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Global attracting set for a class of nonautonomous neutral functional differential equations

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Abstract
In this paper, a class of nonlinear and nonautonomous neutral functional differential equations is considered. By developing a new integral inequality, we obtain sufficient conditions for the existence of a global attracting set of neutral functional differential equations with time-varying delays. The results have extended and improved the related reports in the literature.
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1 Introduction

The asymptotic properties of neutral functional differential equations have attracted considerable attention in the past few decades, and many significant results have been obtained [1–6]. One of the most popular ways to analyze the stability property and asymptotic behavior is the method of Lyapunov functionals [1–3]. However, to construct a suitable Lyapunov functional is not easy for certain equations. The characteristic equation is another important researching tool. It seems to work well for constant delays in neutral equations [4, 5]. Meanwhile, the known approach to the study of differential inequalities for nonautonomous neutral functional differential equations was presented in Azbelev's book [7]. These inequality methods are based on the representation formula of a solution and the analysis of Cauchy, Green's and fundamental matrices. In this way, various assertions about the estimates of solutions, maximum principles and stability on neutral differential equations were obtained. Important results in this direction can be found in [7–10] and in the monograph [11]. Recently, by using differential and integral inequalities, Xu et al. have studied attracting and invariant sets of functional differential systems [12, 13] and impulsive functional differential equations [14]. However, the inequalities mentioned above are ineffective in studying the attracting sets of a class of nonlinear and nonautonomous neutral functional differential equations.

Motivated by the above discussions, in this paper, we will improve the inequality established in [15] so that it is effective for neutral functional differential equations. Combining with the properties of nonnegative matrices, we obtained some sufficient conditions ensuring the global attracting set for a class of nonlinear and nonautonomous neutral differential equations with time-varying coefficients and unbounded delays. The results extend the earlier publications.

2 Preliminaries

In this section, we introduce some notations and recall some basic definitions.

E denotes the $n \times n$ identity matrix, $\mathbb{R}$ is the set of real numbers and $\mathbb{R}_+ = [0, +\infty)$. $A \leq B$ ($A < B$) means that each pair of corresponding elements of A and B satisfies the inequality "$\leq$ ($<$)." Especially, A is called a nonnegative matrix if $A \geq 0$, where 0 denotes the $n \times n$ zero matrix.

$C[X, Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C \triangleq C\ [(-\infty, t_0], \mathbb{R}^n]$ denote the family of all bounded continuous $\mathbb{R}^n$-valued functions ϕ on $(-\infty, t_0]$, here $t_0 \geq 0$.

For $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $\varphi \in C$, $\tau \in C[\mathbb{R}, \mathbb{R}_+]$, we define $[x]^+ = (|x_1|, |x_2|, \ldots, |x_n|)^T$, $[A]^+ = (|a_{ij}|)_{n\times n}$, $[\varphi(t)]^+_{\tau(t)} = (\|\varphi_1(t)\|_{\tau(t)}, \|\varphi_2(t)\|_{\tau(t)}, \ldots, \|\varphi_n(t)\|_{\tau(t)})^T$, $\|\varphi_i(t)\|_{\tau(t)} = \sup_{0\leq s\leq \tau(t)} |\varphi_i(t - s)|$, $i = 1, 2, \ldots, n$.

We consider the following differential equation

$$\begin{cases} \dfrac{d}{dt}\big[x(t) - H\big(t, x\big(t - \tau(t)\big)\big)\big] = A(t)x(t) + F\big(t, x\big(t - \tau(t)\big)\big), & t \geq t_0, \\ x(t) = \varphi(t), \quad -\infty < t \leq t_0, \end{cases} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $H, F \in C[\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n]$, $F(t, 0) \equiv 0$, $\tau \in C[\mathbb{R}, \mathbb{R}_+]$, $\lim_{t\to+\infty}(t - \tau(t)) = +\infty$. We always assume that for any $\varphi \in C$, the system (1) has at least one solution through (t_0, φ) denoted by $x(t, t_0, \varphi)$ or simply $x(t)$ if no confusion should occur.

Definition 2.1 (Xu [16]) $f \in UC_t$ means that $f \in C[\mathbb{R}_+ \times \mathbb{R}, \mathbb{R}_+]$ and for any given α and any $\varepsilon > 0$ there exist positive numbers B, T and A satisfying

$$\int_\alpha^t f(t,s)\,ds \leq B, \qquad \int_\alpha^{t-T} f(t,s)\,ds < \varepsilon, \quad \forall t \geq A. \tag{2}$$

Especially, $f \in UC_t$ if $f(t, s) = f(t - s)$ and $\int_0^\infty f(u)\,du < \infty$.

Definition 2.2 The set $S \subset \mathbb{R}^n$ is called a global attracting set of (1), if for any initial value $\varphi \in C$, the solution $x(t) \triangleq x(t, t_0, \varphi)$ converges to S as $t \to +\infty$. That is,

$$\operatorname{dist}(x(t), S) \to 0 \quad \text{as } t \to +\infty,$$

where $\operatorname{dist}(x, S) = \inf\{|x - a| : a \in S\}$ and $|\cdot|$ is a norm of $\mathbb{R}^n$.

For a nonnegative matrix $A \in \mathbb{R}^{n\times n}$, let $\rho(A)$ denote the spectral radius of A. Then $\rho(A)$ is an eigenvalue of A and its eigenspace is denoted by

$$\Omega_\rho(A) \triangleq \{z \in \mathbb{R}^n | Az = \rho(A)z\},$$

which includes all positive eigenvectors of A provided that the nonnegative matrix A has at least one positive eigenvector (see Refs. [17]).

Lemma 2.1 (Lasalle [18]) *If* $A \geq 0$ *and* $\rho(A) < 1$, *then* $(E - A)^{-1} \geq 0$.

3 Main results

Theorem 3.1 *Let $y \in C[\mathbb{R}, \mathbb{R}_+^n]$ be a solution of the delay integral inequality*

$$y(t) \le G(t,t_0) + B\big[y(t)\big]_{\tau(t)}^+ + \int_{t_0}^t Q(t,s)\big[y(s)\big]_{\tau(s)}^+ \, ds + J, \quad t \ge t_0, \tag{3}$$

$$y(t) \le \varphi(t), \quad \forall t \in (-\infty, t_0], \tag{4}$$

where $G \in C[\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+^n]$, $B \in \mathbb{R}_+^{n\times n}$, $Q \in C[\mathbb{R}_+ \times \mathbb{R}, \mathbb{R}_+^{n\times n}]$, $J = (j_1, \ldots, j_n)^T \ge 0$, $\varphi \in C[(-\infty, t_0], \mathbb{R}_+^n]$, $\lim_{t\to+\infty}(t - \tau(t)) = +\infty$. Assume that the following conditions are satisfied:

($\mathbb{A}_1$) *$G(t,t_0) \to 0$ as $t \to +\infty$, and there exists a constant matrix $\Pi_1 \ge 0$ such that*

$$\int_{t_0}^t Q(t,s)\, ds \le \Pi_1, \quad \forall t \ge t_0. \tag{5}$$

($\mathbb{A}_2$) *Let $\Pi = \Pi_1 + B$, $\rho(\Pi) < 1$.*
Then there exist $z > 0$, $z \in \Omega_\rho(\Pi)$ and a constant $k > 0$, such that

$$y(t) < kz + (E - \Pi)^{-1}J, \quad \textit{for } t \ge t_0. \tag{6}$$

Proof By the condition $\Pi \ge 0$ and the properties of nonnegative matrices, there exists a positive vector z such that $\Pi z = \rho(\Pi)z$. Together with $\rho(\Pi) < 1$ and Lemma 2.1, this implies that $(E - \Pi)^{-1}$ exists and $(E - \Pi)^{-1} \ge 0$.

For the initial conditions $y(t) \le \varphi(t)$, $-\infty < t \le t_0$, we have

$$y(t) \le k_0 z + (E - \Pi)^{-1}J, \quad -\infty < t \le t_0, \tag{7}$$

where $z \in \Omega_\rho(\Pi)$, $z > 0$, $k_0 = \frac{\|\varphi\|}{\min_{1\le i\le n} z_i} \ge 0$, $\| \varphi \| = \max_{1\le i\le n}\{\sup_{-\infty<t\le t_0} |\varphi_i(t)|\}$.

From $\lim_{t\to+\infty} G(t,t_0) = 0$, there must be a constant $T > 0$ such that

$$G(t,t_0) \le \frac{1-\rho(\Pi)}{2} k_0 z, \quad \text{for } t > t_0 + T. \tag{8}$$

By the continuity of $y(t)$, together with (4) and (7), there exists a constant $k > k_0$ such that

$$y(t) < kz + (E - \Pi)^{-1}J, \quad \text{for } t \in (-\infty, t_0 + T]. \tag{9}$$

In the following, we shall prove that

$$y(t) < kz + (E - \Pi)^{-1}J, \quad \text{for } t > t_0 + T. \tag{10}$$

If this is not true, from (9) and the continuity of $y(t)$, then there must be a constant $t_1 > t_0 + T$ and some integer i such that

$$y_i(t_1) = \big\{kz + (E - \Pi)^{-1}J\big\}_i, \tag{11}$$

$$y(t) \le kz + (E - \Pi)^{-1}J, \quad \text{for } t \le t_1, \tag{12}$$

where $\{\cdot\}_i$ denotes the ith component of vector $\{\cdot\}$.

Using (3), (5), (8), (12) and $\rho(\Pi) < 1$, we obtain that

$$\begin{aligned}
y_i(t_1) &\le \left\{ G(t_1,t_0) + B\big[y(t_1)\big]^+_{\tau(t_1)} + \int_{t_0}^{t_1} Q(t_1,s)\big[y(s)\big]^+_{\tau(s)}\,ds + J \right\}_i \\
&\le \left\{ \frac{1-\rho(\Pi)}{2}k_0 z + \left[B + \int_{t_0}^{t_1} Q(t_1,s)\,ds \right]\big[kz + (E-\Pi)^{-1}J\big] + J \right\}_i \\
&\le \left\{ \frac{1-\rho(\Pi)}{2}kz + \Pi\big[kz + (E-\Pi)^{-1}J\big] + (E-\Pi)(E-\Pi)^{-1}J \right\}_i \\
&\le \left\{ \frac{1-\rho(\Pi)}{2}kz + \rho(\Pi)kz + (\Pi + E - \Pi)(E-\Pi)^{-1}J \right\}_i \\
&\le \left\{ \frac{1+\rho(\Pi)}{2}kz + (E-\Pi)^{-1}J \right\}_i \\
&< \big\{ kz + (E-\Pi)^{-1}J \big\}_i.
\end{aligned} \tag{13}$$

This contradicts the equality in (11), and so (10) holds. The proof is complete. □

In order to study the attracting set, we rewrite Eq. (1) as

$$\begin{cases}
x(t) = \Phi(t,t_0)\big[\varphi(t_0) - H\big(t_0,\varphi\big(t_0-\tau(t_0)\big)\big)\big] + H\big(t,x\big(t-\tau(t)\big)\big) \\
\qquad + \displaystyle\int_{t_0}^{t} \Phi(t,s)\big[A(s)H\big(s,x\big(s-\tau(s)\big)\big) + F\big(s,x\big(s-\tau(s)\big)\big)\big]\,ds, \quad t \ge t_0, \\
x(t) = \varphi(t), \quad -\infty < t \le t_0,
\end{cases} \tag{14}$$

where $\Phi(t,t_0)$ is the fundamental matrix of the linear equation $\dot{x}(t) = A(t)x(t)$.

For (1), we suppose the following:

($\mathbb{B}_1$): $[F(t,x(t-\tau(t)))]^+ \le Q(t)[x(t)]^+_{\tau(t)} + J(t)$, $[H(t,x(t-\tau(t)))]^+ \le \hat{H}[x(t)]^+_{\tau(t)}$, where $Q \in C[\mathbb{R},\mathbb{R}_+^{n\times n}]$, $\hat{H} \in \mathbb{R}_+^{n\times n}$, $J(t) = (J_1(t),\ldots,J_n(t))^T \ge 0$, $J_i \in C[\mathbb{R},\mathbb{R}_+]$, $i = 1,2,\ldots,n$.

($\mathbb{B}_2$): $[\Phi(t,s)]^+\{[A(s)]^+\hat{H} + Q(s)\} = (w_{ij}(t,s))_{n\times n}$, $w_{ij} \in UC_t$, $\lim_{t\to+\infty}\Phi(t,t_0) = 0$. For $\forall t \ge t_0$, there exist a constant matrix $\Pi_1 \ge 0$ and a vector $\hat{J} \ge 0$ such that

$$\int_{t_0}^{t} \big[\Phi(t,s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\,ds \le \Pi_1, \qquad \int_{t_0}^{t} \big[\Phi(t,s)\big]^+ J(s)\,ds \le \hat{J}.$$

($\mathbb{B}_3$): Let $\Pi = \Pi_1 + \hat{H}$, $\rho(\Pi) < 1$.

Theorem 3.2 *Assume that ($\mathbb{B}_1$)-($\mathbb{B}_3$) hold. Then $S = \{x \in \mathbb{R}^n \mid [x]^+ \le (E-\Pi)^{-1}\hat{J}\}$ is a global attracting set of (1).*

Proof From (14) and ($\mathbb{B}_1$), we can get for $\forall t \ge t_0$,

$$\begin{aligned}
\big[x(t)\big]^+ &\le \big[\Phi(t,t_0)\big]^+\big[\varphi(t_0) - H\big(t_0,\varphi\big(t_0-\tau(t_0)\big)\big)\big]^+ + \hat{H}\big[x(t)\big]^+_{\tau(t)} \\
&\quad + \int_{t_0}^{t} \big[\Phi(t,s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\big[x(s)\big]^+_{\tau(s)}\,ds + \int_{t_0}^{t} \big[\Phi(t,s)\big]^+ J(s)\,ds.
\end{aligned} \tag{15}$$

By $(\mathbb{B}_2)$-$(\mathbb{B}_3)$ and Theorem 3.1, there exists a constant $k > 0$ such that

$$\big[x(t)\big]^+ < kz + (E - \Pi)^{-1}\hat{J}, \quad \text{for } t \geq t_0, \tag{16}$$

where $z \in \Omega_\rho(\Pi)$, $z > 0$, $k > \frac{\|\varphi\|}{\min_{1\leq i\leq n} z_i}$, $\|\varphi\| = \max_{1\leq i\leq n}\{\sup_{-\infty<t\leq t_0} |\varphi_i(t)|\}$.

From (16), there must be a constant vector $\sigma \geq 0$ such that

$$\overline{\lim_{t\to+\infty}}\big[x(t)\big]^+ = \sigma \leq kz + (E - \Pi)^{-1}\hat{J}. \tag{17}$$

Next, we will show that $\sigma \in S$. From $\lim_{t\to+\infty} \Phi(t, t_0) = 0$, $w_{ij} \in UC_t$, for any $\varepsilon > 0$ and $e = (1, 1, \dots, 1)^T \in \mathbb{R}^n_+$, there exists a positive number $T_1 > t_0$ such that for all $t > T_1$

$$\big[\Phi(t, t_0)\big]^+\big[\varphi(t_0) - H\big(t_0, \varphi\big(t_0 - \tau(t_0)\big)\big)\big]^+ < \frac{\varepsilon e}{2}, \tag{18}$$

$$\int_{t_0}^{t-T_1} \big[\Phi(t, s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\big[kz + (E - \Pi)^{-1}\hat{J}\big]\,ds < \frac{\varepsilon e}{2}. \tag{19}$$

According to the definition of superior limit and $\lim_{t\to+\infty}(t - \tau(t)) = +\infty$, there exists sufficiently large $T_2 \geq T_1$ such that

$$\big[x(t)\big]^+_{r(t)} < \sigma + \varepsilon e, \quad t \geq T_2, \tag{20}$$

where $r(t) = T_1 + \sup_{t-T_1\leq s\leq t} \tau(s)$. Therefore, from $(\mathbb{B}_2)$, (15) and (18)-(19), when $t \geq T_2$, we obtain

$$\begin{aligned}
\big[x(t)\big]^+ &\leq \frac{\varepsilon e}{2} + \hat{H}\big[x(t)\big]^+_{\tau(t)} + \int_{t_0}^{t-T_1} \big[\Phi(t, s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\big[x(s)\big]^+_{\tau(s)}\,ds \\
&\quad + \int_{t-T_1}^{t} \big[\Phi(t, s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\big[x(s)\big]^+_{\tau(s)}\,ds + \hat{J} \\
&\leq \frac{\varepsilon e}{2} + \hat{H}\big[x(t)\big]^+_{\tau(t)} \\
&\quad + \int_{t_0}^{t-T_1} \big[\Phi(t, s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\big[kz + (E - \Pi)^{-1}\hat{J}\big]\,ds \\
&\quad + \int_{t-T_1}^{t} \big[\Phi(t, s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}\big[x(s)\big]^+_{\tau(s)}\,ds + \hat{J} \\
&\leq \varepsilon e + \hat{H}(\sigma + \varepsilon e) + \int_{t-T_1}^{t} \big[\Phi(t, s)\big]^+\big\{\big[A(s)\big]^+\hat{H} + Q(s)\big\}(\sigma + \varepsilon e)\,ds + \hat{J} \\
&\leq \varepsilon e + \Pi(\sigma + \varepsilon e) + \hat{J}.
\end{aligned} \tag{21}$$

Due to (17) and the definition of superior limit, there exists $T_3 \geq T_2$ such that $[x(T_3)]^+ > \sigma - \varepsilon e$. Combining with (21), we get

$$\sigma - \varepsilon e < \big[x(T_3)\big]^+ \leq \varepsilon e + \Pi(\sigma + \varepsilon e) + \hat{J}. \tag{22}$$

Letting $\varepsilon \to 0$, we have $\sigma \leq (E - \Pi)^{-1}\hat{J}$, that is $\sigma \in S$, and the proof is completed. □

Remark 3.1 Theorem 3.2 is a generalization of the results in [12, 13] as $H \equiv 0$ in (1) without the boundedness of $\tau(t)$.

Corollary 3.1 *Suppose that the conditions of Theorem 3.2 hold and $J(t) \equiv 0$. If $x(t) = 0$ is an equilibrium point of System (1), then the equilibrium point $x(t) = 0$ is globally asymptotically stable.*

4 Example

Consider the following scalar equation

$$\frac{d}{dt}\left[x(t) - \frac{1}{4}x\bigl(t - \tau(t)\bigr)\right] = -atx(t) + btx\bigl(t - \tau(t)\bigr) + J(t), \quad t \geq 0, \tag{23}$$

where $a > 0$, b are constants, $J(t) = \sin t$ and $\tau(t) = \frac{1}{2}(t + \sin t)$.

We easily verify that $|\Phi(t,0)| = e^{-a\int_0^t v\,dv}$, $\lim_{t\to+\infty}\Phi(t,0) = 0$. For any $t \geq 0$, we get

$$\begin{aligned}
&\int_0^t e^{-a\int_s^t v\,dv}\left(\frac{1}{4}as + |bs|\right)ds \leq \left(\frac{a}{4} + |b|\right)\int_0^t e^{-a\int_s^t v\,dv}s\,ds \leq \frac{1}{4} + \frac{|b|}{a},\\
&\int_0^t e^{-a\int_s^t v\,dv}|\sin s|\,ds \leq \int_0^t e^{-a\int_s^t v\,dv}s\,ds \leq \frac{1}{a}.
\end{aligned}$$

If $\frac{|b|}{a} < \frac{1}{2}$, we can get $S = \{x \in \mathbb{R}^n \mid |x| \leq \frac{2}{a-2|b|}\}$ is the global attracting set for (23).

Remark 4.1 If $J(t) \equiv 0$ and $\frac{|b|}{a} < \frac{1}{2}$, then every solution of (23) tends to zero at ∞. However, the methods in [4, 6] are inefficient for (23) because the variable coefficients $A(t) = -at$ and $Q(t) = bt$ are unbounded for $t \geq 0$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
LT conceived of the study and drafted the manuscript. LX helped to perform the analysis and give the example. DX participated in its design and coordination. All authors read and approved the final manuscript.

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References

1. Hale, JK, Lunel, SMV: Introduction to Functional Differential Equations. Springer, Berlin (1993)
2. Agarwal, RP, Grace, SR: Asymptotic stability of differential systems of neutral type. Appl. Math. Lett. **13**, 15-19 (2000)
3. Agarwal, RP, Grace, SR: Asymptotic stability of certain neutral differential equations. Math. Comput. Model. **31**, 9-15 (2000)
4. Cao, DQ, He, P, Ge, YM: Simple algebraic criteria for stability of neutral delay-differential systems. J. Franklin Inst. **342**, 311-320 (2005)
5. Dix, JG, Philos, CG, Purnaras, IK: Asymptotic properties of solutions to linear non-autonomous neutral differential equations. J. Math. Anal. Appl. **318**, 296-304 (2006)
6. Zhang, Y: Stability of large scale systems with delays of neutral-type. Sci. China Ser. A **4**, 337-347 (1988) (in Chinese)
7. Azbelev, NV, Maksimov, VP, Rakhmatullina, LF: Introduction to the Theory of Linear Functional Differential Equations. Advanced Series in Math. Science and Engineering, vol. 3. World Federation Publisher Company, Atlanta (1995)

8. Gusarenko, SA, Domoshnitsky, A: Asymptotic and oscillation properties of first order linear scalar functional-differential equations. Differ. Uravn. **25**(12), 2090-2103 (1989)
9. Bainov, D, Domoshnitsky, A: Nonnegativity of the Cauchy matrix and exponential stability of a neutral type system of functional differential equations. Extr. Math. **8**(1), 75-82 (1993)
10. Domoshnitsky, A, Maghakyan, A, Shklyar, R: Maximum principles and boundary value problems for first order neutral functional differential equations. J. Inequal. Appl. (2009). doi:10.1155/2009/141959
11. Agarwal, RP, Berezansky, L, Braverman, E, Domoshnitsky, A: Nonoscillation Theory of Functional Differential Equations with Applications. Springer, Berlin (2012)
12. Zhao, HY: Invariant set and attractor of nonautonomous functional differential systems. J. Math. Anal. Appl. **282**, 437-443 (2003)
13. Xu, DY, Zhao, HY: Invariant and attracting sets of Hopfield neural networks with delay. Int. J. Syst. Sci. **32**(7), 863-866 (2001)
14. Xu, DY, Yang, Z: Attracting and invariant sets for a class of impulsive functional differential equations. J. Math. Anal. Appl. **329**(2), 1036-1044 (2007)
15. Xiang, L, Teng, LY, Wu, H: A new delay vector integral inequality and its application. J. Inequal. Appl. (2010). doi:10.1155/2010/927059
16. Xu, DY: Integro-differential equations and delay integral inequalities. Tohoku Math. J. **44**(3), 365-378 (1992)
17. Horn, RA: Matrix Analysis. Cambridge University Press, Cambridge (1985)
18. Lasalle, JP: The Stability of Dynamical System. SIAM, Philadelphia (1976)

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