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# Approximate homomorphisms and derivations on random Banach algebras

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**Abstract**
The motivation of this paper is to investigate the stability of homomorphisms and derivations on random Banach algebras.
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**Keywords:** random normed algebra; generalized Hyers-Ulam stability; random homomorphism; random derivation; generalized additive functional equation

## 1 Introduction and preliminaries

The study of stability problems originated from a famous talk *Under what condition does there exist a homomorphism near an approximate homomorphism?* given by S. M. Ulam [38] in 1940. Next year, in 1941, D. H. Hyers [15] answered affirmatively the question of Ulam for additive mappings between Banach spaces.

Aoki [3] and Rassias [26] provided a generalization of the Hyers theorem for additive and linear functions respectively, by allowing the Cauchy difference to be unbounded.

**Theorem 1.1** (Th. M. Rassias) *Let $X$ be a normed space, $Y$ be a Banach space and $f : X \to Y$ be a function such that*

$$\left\|f(x+y)-f(x)-f(y)\right\| \leq \varepsilon\left(\|x\|^p+\|y\|^p\right) \tag{1.1}$$

*for all $x, y \in X$, where $\varepsilon$ and $p$ are constants with $\varepsilon > 0$ and $p < 1$. Then there exists a unique additive function $A : X \to Y$ satisfying*

$$\left\|f(x)-A(x)\right\| \leq \varepsilon\|x\|^p/\left(1-2^{p-1}\right) \tag{1.2}$$

*for all $x \in X$. If $p < 0$, then the inequality (1.1) holds for $x, y \neq 0$ and (1.2) for $x \neq 0$. Also, if for each fixed $x \in X$ the function $t \mapsto f(tx)$ is continuous in $t \in \mathbb{R}$, then $A$ is linear.*

The above theorem had a lot of influence on the development of the generalization of the Hyers-Ulam stability concept during the last three decades. This new concept is known as generalized Hyers-Ulam stability or Hyers-Ulam-Rassias stability of functional equations (see [7, 16]). Furthermore, Găvruta [13] provided a generalization of Rassias' theorem which allows the Cauchy difference to be controlled by a general unbounded function.

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During the last three decades, a number of papers and research monographs have been published on various generalizations and applications of the generalized Hyers-Ulam stability to a number of functional equations and mappings (see [4, 6, 8, 9, 12, 17–19, 24] and [27–34]). We also refer the readers to the books [1, 7, 10, 16, 20, 21, 28].

Recently, Khodaei and Rassias [22] introduced the generalized additive functional equation

$$\begin{aligned}&\sum_{k=2}^{n}\left(\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n}\right)f\left(\sum_{i=1,i\neq i_1,\ldots,i_{n-k+1}}^{n}a_ix_i-\sum_{r=1}^{n-k+1}a_{i_r}x_{i_r}\right)\\&\quad+f\left(\sum_{i=1}^{n}a_ix_i\right)=2^{n-1}a_1f(x_1),\end{aligned}\tag{1.3}$$

where $a_1,\ldots,a_n\in\mathbb{Z}\backslash\{0\}$ with $a_1\neq\pm1$, and they established a general solution and the generalized Hyers-Ulam stability for the functional equation (1.3) in various spaces. They proved that a function $f$ between real vector spaces $X$ and $Y$ is a solution of (1.3) if and only if $f$ is additive.

In the sequel we adopt the usual terminology, notations and conventions of the theory of random normed spaces, as in [36, 37]. Throughout this paper, let $\Delta^+$ be the space of distribution functions, that is,

$$\begin{aligned}\Delta^+ := \big\{&F:\mathbb{R}\cup\{-\infty,\infty\}\to[0,1]:F\text{ is left-continuous,}\\&\text{non-decreasing on }\mathbb{R},F(0)=0\text{ and }F(+\infty)=1\big\}\end{aligned}$$

and the subset $D^+\subseteq\Delta^+$ is the set

$$D^+=\big\{F\in\Delta^+:l^-F(+\infty)=1\big\},$$

where, $l^-f(x)$ denotes the left limit of the function $f$ at the point $x$. The space $\Delta^+$ is partially ordered by the usual point-wise ordering of functions, i.e., $F\leq G$ if and only if $F(t)\leq G(t)$ for all $t\in\mathbb{R}$. The maximal element for $\Delta^+$ in this order is the distribution function given by

$$\varepsilon_0(t)=\begin{cases}0, & \text{if } t\leq 0,\\ 1, & \text{if } t>0.\end{cases}$$

**Definition 1.2** ([36]) A function $T:[0,1]\times[0,1]\to[0,1]$ is a continuous triangular norm (briefly, a $t$-norm) if $T$ satisfies the following conditions:

(a) $T$ is commutative and associative;
(b) $T$ is continuous;
(c) $T(a,1)=a$ for all $a\in[0,1]$;
(d) $T(a,b)\leq T(c,d)$ whenever $a\leq c$ and $b\leq d$ for all $a,b,c,d\in[0,1]$.

Typical examples of continuous $t$-norms are $T_P(a,b)=ab$, $T_M(a,b)=\min(a,b)$ and $T_L(a,b)=\max(a+b-1,0)$ (the Łukasiewicz $t$-norm).

Recall (see [14]) that if $T$ is a $t$-norm and $\{x_n\}$ is a given sequence of numbers in $[0,1]$, $T_{i=1}^n x_i$ is defined recurrently by

$$T_{i=1}^n x_i = \begin{cases} x_1, & \text{if } n = 1, \\ T\left(T_{i=1}^{n-1} x_i, x_n\right), & \text{if } n \geq 2. \end{cases}$$

$T_{i=n}^\infty x_i$ is defined as $T_{i=1}^\infty x_{n+i}$.

It is known [14] that for the Łukasiewicz $t$-norm the following implication holds:

$$\lim_{n\to\infty} (T_L)_{i=1}^\infty x_{n+i} = 1 \quad \Longleftrightarrow \quad \sum_{n=1}^\infty (1 - x_n) < \infty.$$

**Definition 1.3** ([37]) A *random normed space* (briefly, RN-space) is a triple $(X, \mu, T)$, where $X$ is a vector space, $T$ is a continuous $t$-norm, and $\mu$ is a function from $X$ into $D^+$ such that, the following conditions hold:

(RN1) $\mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$;
(RN2) $\mu_{\alpha x}(t) = \mu_x(\frac{t}{|\alpha|})$ for all $x \in X$, $\alpha \neq 0$;
(RN3) $\mu_{x+y}(t+s) \geq T(\mu_x(t), \mu_y(s))$ for all $x, y \in X$ and $t, s \geq 0$.

**Definition 1.4** Let $(X, \mu, T)$ be a RN-space.

(1) A sequence $\{x_n\}$ in $X$ is said to be *convergent* to $x$ in $X$ if, for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $N$ such that $\mu_{x_n - x}(\epsilon) > 1 - \lambda$ whenever $n \geq N$.
(2) A sequence $\{x_n\}$ in $X$ is called *Cauchy* if, for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $N$ such that $\mu_{x_n - x_m}(\epsilon) > 1 - \lambda$ whenever $n \geq m \geq N$.
(3) A RN-space $(X, \mu, T)$ is said to be *complete* if and only if every Cauchy sequence in $X$ is convergent to a point in $X$. A complete RN-space is said to be a random Banach space.

**Theorem 1.5** ([36]) *If* $(X, \mu, T)$ *is a RN-space and* $\{x_n\}$ *is a sequence such that* $x_n \to x$, *then* $\lim_{n\to\infty} \mu_{x_n}(t) = \mu_x(t)$ *almost everywhere.*

**Definition 1.6** A random normed algebra is a random normed space with algebraic structure such that (RN4) $\mu_{xy}(ts) = \mu_x(t)\mu_y(s)$ for all $x, y \in X$ and all $t, s > 0$.

**Definition 1.7** Let $(X, \mu, T)$ and $(Y, \mu, T)$ be random normed algebras:

(i) An additive mapping $H : X \to Y$ is called a random homomorphism if $H(xy) = H(x)H(y)$ for all $x, y \in X$.
(ii) An additive mapping $D : X \to Y$ is called a random derivation if $D(xy) = D(x)y - xD(y)$ for all $x, y \in X$.

The theory of random normed spaces is important as a generalization of the deterministic result of linear normed spaces and also in the study of random operator equations. The random normed spaces may also provide us with the appropriate tools to study the geometry of nuclear physics and have an important application in quantum particle physics. The generalized Hyers-Ulam stability of different functional equations in random and fuzzy normed spaces and random and fuzzy normed algebras has been recently studied in Alsina

[2], Miheţ et al. [23], Baktash et al. [5], Saadati et al. [35], Gordji et al. [11], and Park et al. [25].

In this paper, we prove the generalized Hyers-Ulam stability of random homomorphisms and random derivations associated with the generalized additive functional equation (1.3) in random Banach algebras.

## 2 Main results

We use the following abbreviation for a given function $f$:

$$Df(x_1,\dots,x_n,a,b)=\sum_{k=2}^{n}\left(\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n}\right)f\left(\sum_{i=1,i\neq i_1,\dots,i_{n-k+1}}^{n}a_ix_i-\sum_{r=1}^{n-k+1}a_{i_r}x_{i_r}\right)$$
$$+f\left(\sum_{i=1}^{n}a_ix_i\right)-2^{n-1}a_1f(x_1)+f(ab)-f(a)f(b).$$

**Theorem 2.1** *Let $X$ be a real algebra, $(Y,\Lambda,T)$ be a random Banach algebra and $\xi: X^{n+2}\to D^+$ ($n\in\mathbb{N}$, $n\geq 2$ and $\xi(x_1,\dots,x_n,a,b)$ denoted by $\xi_{x_1,\dots,x_n,a,b}$) be a function such that*

$$\lim_{m\to\infty}\xi_{a_1^mx_1,\dots,a_1^mx_n,a_1^ma,a_1^mb}\left(|a_1|^mt\right)=1 \tag{2.1}$$

*for all $x_1,\dots,x_n,a,b\in X$, $t>0$ and*

$$\lim_{m\to\infty}T_{\ell=1}^{\infty}\left(\xi_{a_1^{m+\ell-1}x,0,\dots,0}\left(2^{n-\ell-1}|a_1|^{m+\ell-1}t\right)\right)=1 \tag{2.2}$$

*for all $x\in X$ and all $t>0$. Suppose that $f:X\to Y$ is a function satisfying*

$$\Lambda_{Df(x_1,\dots,x_n,a,b)}(t)\geq\xi_{x_1,\dots,x_n,a,b}(t) \tag{2.3}$$

*for all $x_1,\dots,x_n,a,b\in X$ and $t>0$. Then there exists a unique homomorphism $H:X\to Y$ such that*

$$\Lambda_{f(x)-H(x)}(t)\geq T_{\ell=1}^{\infty}\left(\xi_{a_1^{\ell-1}x,0,\dots,0}\left(2^{n-\ell-1}|a_1|^{\ell}t\right)\right) \tag{2.4}$$

*for all $x\in X$ and $t>0$.*

*Proof* Putting $x_1=x$ and $a=b=x_i=0$ ($i=2,\dots,n$) in (2.3), we obtain that

$$\Lambda_{(\sum_{k=2}^{n}(\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n})f(a_1x)+f(a_1x)-2^{n-1}a_1f(x))}(t)\geq\xi_{x,0,\dots,0}(t)$$

for all $x\in X$ and $t>0$, that is,

$$\Lambda_{\left(\binom{n-1}{n-1}+\binom{n-1}{n-2}+\cdots+\binom{n-1}{1}+1\right)f(a_1x)-2^{n-1}a_1f(x)}(t)\geq\xi_{x,0,\dots,0}(t)$$

for all $x\in X$ and $t>0$. It follows from the last inequality that

$$\Lambda_{\left(1+\sum_{\ell=1}^{n-1}\binom{n-1}{\ell}\right)f(a_1x)-2^{n-1}a_1f(x)}(t)\geq\xi_{x,0,\dots,0}(t)$$

for all $x \in X$ and $t > 0$; hence by using the relation $1 + \sum_{\ell=1}^{n-1} \binom{n-1}{\ell} = 2^{n-1}$, we have

$$\Lambda_{2^{n-1}f(a_1x)-2^{n-1}a_1f(x)}(t) \geq \xi_{x,0,\dots,0}(t)$$

for all $x \in X$ and $t > 0$. So we have

$$\Lambda_{\frac{f(a_1^m x)}{a_1^m}-f(x)}(t) \geq T_{\ell=1}^{m}\left(\xi_{a_1^{\ell-1}x,0,\dots,0}\left(2^{n-\ell-1}|a_1|^{\ell}t\right)\right)$$

for all $x \in X$ and $t > 0$. We can show that the sequence $\{\frac{f(a_1^m x)}{a_1^m}\}$ is convergent. Therefore, one can define the function $H : X \to Y$ by

$$H(x) := \lim_{m\to\infty} \frac{1}{a_1^m} f\left(a_1^m x\right)$$

for all $x \in X$. Now, if we put $a = b = 0$, and replace $x_1, \dots, x_n$ with $a_1^m x_1, \dots, a_1^m x_n$ in (2.3) respectively, it follows that

$$\Lambda_{\frac{Df(a_1^m x_1,\dots,a_1^m x_n,0,0)}{a_1^m}}(t) \geq \xi_{a_1^m x_1,\dots,a_1^m x_n,0,0}\left(|a_1|^m t\right) \tag{2.5}$$

for all $x_1, \dots, x_n \in X$ and all $t > 0$. By letting $m \to \infty$ in (2.5), we have $DH(x_1, \dots, x_n, 0, 0) = 0$; thus $H$ satisfies (1.3). Hence the function $H : X \to Y$ is additive (see also [22]). For the uniqueness property of $H$, see paper [22].

Finally, we show that $H$ is multiplicative. Since $H(a_1^m x) = a_1^m H(x)$ for all $x \in X$ and $m \in \mathbb{N}$, from (2.1) it follows that

$$\begin{aligned}
\Lambda_{H(ab)-H(a)H(b)}(t) &= \Lambda_{\frac{1}{a_1^m}H(a_1^m ab)-H(a)H(b)}(t) \\
&= \lim_{m\to\infty} \Lambda_{\frac{1}{a_1^{2m}}f(a_1^{2m}ab)-\frac{1}{a_1^{2m}}f(a_1^m a)f(a_1^m b)}(t) \\
&= \lim_{m\to\infty} \Lambda_{\frac{Df(0,0,\dots,0,a_1^m a,a_1^m b)}{a_1^{2m}}}(t) \\
&\geq \lim_{m\to\infty} \xi_{0,0,\dots,0,a_1^m a,a_1^m b}\left(|a_1|^{2m}t\right) \\
&= 1
\end{aligned}$$

for all $a, b \in X$ and all $t > 0$. Therefore, there exists a unique random homomorphism $H : X \to Y$ satisfying (2.4). □

In the following theorem, we establish the stability of derivations on random Banach algebras. We use the following abbreviation for a given function $f$:

$$\begin{aligned}
\Delta f(x_1, \dots, x_n, a, b) = {}& \sum_{k=2}^{n}\left(\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n}\right) f\left(\sum_{i=1,i\neq i_1,\dots,i_{n-k+1}}^{n} a_i x_i - \sum_{r=1}^{n-k+1} a_{i_r}x_{i_r}\right) \\
&+ f\left(\sum_{i=1}^{n} a_i x_i\right) - 2^{n-1}a_1 f(x_1) + f(ab) - f(a)b - af(b).
\end{aligned}$$

**Theorem 2.2** *Let $(X, \Lambda, T)$ be a random Banach algebra and $\xi : X^{n+2} \to D^+$ be a function such that (2.1) and (2.2) hold for all $x, x_1, \dots, x_n, a, b \in X$ and all $t > 0$. Suppose that $f : X \to X$ is a function satisfying*

$$\Lambda_{\Delta f(x_1,\dots,x_n,a,b)}(t) \geq \xi_{x_1,\dots,x_n,a,b}(t) \tag{2.6}$$

*for all $x_1, \dots, x_n, a, b \in X$ and $t > 0$. Then there exists a unique derivation $D : X \to X$ such that*

$$\Lambda_{f(x)-D(x)}(t) \geq T_{\ell=1}^{\infty}\left(\xi_{a_1^{\ell-1}x,0,\dots,0}\left(2^{n-\ell-1}|a_1|^{\ell}t\right)\right) \tag{2.7}$$

*for all $x \in X$ and $t > 0$.*

*Proof* By the same reasoning as in the proof of Theorem 2.1, the sequence $\{\frac{f(a_1^m x)}{a_1^m}\}$ is convergent for all $x \in X$, and the function $D : X \to X$ defined by

$$D(x) := \lim_{m\to\infty} \frac{1}{a_1^m} f\left(a_1^m x\right)$$

for all $x \in X$, is a unique additive function which satisfies (2.7). We have to show that $D : X \to X$ is a derivation.

Since $D(a_1^m x) = a_1^m D(x)$ for all $x \in X$ and $m \in \mathbb{N}$, from (2.1) it follows that

$$\begin{aligned}
\Lambda_{D(ab)-D(a)b-aD(b)}(t) &= \Lambda_{\frac{1}{a_1^m}D(a_1^m ab)-\frac{1}{a_1^m}D(a)a_1^m b-a_1^m a\frac{1}{a_1^m}D(b)}(t) \\
&= \lim_{m\to\infty} \Lambda_{\frac{1}{a_1^{2m}}f(a_1^{2m}ab)-\frac{1}{a_1^{2m}}f(a_1^m a)a_1^m b-\frac{1}{a_1^{2m}}a_1^m af(a_1^m b)}(t) \\
&= \lim_{m\to\infty} \Lambda_{\frac{\Delta f(0,0,\dots,0,a_1^m a,a_1^m b)}{a_1^{2m}}}(t) \\
&\geq \lim_{m\to\infty} \xi_{0,0,\dots,0,a_1^m a,a_1^m b}\left(|a_1|^{2m}t\right) \\
&= 1
\end{aligned}$$

for all $a, b \in X$ and all $t > 0$. This means that $D$ is a derivation on $X$. □

**Competing interests**
The authors declare that they have no competing interests.

**Author's contributions**
All authors read and approved the final manuscript.

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