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q-analogue of a new sequence of linear positive operators

Vijay Gupta[1], Taekyun Kim[2*] and Sang-Hun Lee[3]

*Correspondence: tkkim@kw.ac.kr
[2]Department of Mathematics, Kwangwoon University, Seoul 139-701, S. Korea
Full list of author information is available at the end of the article

Abstract
This paper deals with Durrmeyer type generalization of q-Baskakov type operators using the concept of q-integral, which introduces a new sequence of positive q-integral operators. We show that this sequence is an approximation process in the polynomial weighted space of continuous functions defined on the interval $[0,\infty)$. An estimate for the rate of convergence and weighted approximation properties are also obtained.
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1 Introduction

In the year 2003 Agrawal and Mohammad [1] introduced a new sequence of linear positive operators by modifying the well-known Baskakov operators having weight functions of Szasz basis function as

$$\mathcal{D}_n(f,x) = n\sum_{k=1}^{\infty} p_{n,k}(x)\int_0^{\infty} s_{n,k-1}(t)f(t)\,dt + p_{n,0}(x)f(0), \quad x\in[0,\infty), \tag{1.1}$$

where

$$p_{n,k}(x) = \binom{n+k-1}{k}\frac{x^k}{(1+x)^{n+k}}, \qquad s_{n,k}(t) = e^{-nt}\frac{(nt)^k}{k!}.$$

It is observed in [1] that these operators reproduce constant as well as linear functions. Later, some direct approximation results for the iterative combinations of these operators were studied in [14].

A lot of works on q-calculus are available in literature of different branches of mathematics and physics. For systematic study, we refer to the work of Ernst [5], Kim [10, 11], and Kim and Rim [9]. The application of q-calculus in approximation theory was initiated by Phillips [13], who was the first to introduce q-Bernstein polynomials and study their approximation properties. Very recently the q-analogues of the Baskakov operators and their Kantorovich and Durrmeyer variants have been studied in [2, 3] and [7] respectively. We recall some notations and concepts of q-calculus. All of the results can be found in [5] and [8]. In what follows, q is a real number satisfying $0 < q < 1$.

For $n \in \mathbb{N}$,

$$[n]_q := \frac{1-q^n}{1-q},$$

$$[n]_q! := \begin{cases} [n]_q[n-1]_q \cdots [1]_q, & n = 1, 2, \ldots, \\ 1, & n = 0. \end{cases}$$

The q-binomial coefficients are given by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[k]_q![n-k]_q!}, \quad 0 \le k \le n.$$

The q-Beta integral is defined by [12]

$$\Gamma_q(t) = \int_0^{\frac{1}{1-q}} x^{t-1} E_q(-qx)\, d_q x, \quad t > 0, \tag{1.2}$$

which satisfies the following functional equation:

$$\Gamma_q(t+1) = [t]_q \Gamma_q(t), \qquad \Gamma_q(1) = 1.$$

For $f \in C[0,\infty)$, $q > 0$ and each positive integer n, the q-Baskakov operators [2] are defined as

$$\begin{aligned} \mathcal{B}_{n,q}(f,x) &= \sum_{k=0}^{\infty} \begin{bmatrix} n+k-1 \\ k \end{bmatrix}_q q^{\frac{k(k-1)}{2}} \frac{x^k}{(1+x)_q^{n+k}} f\left(\frac{[k]_q}{q^{k-1}[n]_q}\right) \\ &= \sum_{k=0}^{\infty} p_{n,k}^q(x) f\left(\frac{[k]_q}{q^{k-1}[n]_q}\right), \end{aligned} \tag{1.3}$$

where

$$(1+x)_q^n := \begin{cases} (1+x)(1+qx)\cdots\left(1+q^{n-1}x\right), & n = 1, 2, \ldots, \\ 1, & n = 0. \end{cases}$$

Remark 1 The first three moments of the q-Baskakov operators are given by

$$\mathcal{B}_{n,q}(1,x) = 1,$$

$$\mathcal{B}_{n,q}(t,x) = x,$$

$$\mathcal{B}_{n,q}\left(t^2,x\right) = x^2 + \frac{x}{[n]_q}\left(1 + \frac{1}{q}x\right).$$

As the operators $\mathcal{D}_n(f,x)$ have mixed basis functions in summation and integration and have an interesting property of reproducing linear functions, we were motivated to study these operators further. Here we define the q-analogue of the operators as

$$\mathcal{D}_n^q(f,x) = [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{q/(1-q^n)} q^{-k} s_{n,k-1}^q(t) f\left(tq^{-k}\right) d_q t + p_{n,0}^q(x) f(0), \tag{1.4}$$

where $x \in [0, \infty)$ and

$$p_{n,k}^q(x) = \begin{bmatrix} n+k-1 \\ k \end{bmatrix}_q q^{\frac{k(k-1)}{2}} \frac{x^k}{(1+x)_q^{n+k}}, \qquad s_{n,k}^q(t) = E_q\bigl(-[n]_q t\bigr) \frac{([n]_q t)^k}{[k]_q!}.$$

In case $q = 1$, the above operators reduce to the operators (1.1). In the present paper, we estimate a local approximation theorem and the rate of convergence of these new operators as well as their weighted approximation properties.

2 Moment estimation

Lemma 1 *The following equalities hold:*

(i) $\mathcal{D}_n^q(1, x) = 1$,
(ii) $\mathcal{D}_n^q(t, x) = x$,
(iii) $\mathcal{D}_n^q(t^2, x) = x^2 + \frac{x}{[n]_q}(1 + q + \frac{x}{q})$.

Proof The operators $\mathcal{D}_n^q$ are well defined on the function $1, t, t^2$. Then for every $x \in [0, \infty)$, we obtain

$$\mathcal{D}_n^q(1, x) = [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{q/(1-q^n)} q^{-k} \frac{([n]_q t)^{k-1}}{[k-1]_q!} E_q\bigl(-[n]_q t\bigr)\, d_q t + p_{n,0}^q(x).$$

Substituting $[n]_q t = qy$ and using (1.2), we have

$$\begin{aligned} \mathcal{D}_n^q(1, x) &= [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{1/(1-q)} q^{-k} \frac{(qy)^{k-1}}{[k-1]_q!} E_q(-qy) \frac{q\, d_q y}{[n]_q} + p_{n,0}^q(x) \\ &= \sum_{k=1}^{\infty} p_{n,k}^q(x) + p_{n,0}^q(x) = \mathcal{B}_{n,q}(1, x) = 1, \end{aligned}$$

where $\mathcal{B}_{n,q}(f, x)$ is the q-Baskakov operator defined by (1.3).

Next, we have

$$\mathcal{D}_n^q(t, x) = [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{q/(1-q^n)} q^{-k} \frac{([n]_q t)^{k-1}}{[k-1]_q!} E_q\bigl(-[n]_q t\bigr) t q^{-k}\, d_q t.$$

Again substituting $[n]_q t = qy$ and using (1.2), we have

$$\begin{aligned} \mathcal{D}_n^q(t, x) &= [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{1/(1-q)} q^{-k} \frac{(qy)^k}{[k-1]_q! [n]_q} E_q(-qy) \frac{q\, d_q y}{[n]_q q^k} \\ &= \sum_{k=0}^{\infty} p_{n,k}^q(x) q \frac{[k]_q}{[n]_q q^k} = \mathcal{B}_{n,q}(t, x) = x. \end{aligned}$$

Finally,

$$\mathcal{D}_n^q\bigl(t^2, x\bigr) = [n]_q \sum_{k=0}^{\infty} p_{n,k}^q(x) \int_0^{q/(1-q^n)} q^{-k} \frac{([n]_q t)^{k-1}}{[k-1]_q!} E_q\bigl(-[n]_q t\bigr) t^2 q^{-2k}\, d_q t.$$

Again substituting $[n]_q t = qy$, using (1.2) and $[k+1]_q = [k]_q + q^k$, we have

$$\begin{aligned}\mathcal{D}_n^q(t^2,x) &= [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{1/(1-q)} q^{-k} \frac{(qy)^{k+1}}{[k-1]_q![n]_q^2} E_q(-qy) q^{-2k} \frac{q\,d_q y}{[n]_q} \\ &= \sum_{k=1}^{\infty} p_{n,k}^q(x) \frac{[k+1]_q[k]_q}{[n]_q^2 q^{2k-2}} \\ &= \sum_{k=1}^{\infty} p_{n,k}^q(x) \frac{([k]_q + q^k)[k]_q}{[n]_q^2 q^{2k-2}} \\ &= \mathcal{B}_{n,q}(t^2,x) + \frac{q}{[n]_q}\mathcal{B}_{n,q}(t,x) = x^2 + \frac{x}{[n]_q}\left(1+q+\frac{x}{q}\right). \qquad \square\end{aligned}$$

Remark 2 If we put $q = 1$, we get the moments of a new sequence $\mathcal{D}_n(f,x)$ considered in [1] as operators as

$$\begin{aligned}&\mathcal{D}_n(t-x,x) = 0, \\ &\mathcal{D}_n\left((t-x)^2,x\right) = \frac{x(x+2)}{n}.\end{aligned}$$

Lemma 2 *Let $q \in (0,1)$, then for $x \in [0,\infty)$ we have*

$$\mathcal{D}_n^q\left((t-x)^2,x\right) = \frac{x(x+q[2]_q)}{q[n]_q}.$$

3 Direct theorems

By $C_B[0,\infty)$ we denote the space of real valued continuous bounded functions f on the interval $[0,\infty)$; the norm-$\|\cdot\|$ on the space $C_B[0,\infty)$ is given by

$$\|f\| = \sup_{0\le x<\infty} \left|f(x)\right|.$$

The Peetre's K-functional is defined by

$$K_2(f,\delta) = \inf\left\{\|f-g\| + \delta\left\|g''\right\| : g \in W_\infty^2\right\},$$

where $W_\infty^2 = \{g \in C_B[0,\infty) : g', g'' \in C_B[0,\infty)\}$. By [4, pp.177], there exists a positive constant $C > 0$ such that $K_2(f,\delta) \le C\omega_2(f,\delta^{1/2})$, $\delta > 0$ and the second order modulus of smoothness is given by

$$\omega_2\left(f,\sqrt{\delta}\right) = \sup_{0<h\le\sqrt{\delta}} \sup_{0\le x<\infty} \left|f(x+2h) - 2f(x+h) + f(x)\right|.$$

Also, for $f \in C_B[0,\infty)$ a usual modulus of continuity is given by

$$\omega(f,\delta) = \sup_{0<h\le\delta} \sup_{0\le x<\infty} \left|f(x+h) - f(x)\right|.$$

Theorem 1 *Let $f \in C_B[0,\infty)$ and $0 < q < 1$. Then for all $x \in [0,\infty)$ and $n \in N$, there exists an absolute constant $C > 0$ such that*

$$\left|\mathcal{D}_n^q(f,x) - f(x)\right| \le C\omega_2\left(f, \sqrt{\frac{x(x+q[2]_q)}{q[n]_q}}\right).$$

Proof Let $g \in W_\infty^2$ and $x, t \in [0,\infty)$. By Taylor's expansion, we have

$$g(t) = g(x) + g'(x)(t-x) + \int_x^t (t-u)g''(u)\,du.$$

Applying Lemma 2, we obtain

$$\mathcal{D}_n^q(g,x) - g(x) = \mathcal{D}_n^q\left(\int_x^t (t-u)g''(u)\,du, x\right).$$

Obviously, we have $|\int_x^t (t-u)g''(u)\,du| \le (t-x)^2\|g''\|$. Therefore,

$$\left|\mathcal{D}_n^q(g,x) - g(x)\right| \le \mathcal{D}_n^q\left((t-x)^2, x\right)\left\|g''\right\| = \frac{x(x+q[2]_q)}{q[n]_q}\left\|g''\right\|.$$

Using Lemma 1, we have

$$\left|\mathcal{D}_n^q(f,x)\right| \le [n]_q \sum_{k=1}^{\infty} p_{n,k}^q(x) \int_0^{q/(1-q^n)} q^{-k} s_{n,k-1}^q(t)\left|f\left(tq^{-k}\right)\right| d_q t + p_{n,0}^q(x)\left|f(0)\right| \le \|f\|.$$

Thus

$$\begin{aligned}\left|\mathcal{D}_n^q(f,x) - f(x)\right| &\le \left|\mathcal{D}_n^q(f-g,x) - (f-g)(x)\right| + \left|\mathcal{D}_n^q(g,x) - g(x)\right| \\ &\le 2\|f-g\| + \frac{x(x+q[2]_q)}{q[n]_q}\left\|g''\right\|.\end{aligned}$$

Finally, taking the infimum over all $g \in W_\infty^2$ and using the inequality $K_2(f,\delta) \le C\omega_2(f,\delta^{1/2})$, $\delta > 0$, we get the required result. This completes the proof of Theorem 1. □

We consider the following class of functions:

Let $H_{x^2}[0,\infty)$ be the set of all functions f defined on $[0,\infty)$ satisfying the condition $|f(x)| \le M_f(1+x^2)$, where M_f is a constant depending only on f. By $C_{x^2}[0,\infty)$, we denote the subspace of all continuous functions belonging to $H_{x^2}[0,\infty)$. Also, let $C_{x^2}^*[0,\infty)$ be the subspace of all functions $f \in C_{x^2}[0,\infty)$, for which $\lim_{|x|\to\infty} \frac{f(x)}{1+x^2}$ is finite. The norm on $C_{x^2}^*[0,\infty)$ is $\|f\|_{x^2} = \sup_{x\in[0,\infty)} \frac{|f(x)|}{1+x^2}$. We denote the modulus of continuity of f on closed interval $[0,a]$, $a > 0$ as by

$$\omega_a(f,\delta) = \sup_{|t-x|\le\delta} \sup_{x,t\in[0,a]} \left|f(t) - f(x)\right|.$$

We observe that for function $f \in C_{x^2}[0,\infty)$, the modulus of continuity $\omega_a(f,\delta)$ tends to zero.

Theorem 2 *Let $f \in C_{x^2}[0,\infty)$, $q \in (0,1)$ and $\omega_{a+1}(f,\delta)$ be its modulus of continuity on the finite interval $[0,a+1] \subset [0,\infty)$, where $a > 0$. Then for every $n > 2$,*

$$\left\| \mathcal{D}_n^q(f) - f \right\|_{C[0,a]} \le \frac{6M_f a(1+a^2)(2+a)}{q[n]_q} + 2\omega\left(f, \sqrt{\frac{a(a+q[2]_q)}{q[n]_q}}\right).$$

Proof For $x \in [0,a]$ and $t > a+1$, since $t - x > 1$, we have

$$\begin{aligned}\left|f(t) - f(x)\right| &\le M_f\left(2 + x^2 + t^2\right)\\ &\le M_f\left(2 + 3x^2 + 2(t-x)^2\right)\\ &\le 6M_f\left(1+a^2\right)(t-x)^2.\end{aligned} \tag{3.1}$$

For $x \in [0,a]$ and $t \le a+1$, we have

$$\left|f(t) - f(x)\right| \le \omega_{a+1}\left(f, |t-x|\right) \le \left(1 + \frac{|t-x|}{\delta}\right)\omega_{a+1}(f,\delta) \tag{3.2}$$

with $\delta > 0$.

From (3.1) and (3.2) we can write

$$\left|f(t) - f(x)\right| \le 6M_f\left(1+a^2\right)(t-x)^2 + \left(1 + \frac{|t-x|}{\delta}\right)\omega_{a+1}(f,\delta) \tag{3.3}$$

for $x \in [0,a]$ and $t \ge 0$. Thus

$$\begin{aligned}\left|\mathcal{D}_n^q(f,x) - f(x)\right| &\le \mathcal{D}_n^q\left(\left|f(t) - f(x)\right|, x\right)\\ &\le 6M_f\left(1+a^2\right)\mathcal{D}_n^q\left((t-x)^2, x\right)\\ &\quad + \omega_{a+1}(f,\delta)\left(1 + \frac{1}{\delta}\mathcal{D}_n^q\left((t-x)^2, x\right)\right)^{\frac{1}{2}}.\end{aligned}$$

Hence, by using Schwarz inequality and Lemma 2, for every $q \in (0,1)$ and $x \in [0,a]$

$$\begin{aligned}\left|\mathcal{D}_n^q(f,x) - f(x)\right| &\le \frac{6M_f(1+a^2)x(q[2]_q + x)}{q[n]_q}\\ &\quad + \omega_{a+1}(f,\delta)\left(1 + \frac{1}{\delta}\sqrt{\frac{x(q[2]_q + x)}{q[n]_q}}\right)\\ &\le \frac{6M_f a(1+a^2)(2+a)}{q[n]_q} + \omega_{a+1}(f,\delta)\left(1 + \frac{1}{\delta}\sqrt{\frac{a(a+q[2]_q)}{q[n]_q}}\right).\end{aligned}$$

By taking $\delta = \sqrt{\frac{a(q[2]_q + a)}{q[n]_q}}$ we get the assertion of our theorem. □

4 Higher order moments and an asymptotic formula

Lemma 3 ([6]) *Let $0 < q < 1$, we have*

$$\mathcal{B}_{n,q}\left(t^3, x\right) = \frac{1}{[n]_q}x + \frac{1+2q}{q^2}\frac{[n+1]_q}{[n]_q^2}x^2 + \frac{1}{q^3}\frac{[n+1]_q[n+2]_q}{[n]_q^2}x^3,$$

$$\begin{aligned}\mathcal{B}_{n,q}\left(t^4,x\right) &= \frac{1}{[n]_q^3}x + \frac{1}{q^3}\left(1+3q+3q^2\right)\frac{[n+1]_q}{[n]_q^3}x^2 \\ &\quad + \frac{1}{q^5[2]_q}\left(1+3q+5q^2+3q^3\right)\frac{[n+1]_q[n+2]_q}{[n]_q^3}x^3 \\ &\quad + \frac{1}{q^6[2]_q[3]_q[4]_q}\left(1+3q+5q^2+6q^3+5q^4+3q^5+q^6\right) \\ &\quad \times \frac{[n+1]_q[n+2]_q[n+3]_q}{[n]_q^3}x^4.\end{aligned}$$

Now, we present higher order moments for the operators (1.4).

Lemma 4 *Let* $0 < q < 1$, *we have*

$$\begin{aligned}\mathcal{D}_n^q\left(t^3,x\right) &= \frac{[n+1]_q[n+2]_q}{q^3[n]_q^2}x^3 + \left(\frac{1+2q}{q^2}\frac{[n+1]_q}{[n]_q^2} + \frac{(2+q)q}{[n]_q} + \frac{(2+q)}{[n]_q}\right)x^2 \\ &\quad + \left(\frac{1}{[n]_q^2} + \frac{(2+q)q}{[n]_q^2} + \frac{(1+q)q^2}{[n]_q^2}\right)x,\end{aligned}$$

$$\begin{aligned}&\mathcal{D}_n^q\left(t^4,x\right) \\ &= \left(\frac{(1+3q+5q^2+6q^3+5q^4+3q^5+q^6)}{q^6[2]_q[3]_q[4]_q}\frac{[n+1]_q[n+2]_q[n+3]_q}{[n]_q^3}\right)x^4 \\ &\quad + \left(\frac{(1+3q+5q^2+3q^3)}{q^5[2]_q}\frac{[n+1]_q[n+2]_q}{[n]_q^3} + \frac{q(3+2q+q^2)}{[n]_q^3}[n+1]_q[n+2]_q\right)x^3 \\ &\quad + \left(\frac{(1+3q+3q^2)}{q^3}\frac{[n+1]_q}{[n]_q} + \frac{(1+2q)(3+2q+q^2)}{q[n]_q^3}[n+1]_q\right. \\ &\quad \left. + \frac{q^2(3+4q+3q^2+q^3)}{[n]_q^2} + \frac{q(3+4q+3q^2+q^3)}{[n]_q^3}\right)x^2 \\ &\quad + \left(\frac{1}{[n]_q} + \frac{q(3+2q+q^2)}{[n]_q^3}\right. \\ &\quad \left. + \frac{q^2(3+4q+3q^2+q^3)}{[n]_q^3} + \frac{q^3(1+2q+2q^2+q^3)}{[n]_q^3}\right)x.\end{aligned}$$

The proof of Lemma 4 can be obtained by using Lemma 3.

We consider the following classes of functions:

$$\begin{aligned}C_m[0,\infty) &:= \left\{f \in C[0,\infty) : \exists M_f > 0\ \left|f(x)\right| < M_f\left(1+x^m\right) \text{ and } \|f\|_m := \sup_{x\in[0,\infty)}\frac{|f(x)|}{1+x^m}\right\}, \\ C_m^*[0,\infty) &:= \left\{f \in C_m[0,\infty) : \lim_{x\to\infty}\frac{|f(x)|}{1+x^m} < \infty\right\}, \quad m \in \mathbb{N}.\end{aligned}$$

Theorem 3 *Let* $q_n \in (0,1)$, *then the sequence* $\{\mathcal{D}_n^{q_n}(f)\}$ *converges to* f *uniformly on* $[0,A]$ *for each* $f \in C_2^*[0,\infty)$ *if and only if* $\lim_{n\to\infty} q_n = 1$.

Theorem 4 *Assume that $q_n \in (0,1)$, $q_n \to 1$ and $q_n^n \to a$ as $n \to \infty$. For any $f \in C_2^*[0,\infty)$ such that $f', f'' \in C_2^*[0,\infty)$ the following equality holds*

$$\lim_{n\to\infty} [n]_{q_n} \big(\mathcal{D}_n^{q_n}(f;x) - f(x)\big) = \big(x^2 + 2x\big) f''(x)$$

uniformly on any $[0,A]$, $A > 0$.

Proof Let $f, f', f'' \in C_2^*[0,\infty)$ and $x \in [0,\infty)$ be fixed. By using Taylor's formula, we may write

$$f(t) = f(x) + f'(x)(t-x) + \frac{1}{2} f''(x)(t-x)^2 + r(t;x)(t-x)^2, \tag{4.1}$$

where $r(t;x)$ is the Peano form of the remainder, $r(\cdot;x) \in C_2^*[0,\infty)$ and $\lim_{t\to x} r(t;x) = 0$. Applying $\mathcal{D}_n^{q_n}$ to (4.1), we obtain

$$[n]_{q_n}\big(\mathcal{D}_n^{q_n}(f;x) - f(x)\big) = \frac{1}{2} f''(x)[n]_{q_n} \mathcal{D}_n^{q_n}\big((t-x)^2;x\big) + [n]_{q_n} \mathcal{D}_n^{q_n}\big(r(t;x)(t-x)^2;x\big).$$

By the Cauchy-Schwarz inequality, we have

$$\mathcal{D}_n^{q_n}\big(r(t;x)(t-x)^2;x\big) \le \sqrt{\mathcal{D}_n^{q_n}\big(r^2(t;x);x\big)}\sqrt{\mathcal{D}_n^{q_n}\big((t-x)^4;x\big)}. \tag{4.2}$$

Observe that $r^2(x;x) = 0$ and $r^2(\cdot;x) \in C_2^*[0,\infty)$. Then it follows from Theorem 3 and Lemma 4, that

$$\lim_{n\to\infty} \mathcal{D}_n^{q_n}\big(r^2(t;x);x\big) = r^2(x;x) = 0 \tag{4.3}$$

uniformly with respect to $x \in [0,A]$. Now from (4.2), (4.3) and Remark 2, we get immediately

$$\lim_{n\to\infty} [n]_{q_n} \mathcal{D}_n^{q_n}\big(r(t;x)(t-x)^2;x\big) = 0.$$

Then, we get the following

$$\begin{aligned}
&\lim_{n\to\infty} [n]_{q_n}\big(\mathcal{D}_n^{q_n}(f;x) - f(x)\big) \\
&\quad = \lim_{n\to\infty}\left(\frac{1}{2} f''(x)[n]_{q_n}\mathcal{D}_n^{q_n}\big((t-x)^2;x\big) + [n]_{q_n}\mathcal{D}_n^{q_n}\big(r(t;x)(t-x)^2;x\big)\right) \\
&\quad = \big(x^2+2x\big) f''(x).
\end{aligned}$$

□

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed in preparing the manuscript equally.

Author details
[1]School of Applied Sciences, Netaji Subhas Institute of Technology, Sector 3 Dwarka, New Delhi 110078, India.
[2]Department of Mathematics, Kwangwoon University, Seoul 139-701, S. Korea. [3]Division of General Education-Mathematics, Kwangwoon University, Seoul 139-701, S. Korea.

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