(t)}+\sum_{n>(1+\varepsilon(t))\lambda(t)}\biggr)\mathbb{P}\bigl(S_n-\mu\lambda(t)>x\bigr)\mathbb{P}\bigl(N(t)=n\bigr)\\
&\quad=: I_1+I_2+I_3,
\end{aligned} \tag{3.3}$$

where $\varepsilon(t)$ is some positive function satisfying $\varepsilon(t)\to 0$ and $t\varepsilon^2(t)\nearrow\infty$. We proceed with a series of lemmas below to prove Theorem 1.1.

Lemma 3.3 *Assume that Condition* A, (1.2) *and* (1.3) *hold. Let* $\varepsilon(t)=c_1\log t/\sqrt{t}$ *for some* $c_1>0$. *Then for any* $\gamma>0$,

$$I_1\sim\lambda t\overline{B}(x) \tag{3.4}$$

holds uniformly for all $x\geq\gamma\lambda(t)$.

Proof Along the line of Baltrūnas *et al.* [7], we rewrite

$$\begin{aligned}
I_1&=\overline{B}(x)\sum_{|n-\lambda(t)|\leq\varepsilon(t)\lambda(t)}\mathbb{P}\bigl(N(t)=n\bigr)\cdot\frac{\overline{B}(x+\mu\lambda(t)-n\mu)}{\overline{B}(x)}\\
&\quad\times\frac{\mathbb{P}(S_n-n\mu>x+\mu\lambda(t)-n\mu)}{\overline{B}(x+\mu\lambda(t)-n\mu)}.
\end{aligned} \tag{3.5}$$

If $\gamma \geq -\mu > 0$, then $x + \mu\lambda(t) - n\mu \geq (\gamma + \mu)\lambda(t) - n\mu \geq n(\gamma - \mu\varepsilon(t))/(1 + \varepsilon(t))$; analogously, if $0 < \gamma < -\mu$, then $x + \mu\lambda(t) - n\mu \geq n(\gamma + \mu\varepsilon(t))/(1 - \varepsilon(t))$. Taking account of the vanishing of $\varepsilon(t)$, we have that $x + \mu\lambda(t) - n\mu \geq \gamma n/2$ for sufficiently large t. Since $Q(u)/u$ decreases eventually (Lemma 3.2(2)), we derive that for any $\epsilon > 0$ satisfying $r + \epsilon < 1/2$, sufficiently large t and $|n - \lambda(t)| \leq \varepsilon(t)\lambda(t)$, by Lemma 3.2(3)

$$\begin{aligned} \sqrt{n} \sup_{u \geq x + \mu\lambda(t) - n\mu} \frac{Q(u)}{u} &\leq \sqrt{n} \frac{Q(\frac{\gamma}{2}n)}{\frac{\gamma}{2}n} \\ &\leq c_\epsilon \sqrt{n} \left(\frac{\gamma}{2}n\right)^{r+\epsilon-1} \\ &\to 0, \quad n \to \infty \text{ (or equivalently, } t \to \infty), \end{aligned} \tag{3.6}$$

which means that (3.2) is fulfilled. Using Lemma 3.1, we can obtain that

$$\begin{aligned} \mathbb{P}\bigl(S_n - \mu\lambda(t) > x\bigr) &= \mathbb{P}\bigl(S_n - n\mu > x + \mu\lambda(t) - n\mu\bigr) \\ &\sim n\overline{B}\bigl(x + \mu\lambda(t) - n\mu\bigr) \end{aligned} \tag{3.7}$$

holds uniformly for $|n - \lambda(t)| \leq \varepsilon(t)\lambda(t)$. If $(n - \lambda(t))\mu \geq 0$, according to the mean value theorem, there exists some constant $c_2 = c_2(x) \in (x - (n - \lambda(t))\mu, x)$ such that for the above fixed $\epsilon > 0$ satisfying $r + \epsilon < 1/2$ and sufficiently large t,

$$\begin{aligned} 1 &\leq \frac{\overline{B}(x + \mu\lambda(t) - n\mu)}{\overline{B}(x)} \\ &= \exp\bigl\{Q(x) - Q\bigl(x + \mu\lambda(t) - n\mu\bigr)\bigr\} \\ &= \exp\left\{\bigl(n - \lambda(t)\bigr)\mu \cdot \frac{c_2 q(c_2)}{Q(c_2)} \cdot \frac{Q(c_2)}{c_2}\right\} \\ &\leq \exp\left\{(r + \epsilon)\bigl(n - \lambda(t)\bigr)\mu \cdot \frac{Q(c_2)}{c_2}\right\}. \end{aligned} \tag{3.8}$$

Note that by (1.3), $c_2 > x + (n - \lambda t)|\mu| \geq x - \varepsilon(t)|\mu|\lambda(t) \geq (\gamma - \varepsilon(t)|\mu|)\lambda(t) \geq \gamma\lambda t/2$, and $0 \leq (n - \lambda t)\mu \leq \varepsilon(t)|\mu|\lambda(t) \leq C\sqrt{t}\log t$. Hence, by (3.8) and Lemma 3.2, we have that for sufficiently large t,

$$\begin{aligned} 1 &\leq \frac{\overline{B}(x + \mu\lambda(t) - n\mu)}{\overline{B}(x)} \\ &\leq \exp\left\{C\sqrt{t}\log t \cdot \frac{Q(\frac{\gamma}{2}\lambda t)}{\frac{\gamma}{2}\lambda t}\right\} \\ &\leq \exp\bigl\{Ct^{r+\epsilon-\frac{1}{2}}\log t\bigr\} \to 1. \end{aligned} \tag{3.9}$$

If $(n - \lambda t)\mu < 0$, the proof of (3.9) is analogous. Clearly, the condition (1.2) is equivalent to

$$\lim \mathbb{P}\bigl(|N(t) - \lambda(t)| \leq \varepsilon(t)\lambda(t)\bigr) = 1. \tag{3.10}$$

Therefore, it follows from (3.5)-(3.10), (1.3) and the dominated convergence theorem that

$$\begin{aligned} I_1 &\sim \overline{B}(x) \sum_{|n-\lambda(t)|\le\varepsilon(t)\lambda(t)} n\mathbb{P}\bigl(N(t)=n\bigr) \\ &\sim \lambda(t)\overline{B}(x)\mathbb{P}\bigl(\bigl|N(t)-\lambda(t)\bigr| \le \varepsilon(t)\lambda(t)\bigr) \\ &\sim \lambda t\overline{B}(x) \end{aligned}$$

holds uniformly for all $x \ge \gamma\lambda(t)$. It completes the proof of the lemma. □

Lemma 3.4 *Assume that Condition* A *and* (1.2) *hold. Let* $\varepsilon(t)$ *be any positive function satisfying* $\varepsilon(t) \to 0$ *and* $t\varepsilon^2(t) \nearrow \infty$. *Then for any* $\gamma > |\mu|$,

$$I_3 = o\bigl(\lambda(t)\overline{B}(x)\bigr) \tag{3.11}$$

holds uniformly for all $x \ge \gamma\lambda(t)$.

Proof Note that $0 < \varepsilon < 1$ for all sufficiently large t, then by the dominated convergence theorem and (1.2), we have that

$$\lim \mathbb{E}\left(\frac{N(t)}{\lambda(t)}\mathbf{1}_{\{N(t)\le(1+\varepsilon)\lambda(t)\}}\right) = 1,$$

which implies that

$$\mathbb{E}N(t)\mathbf{1}_{\{N(t)>(1+\varepsilon)\lambda(t)\}} = o\bigl(\lambda(t)\bigr). \tag{3.12}$$

Similarly to (3.6), (3.2) is satisfied for $t_n = x + \mu\lambda(t) - n\mu$ with $n > (1+\varepsilon(t))\lambda(t)$. Note that $x + \mu\lambda(t) - n\mu \ge \max(x, (\gamma+\mu)\lambda(t) + n|\mu|) \ge \max(x, n|\mu|)$. So, from Lemma 3.1 and (3.12), we obtain that

$$\begin{aligned} I_3 &\sim \sum_{n>(1+\varepsilon(t))\lambda(t)} n\mathbb{P}\bigl(N(t)=n\bigr)\overline{B}\bigl(x+\mu\lambda(t)-n\mu\bigr) \\ &\le \overline{B}(x)\mathbb{E}N(t)\mathbf{1}_{\{N(t)>(1+\varepsilon(t))\lambda(t)\}} \\ &= o\bigl(\lambda(t)\overline{B}(x)\bigr) \end{aligned}$$

holds uniformly for all $x \ge \gamma\lambda(t)$. □

Lemma 3.5 *Assume that Condition* A *and* (1.2) *hold. Let* $\varepsilon(t)$ *be any positive function satisfying* $\varepsilon(t) \to 0$ *and* $t\varepsilon^2(t) \nearrow \infty$. *If* $B \in \mathcal{D}$, *then for any* $\gamma > |\mu|$,

$$I_2 = o\bigl(\lambda(t)\overline{B}(x)\bigr) \tag{3.13}$$

holds uniformly for all $x \ge \gamma\lambda(t)$.

Proof For any $\epsilon > 0$, by Lemma 3.1, there exists some sufficiently large integer n_0 such that for all $u \ge t_n$ and $n \ge n_0$,

$$\left|\frac{\mathbb{P}(S_n - n\mu > u)}{n\overline{B}(u)} - 1\right| \le \epsilon. \tag{3.14}$$

Now we divide I_2 into two parts.

$$\begin{aligned} I_2 &= \Bigg(\sum_{n\le n_0} + \sum_{n_0<n\le(1-\varepsilon(t))\lambda(t)}\Bigg)\mathbb{P}\big(N(t)=n\big)\mathbb{P}\big(S_n-\mu\lambda(t)>x\big) \\ &\equiv I_{21}+I_{22}. \end{aligned} \tag{3.15}$$

We firstly estimate I_{21}. By $r<1$ and Lemma 3.2(1), we know that $B\in\mathscr{S}$. According to Lemma 4.3(b) of Baltrūnas*et al.* [10], it holds that $\overline{F}(t)\sim\overline{B}(t)$, which implies $F\in\mathscr{S}$. Hence, by the subexponentiality and $B\in\mathscr{D}$, we have

$$\begin{aligned} I_{21} &\sim \overline{F}\big(x+\mu\lambda(t)\big)\sum_{n\le n_0} n\mathbb{P}\big(N(t)=n\big) \\ &\sim \overline{B}\big(x+\mu\lambda(t)\big)\mathbb{E}N(t)\mathbf{1}_{\{N(t)\le n_0\}} \\ &\le C(n_0)\overline{B}(x) \\ &= o\big(\lambda(t)\overline{B}(x)\big) \end{aligned} \tag{3.16}$$

holds uniformly for all $x\ge\gamma\lambda(t)$. As for I_{22}, noting that $x+\mu\lambda(t)-n\mu\ge(\gamma+\mu)\lambda(t)-n\mu\ge n(\gamma+\mu\varepsilon(t))/(1-\varepsilon(t))\ge\gamma n/2\ge t_n$, and from (3.14), $0<-\mu<\gamma$, $B\in\mathscr{D}$, (3.10), we obtain that

$$\begin{aligned} I_{22} &\le (1+\epsilon)\sum_{n_0<n\le(1-\varepsilon(t))\lambda(t)}\mathbb{P}\big(N(t)=n\big)n\overline{B}\big(x+\mu\lambda(t)-n\mu\big) \\ &\le (1+\epsilon)\big(1-\varepsilon(t)\big)\lambda(t)\overline{B}\bigg(\frac{\gamma+\mu}{\gamma}x\bigg)\mathbb{P}\big(n_0<N(t)\le\big(1-\varepsilon(t)\big)\lambda(t)\big) \\ &\le (1+\epsilon)\big(1-\varepsilon(t)\big)\lambda(t)C\overline{B}(x)\mathbb{P}\big(n_0<N(t)\le\big(1-\varepsilon(t)\big)\lambda(t)\big) \\ &= o\big(\lambda(t)\overline{B}(x)\big) \end{aligned} \tag{3.17}$$

holds uniformly for all $x\ge\gamma\lambda(t)$. Therefore, the desired (3.13) follows from (3.15)-(3.17). □

Combining above Lemmas 3.3, 3.4 and 3.5, we complete the proof of Theorem 1.1.

Proof of Corollary 1.1 Clearly, $\limsup tq(t)<\infty$ implies $r=0$. By Lemma 3.6(b) of Baltrūnas *et al.* [10], we have $\kappa\ge\liminf tq(t)>2=\alpha(r)$. Hence, it only remains to prove $B\in\mathscr{D}$. Indeed, there exists some constant $C>0$ such that $q(t)\le Ct^{-1}$ for sufficiently large t. For any $0<\theta<1$ and sufficiently large t, we have that

$$\begin{aligned} \frac{\overline{B}(\theta t)}{\overline{B}(t)} &= \exp\bigg\{\int_{\theta t}^{t} q(u)\,\mathrm{d}u\bigg\} \\ &\le \exp\bigg\{C\int_{\theta t}^{t} u^{-1}\,\mathrm{d}u\bigg\} \\ &= \theta^{-C}<\infty, \end{aligned}$$

which implies $B\in\mathscr{D}$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The first author carried out the proofs of the main theorems and lemmas. All authors conceived of the study and participated in its design and writing. All authors read and approved the final manuscript.

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References
1. Klüppelberg, C, Mikosch, T: Large deviations of heavy-tailed random sums with applications in insurance and finance. J. Appl. Probab. **34**, 293-308 (1997)
2. Tang, Q, Su, C, Jiang, T, Zhang, J: Large deviations for heavy-tailed random sums in compound renewal model. Stat. Probab. Lett. **52**, 91-100 (2001)
3. Ng, KW, Tang, Q, Yan, J, Yang, H: Precise large deviations for sums of random variables with consistently varying tails. J. Appl. Probab. **41**, 93-107 (2004)
4. Liu, Y: Precise large deviations for negatively associated random variables with consistently varying tails. Stat. Probab. Lett. **77**, 181-189 (2007)
5. Wang, Y, Wang, K, Cheng, D: Precise large deviations for sums of negatively associated random variables with common dominatedly varying tails. Acta Math. Sin. Engl. Ser. **22**, 1725-1734 (2006)
6. Yang, Y, Wang, K: Precise large deviations for dependent random variables with applications to the compound renewal risk model. Rocky Mt. J. Math. (2010, in press)
7. Baltrūnas, A, Leipus, R, Šiaulys, J: Precise large deviation results for the total claim amount under subexponential claim sizes. Stat. Probab. Lett. **78**, 1206-1214 (2008)
8. Yang, Y, Wang, Y: Large deviations for random variables with two-sided distributions. Acta Math. Sin. Chin. Ser. **52**, 289-300 (2009)
9. Chen, Y, Zhang, W: Large deviations for random sums of negatively dependent random variables with consistently varying tails. Stat. Probab. Lett. **77**, 530-538 (2007)
10. Baltrūnas, A, Daley, DJ, Klüppelberg, C: Tail behaviour of the busy period of a GI/GI/1 queue with subexponential service times. Stoch. Process. Appl. **111**, 237-258 (2004)
11. Yang, Y: The study of some topics on heavy-tailed risk models. PhD thesis, Soochow University (2008)
12. Embrechts, P, Klüppelberg, C, Mikosch, T: Modelling Extremal Events. Springer, Berlin (1997)
13. Wang, Y, Cheng, D: Basic renewal theorems for a random walk with widely dependent increments and their applications. J. Math. Anal. Appl. **384**, 597-606 (2011)
14. Mikosch, T, Nagaev, AV: Large deviations of heavy-tailed sums with applications in insurance. Extremes **1**, 81-110 (1998)
15. Ng, KW, Tang, Q, Yan, J, Yang, H: Precise large deviations for the prospective-loss process. J. Appl. Probab. **40**, 391-400 (2003)
16. Daley, DJ, Klüppelberg, C, Yang, Y: Corrigendum to Baltrūnas, Daley and Klüppelberg 'Tail behaviour of the busy period of a GI/GI/1 queue with subexponential service times' [Stochastic Process. Appl. 111 (2004) 237-258]. Stoch. Process. Appl. **121**, 2186-2187 (2011)

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