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Some identities on Bernoulli and Euler polynomials arising from orthogonality of Legendre polynomials

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Abstract

The purpose of this paper is to investigate some interesting identities on the Bernoulli and Euler polynomials arising from the orthogonality of Legendre polynomials in the inner product space \mathbb{P}_n .

1 Introduction

As is well known, the Legendre polynomial $P_n(x)$ is a solutions of the following differential equation:

$$(1-x^2)u'' - 2xu' + n(n+1)u = 0 \quad (\text{see [1-7]}),$$

where $n = 0, 1, 2, \dots$

It is a polynomial of degree n . If n is even or odd, then $P_n(x)$ is accordingly even or odd. They are determined up to constant and normalized so that $P_n(1) = 1$.

Rodrigues' formula is given by

$$P_n(x) = \frac{1}{2^n n!} \left\{ \left(\frac{d}{dx} \right)^n (x^2 - 1)^n \right\}, \quad n \in \mathbb{Z}_+. \quad (1.1)$$

Integrating by parts, we can derive

$$\int_{-1}^1 P_m(x)P_n(x) dx = \frac{2}{2n+1} \delta_{m,n} \quad (\text{see [1-7]}), \quad (1.2)$$

where $\delta_{m,n}$ is the Kronecker symbol.

By (1.1), we get

$$P_n(x) = \frac{1}{2^n} \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} (-1)^k \binom{n}{k} \binom{2n-2k}{n} x^{n-2k}. \quad (1.3)$$

The generating function is given by

$$(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)t^n. \quad (1.4)$$

The Bernoulli polynomial is defined by a generating function to be

$$\frac{t}{e^t - 1} e^{xt} = e^{B(x)t} = \sum_{n=0}^{\infty} B_n(x) \frac{t^n}{n!} \quad (\text{see [8-13]}) \tag{1.5}$$

with the usual convention about replacing $B^n(x)$ by $B_n(x)$.

In the special case, $x = 0$, $B_n(0) = B_n$ are called the *Bernoulli numbers*.

From (1.5), we have

$$B_n(x) = \sum_{l=0}^n \binom{n}{l} B_{n-l} x^l \quad (\text{see [10-26]}). \tag{1.6}$$

As is well known, the *Euler numbers* are defined by

$$E_0 = 1, \quad (E + 1)^n + E_n = 2\delta_{0,n} \quad (\text{see [10-13]}) \tag{1.7}$$

with the usual convention about replacing E^n by E_n .

The Euler polynomials are defined as

$$E_n(x) = \sum_{l=0}^n \binom{n}{l} E_{n-l} x^l \quad (\text{see [27-31]}). \tag{1.8}$$

Let $\mathbb{P}_n = \{p(x) \in \mathbb{O}[x] \mid \deg p(x) \leq n\}$. Then \mathbb{P}_n is an inner product space with respect to the inner product $\langle \cdot, \cdot \rangle$ with

$$\langle q_1(x), q_2(x) \rangle = \int_{-1}^1 q_1(x)q_2(x) dx,$$

where $q_1(x), q_2(x) \in \mathbb{P}_n$.

From (1.2), we can show that $\{P_0(x), P_1(x), \dots, P_n(x)\}$ is an orthogonal basis for \mathbb{P}_n . In this paper, we derive some interesting identities on the Bernoulli and Euler polynomials from the orthogonality of Legendre polynomials in \mathbb{P}_n .

2 Some identities on the Bernoulli and Euler polynomials

For $q(x) \in \mathbb{P}_n$, let

$$q(x) = \sum_{k=0}^n C_k P_k(x). \tag{2.1}$$

Then, from (1.2), we have

$$\begin{aligned} \langle q(x), P_k(x) \rangle &= C_k \langle P_k(x), P_k(x) \rangle \\ &= C_k \int_{-1}^1 \{P_k(x)\}^2 dx \\ &= \frac{2}{2k + 1} C_k. \end{aligned} \tag{2.2}$$

By (2.2), we get

$$\begin{aligned}
 C_k &= \frac{2k+1}{2} \langle q(x), P_k(x) \rangle = \frac{2k+1}{2} \int_{-1}^1 P_k(x) q(x) dx \\
 &= \left(\frac{2k+1}{2} \right) \frac{1}{2^{k+1}k!} \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2-1)^k \right) q(x) dx \\
 &= \left(\frac{2k+1}{2^{k+1}k!} \right) \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2-1)^k \right) q(x) dx.
 \end{aligned} \tag{2.3}$$

Therefore, by (2.1) and (2.3), we obtain the following proposition.

Proposition 2.1 For $q(x) \in \mathbb{P}_n$, let

$$q(x) = \sum_{k=0}^n C_k P_k(x).$$

Then

$$C_k = \frac{2k+1}{2^{k+1}k!} \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2-1)^k \right) q(x) dx.$$

Let us assume that $q(x) = x^n \in \mathbb{P}_n$.

From Proposition 2.1, we have

$$\begin{aligned}
 C_k &= \frac{2k+1}{2^{k+1}k!} \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2-1)^k \right) x^n dx \\
 &= \frac{2k+1}{2^{k+1}} (-1)^k \binom{n}{k} \int_{-1}^1 (x^2-1)^k x^{n-k} dx \\
 &= \frac{2k+1}{2^{k+1}} \binom{n}{k} (1 + (-1)^{n-k}) \int_0^1 (1-x^2)^k x^{n-k} dx.
 \end{aligned} \tag{2.4}$$

For $n - k \equiv 0 \pmod{2}$, by (2.4), we get

$$\begin{aligned}
 C_k &= \frac{2k+1}{2^{k+1}} \binom{n}{k} \int_0^1 (1-y)^k y^{\frac{n-k-1}{2}} dy \\
 &= \frac{2k+1}{2^{k+1}} \binom{n}{k} B\left(k+1, \frac{n-k+1}{2}\right) \\
 &= \frac{2k+1}{2^{k+1}} \binom{n}{k} \frac{\Gamma(k+1)\Gamma(\frac{n-k+1}{2})}{\Gamma(\frac{n+k+1}{2}+1)} \\
 &= \frac{2k+1}{2^{k+1}} \binom{n}{k} \frac{k! \Gamma(\frac{n-k+1}{2})}{(\frac{n+k+1}{2})(\frac{n+k-1}{2}) \dots (\frac{n-k+1}{2}) \Gamma(\frac{n-k+1}{2})} \\
 &= \frac{2k+1}{2^{k+1}} \binom{n}{k} k! 2^{k+1} \frac{(n-k)!(n+k+2)(n+k) \dots (n-k+2)}{(n+k+2)!} \\
 &= \frac{(2k+1)2^{k+1}}{(n+k+2)!} \times \frac{n! \binom{n+k+2}{2}}{\binom{n-k}{2}!}.
 \end{aligned} \tag{2.5}$$

Here the beta function $B(x, y)$ is defined by

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt \quad (\operatorname{Re}(x), \operatorname{Re}(y) > 0),$$

and it is well known that

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},$$

where $\Gamma(s) = \int_0^\infty t^{s-1}e^{-t} dt$ ($\operatorname{Re}(s) > 0$) is the gamma function.

By Proposition 2.1 and (2.5), we get

$$x^n = \sum_{0 \leq k \leq n, n-k \equiv 0 \pmod{2}} \frac{(2k+1)n!2^{k+1}(\frac{n+k+2}{2})!}{(n+k+2)!(\frac{n-k}{2})!} P_k(x). \tag{2.6}$$

From (1.5), we can easily derive the following equation (2.7):

$$x^n = \frac{1}{n+1} \sum_{l=0}^n \binom{n+1}{l} B_l(x) \quad (n \in \mathbb{Z}_+). \tag{2.7}$$

Therefore, by (2.6) and (2.7), we obtain the following Proposition 2.2.

Proposition 2.2 *For $n \in \mathbb{Z}_+$, we have*

$$\sum_{l=0}^n \frac{B_l(x)}{(n+1-l)!} = \sum_{0 \leq k \leq n, n-k \equiv 0 \pmod{2}} \frac{(2k+1)2^{k+1}(\frac{n+k+2}{2})!}{(n+k+2)!(\frac{n-k}{2})!} P_k(x).$$

Let us take $q(x) = B_n(x) \in \mathbb{P}_n$. By Proposition 2.1, we get

$$\begin{aligned} C_k &= \frac{2k+1}{2^{k+1}k!} \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2-1)^k \right) B_n(x) dx \\ &= \frac{(-1)^k(2k+1)}{2^{k+1}} \binom{n}{k} \int_{-1}^1 (x^2-1)^k B_{n-k}(x) dx \\ &= \frac{(-1)^k(2k+1)}{2^{k+1}} \binom{n}{k} \sum_{l=0}^{n-k} \binom{n-k}{l} B_{n-k-l} \int_{-1}^1 (x^2-1)^k x^l dx \\ &= \frac{2k+1}{2^{k+1}} \binom{n}{k} \sum_{l=0}^{n-k} \binom{n-k}{l} B_{n-k-l} (1+(-1)^l) \int_0^1 (1-x^2)^k x^l dx. \end{aligned} \tag{2.8}$$

For $l \in \mathbb{Z}_+$ with $l \equiv 0 \pmod{2}$, we have

$$\begin{aligned} C_k &= \frac{2k+1}{2^{k+1}} \binom{n}{k} \sum_{0 \leq l \leq n-k, l \text{ is even}} \binom{n-k}{l} B_{n-k-l} \int_0^1 (1-y)^k y^{\frac{l-1}{2}} dy \\ &= \frac{2k+1}{2^{k+1}} \binom{n}{k} \sum_{0 \leq l \leq n-k, l \text{ is even}} \binom{n-k}{l} B_{n-k-l} \frac{\Gamma(k+1)\Gamma(\frac{l+1}{2})}{\Gamma(\frac{2k+l+1}{2}+1)} \\ &= (2k+1)2^{k+1}n! \sum_{0 \leq l \leq n-k, l \equiv 0 \pmod{2}} \frac{B_{n-k-l}}{(n-k-l)!} \times \frac{(\frac{2k+l+2}{2})!}{(2k+l+2)!(\frac{l}{2})!}. \end{aligned} \tag{2.9}$$

In [14], we showed that

$$B_n(x) = \sum_{k=0}^{n-2} \binom{n}{k} B_{n-k} E_k(x) + E_n(x) = \sum_{k=0, k \neq n-1}^n \binom{n}{k} B_{n-k} E_k(x). \quad (2.10)$$

Therefore, by Proposition 2.1, (2.9) and (2.10), we obtain the following theorem.

Theorem 2.3 For $n \in \mathbb{Z}_+$, we have

$$\frac{1}{n!} \sum_{k=0, k \neq n-1}^n \binom{n}{k} B_{n-k} E_k(x) = \sum_{k=0}^n \left(\sum_{0 \leq l \leq n-k, l \equiv 0 \pmod{2}} \frac{(2k+1)2^{k+1} \binom{l+2k+2}{2}! B_{n-k-l}}{(n-k-l)!(l+2k+2)!(\frac{l}{2})!} \right) P_k(x).$$

By the same method of Theorem 2.3, we easily see that

$$\frac{E_n(x)}{n!} = \sum_{k=0}^n \left(\sum_{0 \leq l \leq n-k, l \equiv 0 \pmod{2}} \frac{(2k+1)2^{k+1} \binom{l+2k+2}{2}! B_{n-k-l}}{(n-k-l)!(l+2k+2)!(\frac{l}{2})!} \right) P_k(x). \quad (2.11)$$

Let us take $q(x) = \sum_{k=0}^n B_k(x) B_{n-k}(x) \in \mathbb{P}_n$. Then we see that

$$\begin{aligned} & \sum_{k=0}^n B_k(x) B_{n-k}(x) \\ &= (n+1) \sum_{k=0}^n \frac{\binom{n}{k}}{n-k+1} \left\{ \sum_{l=k}^n B_{l-k} B_{n-l} + B_{n-1-k} \right\} E_k(x) + \frac{(n^2-1)n}{12} E_{n-2}(x). \end{aligned} \quad (2.12)$$

The equation (2.12) was proved in [14].

By (2.12) and Proposition 2.2, we have

$$\begin{aligned} C_k &= \frac{2k+1}{2^{k+1}k!} \left\{ (n+1) \sum_{l=0}^n \frac{\binom{n}{l}}{n-l+1} \left(\sum_{m=l}^n B_{m-l} B_{n-m} + B_{n-1-l} \right) \right. \\ &\quad \times \int_{-1}^1 E_l(x) \left(\frac{d^k}{dx^k} (x^2-1)^k \right) dx \\ &\quad \left. + \frac{(n^2-1)n}{12} \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2-1)^k \right) E_{n-2}(x) dx \right\}. \end{aligned} \quad (2.13)$$

Integrating by parts, we get

$$\begin{aligned} & \int_{-1}^1 E_l(x) \left(\frac{d^k}{dx^k} (x^2-1)^k \right) dx \\ &= \sum_{j=0}^l \binom{l}{j} E_{l-j} \int_{-1}^1 x^j \frac{d^k}{dx^k} (x^2-1)^k dx \\ &= \sum_{j=k}^l \binom{l}{j} E_{l-j} \frac{(-1)^k j!}{(j-k)!} \int_{-1}^1 x^{j-k} (x^2-1)^k dx \\ &= \sum_{j=k}^l \binom{l}{j} E_{l-j} \frac{j!}{(j-k)!} (1 + (-1)^{j-k}) \int_0^1 x^{j-k} (-x^2+1)^k dx. \end{aligned} \quad (2.14)$$

Then we see that

$$\begin{aligned}
 & \int_{-1}^1 E_l(x) \left(\frac{d^k}{dx^k} (x^2 - 1)^k \right) dx \\
 &= \sum_{j=k, j-k \equiv 0 \pmod{2}}^l \binom{l}{j} E_{l-j} \frac{j!}{(j-k)!} \int_0^1 t^{\frac{j-k-1}{2}} (1-t)^k dt \\
 &= \sum_{j=k, j-k \equiv 0 \pmod{2}}^l \binom{l}{j} E_{l-j} \frac{j!}{(j-k)!} \frac{\Gamma(\frac{j-k+1}{2}) \Gamma(k+1)}{\Gamma(\frac{j+k+1}{2} + 1)} \\
 &= \sum_{j=k, j-k \equiv 0 \pmod{2}}^l \binom{l}{j} E_{l-j} \frac{j! k!}{(j-k)!} \times \frac{(j-k)! 2^{2k+2}}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!} \\
 &= \sum_{j=k, j-k \equiv 0 \pmod{2}}^l \binom{l}{j} E_{l-j} \frac{j! k! 2^{2k+2}}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!}. \tag{2.15}
 \end{aligned}$$

It is easy to show that

$$\begin{aligned}
 & \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2 - 1)^k \right) E_{n-2}(x) dx \\
 &= \sum_{j=0}^{n-2} \binom{n-2}{j} E_{n-2-j} \int_{-1}^1 \left(\frac{d^k}{dx^k} (x^2 - 1)^k \right) x^j dx \\
 &= \sum_{j=k}^{n-2} \binom{n-2}{j} E_{n-2-j} (1 + (-1)^{j-k}) (-1)^k \frac{j!}{(j-k)!} \int_0^1 (x^2 - 1)^k x^{j-k} dx \\
 &= \sum_{k \leq j \leq n-2, j-k \equiv 0 \pmod{2}} \binom{n-2}{j} E_{n-2-j} \frac{j! k! 2^{2k+2}}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!}. \tag{2.16}
 \end{aligned}$$

Therefore, by (2.13), (2.14), (2.15) and (2.16), we get

$$\begin{aligned}
 C_k &= (2k+1)2^{k+1} \left\{ (n+1) \sum_{l=k}^n \frac{\binom{n}{k}}{n-l+1} \left(\sum_{m=l}^n B_{m-l} B_{n-m} + B_{n-1-l} \right) \right. \\
 &\quad \times \sum_{k \leq j \leq l, j-k \equiv 0 \pmod{2}} \binom{l}{j} E_{l-j} \frac{j!}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!} \\
 &\quad \left. + \frac{(n^2-1)n}{12} \sum_{k \leq j \leq n-2, j-k \equiv 0 \pmod{2}} \binom{n-2}{j} E_{n-2-j} \frac{j!}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!} \right\}. \tag{2.17}
 \end{aligned}$$

Therefore, by Proposition 2.1 and (2.17), we obtain the following theorem.

Theorem 2.4 For $n \in \mathbb{Z}_+$, we have

$$\begin{aligned}
 & \sum_{k=0}^n B_k(x) B_{n-k}(x) \\
 &= \sum_{k=0}^n (2k+1)2^{k+1} \left\{ (n+1) \sum_{l=k}^n \frac{\binom{n}{k}}{n-l+1} \left(\sum_{m=l}^n B_{m-l} B_{n-m} + B_{n-1-l} \right) \right.
 \end{aligned}$$

$$\begin{aligned} & \times \sum_{k \leq j \leq l, j-k \equiv 0 \pmod{2}} \binom{l}{j} E_{l-j} \frac{j!}{(j+k+2)!} \times \frac{\left(\frac{j+k+2}{2}\right)!}{\left(\frac{j-k}{2}\right)!} \\ & + \frac{(n^2-1)n}{12} \sum_{k \leq j \leq n-2, j-k \equiv 0 \pmod{2}} \binom{n-2}{j} E_{n-2-j} \frac{j!}{(j+k+2)!} \times \frac{\left(\frac{j+k+2}{2}\right)!}{\left(\frac{j-k}{2}\right)!} \Big\} P_k(x). \end{aligned}$$

Remark 2.5 The extended Laguerre polynomials are given by

$$L_n^\alpha(x) = \sum_{r=0}^n \frac{(-1)^r}{r!} \binom{n+\alpha}{n-r} x^r \quad (\alpha > -1).$$

By the same method, we get

$$L_n^\alpha(x) = \sum_{k=0}^n \sum_{0 \leq l \leq n-k, l \equiv 0 \pmod{2}} \frac{(-1)^{k+l} (2k+1) 2^{k+1} \binom{n+\alpha}{n-k-l} \left(\frac{l+2k+2}{2}\right)!}{(l+2k+2)! \left(\frac{l}{2}\right)!} P_k(x)$$

and

$$H_n(x) = \sum_{k=0}^n \sum_{0 \leq l \leq n-k, l \equiv 0 \pmod{2}} \frac{(2k+1) 2^{2k+l+1} n! \left(\frac{l+2k+2}{2}\right)! H_{n-k-l}}{(n-k-l)! (l+2k+2)! \left(\frac{l}{2}\right)!} P_k(x),$$

where $H_n(x)$ is the Hermite polynomial of degree n (see [7]).

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally in this paper. They read and approved the final manuscript.

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