Huang *Journal of Inequalities and Applications* 2012, **2012**:132
http://www.journalofinequalitiesandapplications.com/content/2012/1/132

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Random attractor of stochastic partly dissipative systems perturbed by Lévy noise

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Abstract

The current paper is devoted to random dynamics of stochastic partly dissipative systems perturbed by Lévy noise. By the technique of dissipative in probability and multivalued random dynamical systems (MRDS), the existences of random attractor for MRDS generated by the stochastically perturbed partly dissipative systems are provided, both the weaker restrictions and stronger restrictions on the coefficients of Lévy noise respectively.

Mathematics Subject Classification: 35R60; 60H15; 35K57.

Keywords: multivalued random dynamical systems, random attractor, dissipative in probability, Lévy noise

1 Introduction

Global attractors play an important role in the study of asymptotic behavior of various nonlinear systems. There is a great amount of works toward the global attractors for dissipative autonomous as well as nonautonomous and random equations, see [1-7], etc. Dynamical systems driven by non-Gaussian processes, such as Lévy processes, have attracted a lot of attention recently. Stochastic differential equations driven by Lévy processes have been summarized in [8]. Peszat and Zabczyk [9] have presented a basic framework for partial differential equations driven by Lévy processes, which extended several results known for stochastic partial differential equations (SPDEs) driven by Wiener processes. For more works on SPDEs driven by Lévy processes, see [9] and references therein.

Recently, the authors in [6] developed the new frame of the random attractor for infinite dimensional systems, in which, the solution is not necessarily required to be unique, but the corresponding multivalued random dynamical systems (MRDS) is dissipative with probability one. Kapustyan et al. [10] follows the idea of the one in [6], and study the random attractor of reaction diffusion systems perturbed by *càdlàg* process.

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = a\Delta u(t,x) - f(u(t,x)) + h(x) + g(u(t,x))\eta(t,\omega), \\ u|_{\partial Q} = 0. \quad u_{|t=0} = u_0(x). \end{cases} \tag{1.1}$$

where $a > 0$, $Q \subset R^n$ is a bounded domain with smooth boundary, $f, g \in C(R)$, $h \in L^2(Q)$, f and g satisfy some appropriate assumptions, and $\eta(t, \omega)$ is a stochastic *càdlag* process with right continuity and left limits.

Global attractors for deterministic FitzHugh-Nagumo systems and partly dissipative deterministic reaction diffusion systems have been investigated in [11] for the bounded domain case and in [7] for the unbounded domain case. Regarding the stochastic FitzHugh-Nagumo system as well as the following general partly dissipative random reaction diffusion system

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = \Delta u(t,x) + h(\theta_t\omega, x, u) + f(\theta_t\omega, x, u, v), \\ \frac{\partial v(t,x)}{\partial t} = -\sigma v(t,x) + g(\theta_t\omega, x, u), \\ u|_{\partial Q} = 0. \end{cases} \tag{1.2}$$

where $D \subset R^n$ is a smooth bounded domain, $h : \Omega \times D \times R \to R, f : \Omega \times D \times R^2 \to R$ and $g : \Omega \times D \times R \to R$ are measurable and satisfy some appropriate hypotheses. The authors in [5] show the existence of random attractor for the general random partly dissipative reaction diffusion equation (1.2), and provide the Hausdorff dimension of random attractor of stochastic FitzHugh-Nagumo systems, [12] shows the existence of pullback attractor of the non-autonomous FitzHugh-Nagumo systems on unbounded domains. The author in [13] study the stochastic partly FitzHugh-Nagumo systems driven by Gaussian white noise, and show the existence of random attractor by uniform estimates on solution for large space and time variable via a cut-off technique.

Motivated by Kapustyan et al. [10], we consider the following partly dissipative reaction diffusion systems perturbed by Lévy noise.

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} - d\Delta u(t,x) + h(x,u) + f(x,u,v) = k_1(u)L(t,\omega), \quad (x,t) \in D \times R^+, \\ \frac{\partial v(t,x)}{\partial t} + \sigma(x)v + g(x,u) = k_2(v)L(t,\omega), \quad (x,t) \in D \times R^+, \\ u|_{\partial D} = 0. \\ u_{|t=0} = u_0(x), v_{|t=0} = v_0(x), \end{cases} \tag{1.3}$$

where $d > 0, L(t, \omega)$ is a stochastic Lévy process which trajectories are right-continuous and have left limits. Functions h, f, g, k_1, k_2 and σ are twice continuously differentiable in all variables and satisfy

(H1) $c_1|u|^p - c_3 \le h(x, u)u \le c_2|u|^p + c_3, p > 2.$

(H2) $|f(x,u,v)| \le c_4(1 + |u|^{p_1} + |v|)$, $0 < p_1 < p - 1$.

(H3) $\delta(x) \ge \delta > 0$.

(H4) $|g'(u)| \le c_5$, $|g'_{x_i}(x,u)| \le c_5(1 + |u|)$, $i = 1, \ldots, n$, where $\delta_i > 0$, $i = 1, \ldots, 5$.

(H5) $(h'_u(x,u) + f'_u(x,u,v))\xi_1^2 + f'_v(x,u,v)\xi_1\xi_2 \ge -c_6(\xi_1^2 + \xi_2^2)$, $i = 1, \ldots, n$, where $\delta_6 > 0$.

(H6) there exist some positive constants a_1, a_2, b_1 and b_2 such that

$$|k_1(u)| \le a_1 |u| + b_1, \quad |k_2(v)| \le a_2 |v| + b_2.$$

In current paper, we study the long time behavior of stochastic partly dissipative equations (1.3) perturbed by Lévy noise. By the technique of dissipative in probability and MRDS, the existences of random attractor for MRDS generated by the perturbed partly dissipative systems (1.3) are provided, both the weaker restrictions and stronger restrictions on the coefficients of Lévy noise, respectively.

It is necessary to point out that, comparing with the main result for (1.1) in [10], the two difficulties we need to tackle are the measurability and the asymptotic compactness of the solutions operators of stochastic partly dissipative systems (1.3). By the similar arguments in [4], the measurability for stochastic partly dissipative systems perturbed by Lévy noise, so we will use the technique of dissipative in probability and MRDS, to show the existences of random attractor for MRDS generated by the stochastically perturbed partly dissipative systems (1.3).

The rest of the paper is organized as follows. In Section 2, we present some definitions and theorems about MRDS. Section 3 is devoted to the study of the existence of random attractor of partly dissipative random reaction diffusion systems perturbed by Lévy noise in terms of dissipative in probability, the weaker restrictions and stronger restrictions on the coefficients of Lévy noise, respectively.

2 Multivalued random dynamical systems

In this section, we recall the definitions of multivalued random dynamical systems and random attractors, the reader is referred to [6,10] for details.

Definition 2.1. [9]*Let E be a Banach space, and let $X = (X(t), t \geq 0)$ be a E valued stochastic process defined on a probability space $(\Omega, \mathcal{F}, P)$. It is called as a Lévy process if*

(L1) $X(0) = 0$, *a.s.*;

(L2) *X has independent and stationary increments; and*

(L3) *X is stochastically continuous, i.e. for all $\delta > 0$ and for all $s \geq 0$*

$$\lim_{t\to s} P(|X(t) - X(s)| > \delta) = 0.$$

We can choose the sample space $\Omega = D(R)$ of Lévy process as

$$\Omega = D(R) = \{\omega(\cdot) : R \to R, \quad \forall t \in R, \lim_{s\to t-} \omega(s) = \omega(t-), \lim_{s\to t+} \omega(s) = \omega(t)\}$$

It is easy to check that $\Omega = D(R)$ is the Skorokhod metric space with Borel σ-algebra Φ.

$$\theta_t\omega(\cdot) = \omega(t + \cdot),$$

P is θ_t-invariant probability measure. The matric ρ can be defined by

$$\rho(\omega_1, \omega_2) = \Sigma_{i=1}^{\infty} \frac{1}{2^i} \frac{\rho_i(\omega_1, \omega_2)}{1 + \rho_i(\omega_1, \omega_2)},$$

where

$$\rho_i(\omega_1, \omega_2) = \inf_{\lambda\in\Lambda} \left(\sup_{t\in[-i,i]} |\omega_1(t) - \omega_2(\lambda(t))| + \sup_{t\in[-i,i]} |t - \lambda(t)| \right),$$

$\Lambda = \{\lambda(\cdot)| : \lambda(\cdot) : [-i, i] \to [-i, i], \lambda(-i) = -i, \lambda(i) = i, \lambda(\cdot)$ is continuous and monotonically increasing function$\}$.

Remark 2.1. *It follows from* [9]*that Lévy process $\eta(t, \omega)$ is a stochastic càdlàg process with trajectories without discontinuities of the second kind, that is to say, the sample path is right continuous with left limits.*

In this paper, we just use the càdlàg property of the Lévy process, and $L(t, \omega) = \omega(t) = \pi(\theta_t\omega)$, where $\pi : \Omega \to R$, and $\pi(\omega) = \omega(0)$.

Let $C(X)$ be the family of all nonempty closed subset of X, $\beta(X)$ be the family of all nonempty and bounded sunset of X. In the sequel, we will introduce the definition of MRDS.

Definition 2.2. [10]*A multivalued mapping* $G : R_+ \times \Omega \times X \to C(X)$ *is called as a MRDS if*

(1) *the mapping* $(t, x) \to G(t, \omega)x$ *is measurable for all* $x \in X$
(2) $G(0, \omega)x = x$ *and* $G(t + s, \omega)x \subset G(t, \theta_s\omega)G(s, \omega)x$ *for all* $t, s \in R_+, x \in X, \omega \in \Omega$.

Definition 2.3. [10]*A measurable set* $A(\omega)$ *is called a random attractor for a MRDS G if for P-almost all* $\omega \in \Omega$,

(1) $A(\theta_t\omega) \subset G(t, \omega)A(\omega)$ *for all* $t \in R_+$;
(2) *for all* $B \in \beta(X)$, $dist(G(t, \theta_{-t}\omega))B, A(\omega)) \to 0$, $t \to +\infty$;
(3) $A(\omega)$ *is a compact set in X.*

The following hypotheses is developed by Kapustyan et al. [10], which are key tools to show the existence of random attractor for MRDS generated by stochastic equations (1.3).

(G1) the mapping $(t,\omega) \to \overline{G(t,\omega)B}$ is measurable for all $B \in \beta(X)$

(G2) for all $\epsilon > 0$, there exists $R = R(\epsilon)$ such that for all $B \in \beta(X)$, there exists $T = T(B, R, \epsilon)$ for which

$$P\left\{\sup_{t\geq T}||G(t,\theta_{-t}\omega)B|| > R\right\} < \varepsilon.$$

As the authors pointed out in the paper [10] that, the condition (G1) is used to show the measurability of the map, and the condition (G2) is used to show the MRDS is dissipative in probability.

For a given $B \in \mathcal{B}(X)$, define

$$\wedge_{B_n}(\omega) = \bigcap_{T>0}\overline{\bigcup_{t\geq T} G(t,\theta_{-t}\omega)B}, \quad \mathcal{A} = \overline{\bigcup_{B\in\mathcal{B}(X)} \wedge_{B_n}(\omega)} = \overline{\bigcup_{k=1}^{\infty} \wedge_{B_n}(\omega)}.$$

Then, $\wedge_{B_n}(\omega)$ consists of the limit of all convergent sequences $\{\xi_n\}$, where $\xi_n \in G(t_n,\theta_{-t_n}\omega)B$, and $t_n \to \infty$.

Theorem 2.1. ([10], Theorem 1)

Let a mapping $x \to G(t, \omega)x$ *be upper semicontinuous and compact-valued for all* $t \in R_+$ *and* $\omega \in \Omega$, *let conditions* (G1) *and* (G2) *hold for a MRDS G and let the set* $G(t, \omega)B_R$ *be precompact in X for all* $\omega \in \Omega$, $t > 0$ *and* $R > 0$. *Then,*

$$\mathcal{A}(\omega) = \overline{\bigcup_{n=1}^{\infty} \wedge_{B_n}(\omega)}$$

is a random attractor for G. Therefore, the attractor is unique, it is a minimal set among closed attracting sets, and it is a maximal set among compact, measurable, semiinvariant sets.

3 Random attractor for MRDS of (1.3)

In this section, we will prove the solution of equation (1.3) can generate a MRDS, and the MRDS posses a random attractor.

Lemma 3.1. *Under the assumptions* (H1)-(H5), *there exists the positive constants* m_1, m_2, δ_3, c_a, c_r *such that the solution* $((u(t), v(t))$ *of the equations* (1.3) *satisfy the following estimates.*

$$\begin{aligned}\left|u(t)\right|^2+\left|v(t)\right|^2 \quad &\leq \quad \left(\left|u(0)\right|^2+\left|v(0)\right|^2\right)e^{\left(\int_0^t (m_1|\omega(p)|-m_2)\mathrm{d}p\right)} \\ &\quad + \int_0^t \left(c_3\,|D|+\delta_3+(c_a+c_r)\left|\omega(s)\right|\right)e^{\left(\int_s^t (m_1|\omega(p)|-m_2)\mathrm{d}p\right)}\mathrm{d}s.\end{aligned}$$

Proof. Multiplying the first equation of system (1.3) by u and the second equation of system (1.3) by v, Integrating them over D, we have

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(|u|^2+|v|^2\right)+d\|u\|^2+\int_D \sigma(x)v^2\mathrm{d}x+\int_D h(x,u)u\mathrm{d}x+\int_D \left[f(x,u,v)u+g(x,v)v\right]\mathrm{d}x \\ &=\int_D k_1(x,u)\omega(t)u\mathrm{d}x+\int_D k_2(x,v)\omega(t)v\mathrm{d}x.\end{aligned}$$

It follows from (H6) and Young inequality that there exist two constants $a > 0$ and $r > 0$ such that

$$\begin{aligned}&2(k_1(u)\omega(t),u)\leq(2a_1+r)\,|\omega|\;\;|u|^2+c_r\,|\omega|\,, \\ &2(k_2(v)\omega(t),v)\leq(2a_2+a)\,|\omega|\;\;|v|^2+c_a\,|\omega|\,.\end{aligned}$$

Due to (H4), there exists a constant $c_7 > 0$ such that

$$\left|g(x,\xi)\right|\leq c_7(1+|\xi|),\quad \forall\xi\in R,\; x\in D.$$

We deduce from (H1) to (H3) that

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left(|u|^2+|v|^2\right)+2d\|u\|^2+2\delta|v|^2+2c_1\int_D |u|^p\mathrm{d}x \\ &\leq 2c_3\,|D|+2(c_4+c_7)\int_D\left(|u|+|u|^{p_1+1}\right)\mathrm{d}x+2(c_4+c_7)\int_D |v|\,(1+|u|)\,\mathrm{d}x \\ &+(c_r+c_a)\,|\omega|+(2a_1+r)\,|\omega|\;\;\|u\|^2+(2a_2+a)\,|\omega|\;\;\|v\|^2.\end{aligned}$$

Notice that

$$(c_4+c_7)\int_D(|v|\,(1+|u|)\mathrm{d}x\leq\frac{\delta}{2}\int_D |v|^2\mathrm{d}x+\frac{(c_4+c_7)^2}{2\delta}\int_D(1+|u|)^2\mathrm{d}x.$$

Let $q = \max\{p_1 + 1, 2\}$, then there exists a constant $\delta_2 > 0$ such that

$$(c_4+c_7)\left(|\xi|+|\xi|^{p_1+1}+\frac{(c_4+c_7)^2}{2\delta}\left(1+|\xi|^2\right)\leq\delta_2\left(|\xi|^q+1\right)\right)$$

Thus,

$$(c_4+c_7)\int_D\left(|u|+|u|^{p_1+1}+\frac{(c_4+c_7)^2}{2\delta}(1+|u|)^2\right)\mathrm{d}x\le\frac{c_1}{4}\int_D|u|^p\mathrm{d}x+\delta_3.$$

Hence

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(|u|^2+|v|^2)+2d\|u\|^2+\delta|v|^2+\frac{3}{2}c_1\,|u|_p^p\\&\quad\le c_3\,|D|+\delta_3+(c_a+c_r)\,|\omega|+(2a_1+r)\,|\omega|\,|u|^2+(2a_2+a)\,|\omega|\,|v|^2.\end{aligned}\tag{3.1}$$

Let $m_1=\max\{2a_1+r,\,2a_2+a\}$, $m_2=\min\{\delta,\,2d\lambda_1\}$, where $\lambda_1>0$ is the first eigenvalue of -Δ in $H_0^1(D)$. By the inequality (3.1), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(|u|^2+|v|^2\right)\le c_3\,|D|+\delta_3+(c_a+c_r)\,|\omega|+(m_1|\omega(t)|-m_2)\left(|u|^2+|v|^2\right).\tag{3.2}$$

Integrating (3.2) on the interval $[s,\,t]$ $(t\ge s\ge 0)$, we obtain that

$$\begin{aligned}\left|u(t)\right|^2+\left|v(t)\right|^2\quad\le\quad&\left|u(s)\right|^2+\left|v(s)\right|^2+(c_3\,|D|+\delta_3)\,(t-s)+(c_a+c_r)\int_s^t|\omega(p)|\mathrm{d}p\\&+\int_s^t(m_1\,|\omega(p)|-m_2)\left(\left|u(p)\right|^2+\left|v(p)\right|^2\right)\mathrm{d}p.\end{aligned}\tag{3.3}$$

Using the Gronwall's Lemma,

$$\begin{aligned}\left|u(t)\right|^2+\left|v(t)\right|^2\quad\le\quad&\left(\left|u(0)\right|^2+\left|v(0)\right|^2\right)e^{\left(\int_0^t(m_1|\omega(p)|-m_2)\mathrm{d}p\right)}\\&+\int_0^t\left(c_3\,|D|+\delta_3+(c_a+c_r)\,\left|\omega(s)\right|\right)e^{\left(\int_s^t(m_1|\omega(p)|-m_2)\mathrm{d}p\right)}\mathrm{d}s.\end{aligned}\tag{3.4}$$

for all $t>0$. The proof of Lemma 3.1 is completed.

Lemma 3.2. *Let* $\{(u_n,v_n)=(u_n(t,\omega_n)(u_n^0,v_n^0),v_n(t,\omega_n)(u_n^0,v_n^0),)\}\subset(L^2(0,T;H_0^1(D))\bigcap L^p(D\times(0,T)\bigcap C(0,\infty;L^2(D))\times C(0,\infty;L^2(D))$ *be an arbitrary sequence of solution of equation* (1.1), *where* $\omega_n\to\omega_0$ *in* Ω, $u_n^0\to u_0$ *weakly in* $L^2\times L^2$, *and* $t_n\to t_0>0$. *Then,*

$$(u_n(t_n,\omega_n),v_n(t_n,\omega_n)(u_n^0,v_n^0)\to(u(t_0,\omega_0),v(t_0,\omega_0)(u_0,v_0).$$

in $L^2\times L^2$ *at least along some subsequence where* $(u,v)=(u(t,\omega_0),v(t,\omega_0)(u_0,v_0)\in(L^2(0,T;H_0^1(D))\bigcap L^p(D\times(0,T)\bigcap C(0,\infty;L^2(D))\times C(0,\infty;L^2(D))$ *is the solution of equation* (1.3).

Proof. Let $T>0$. It follows from (3.1) that the sequence $\{((u_n,\,v_n))\}$ is bounded in $\left(L^p(0,T,L^p(D))\bigcap L^2(0,T,H_0^1(D))\bigcap L^\infty(0,T,L^2(D))\right)\times C(0,T,L^2(D))$. By the similar argument of Proposition 1.1 that there exists a subsequence $u_n(\cdot,\omega_n)(u_n^0,v_n^0))\to(u(\cdot),$ $v(\cdot))$ in $L^2(0,\,T,\,L^2(D))\times L^2(0,\,T,\,L^2(D))$, $(u_n(t,\,\omega_n),\;v_n(t,\omega_n))(u_n^0,v_n^0)\to(u(t),v(t))$ strongly in $L^2(D)\times L^2(D))$ for almost all $t\in(0,\,T)$ and weakly in $L^2(D)\times L^2(D))$ uniformly in $t\in[0,\,T]$.

Let

$$L_n(p) = \max\{2a_1 + r, 2a_2 + \alpha\} \left|\omega_n(p)\right| - \min\{\delta, 2d\lambda_1\} = m_1 \left|\omega_n(p)\right| - m_2,$$
$$L_0(p) = \max\{2a_1 + r, 2a_2 + \alpha\} \left|\omega_0(p)\right| - \min\{\delta, 2d\lambda_1\} = m_1 \left|\omega_0(p)\right| - m_2,$$

and denote

$$J_n(t, \omega_n) = \left(\left|u_n(t)\right|^2 + \left|v_n(t)\right|^2\right) - (c_3 \left|D\right| + \delta_3)\, t - \left(c_\alpha + c_\gamma\right) \int_0^t \left|\omega_n(p)\right| \mathrm{d}p$$
$$- \int_0^t L_n(p) \left(\left|u_n(p)\right|^2 + \left|v_n(p)\right|^2\right) \mathrm{d}p,$$
$$J_0(t, \omega_0) = \left(\left|u(t)\right|^2 + \left|v(t)\right|^2\right) - (c_3 \left|D\right| + \delta_3)\, t - \left(c_\alpha + c_\gamma\right) \int_0^t \left|\omega_0(p)\right| \mathrm{d}p$$
$$- \int_0^t L_0(p) \left(\left|u(p)\right|^2 + \left|v(p)\right|^2\right) \mathrm{d}p.$$

It follows from (3.3) that

$$J(t, \omega_0) \le J(s, \omega_0), \quad J_n(t, \omega_n) \le J_n(s, \omega_n)$$

for all $t \ge s$, $t, s \in [0, T]$, and $J_n(t, \omega) \to J(t, \omega_0)$ for almost all $t \in (0, T)$. It is easy to check that $J_n(t, \omega_n) \to J(t, \omega_0)$ uniformly on an arbitrary interval $[a, b] \subset (0, T)$. The convergence $\omega_n \to \omega_0$ implies that $L_n(\cdot) \to L_0(\cdot)$ in $L^1(0, t)$ and $L_n(\cdot)$, $n \ge 1$ are bounded in $L^\infty(0, t)$. Then

$$\int_0^t L_n(p) \left(\left|u_n(p)\right|^2 + \left|v_n(p)\right|^2\right) \mathrm{d}p \to \int_0^t L_0(p) \left(\left|u(p)\right|^2 + \left|v_n(p)\right|^2\right) \mathrm{d}p.$$

Hence,

$$\liminf_{n\to\infty} J_n(t_n, \omega_n) = J(t_0, \omega_0)$$
$$\ge \liminf_{n\to\infty} \left(|u_n|^2 + |v_n|^2\right) - (c_3 \left|D\right| + \delta_3)\, t_0 - \left(c_\alpha + c_\gamma\right) \int_0^{t_0} \left|\omega_0(p)\right| \mathrm{d}p$$
$$- \int_0^{t_0} L_0(p) \left(\left|u(p)\right|^2 + \left|v(p)\right|^2\right) \mathrm{d}p$$

for all $t_n \ge t_0 > 0$.

Hence

$$\left|u(t_0)\right| + \left|v(t_0)\right| \ \ge \liminf_{n\to\infty} \left(\left|u_n(t_n)\right| + \left|v_n(t_n)\right|\right).$$

Since $(u_n(t_n), v_n(t_n)) \to (u(t_0), v(t_0))$ weakly in $L^2(D) \times L^2(D)$. The converse inequality also holds and therefore

$$\liminf_{n\to\infty} \left(\left|u_n(t_n)\right| + \left|v_n(t_n)\right|\right) \le \left(\left|u(t_0)\right| + \left|v(t_0)\right|\right) \le \liminf_{n\to\infty} \left(\left|u_n(t_n)\right| + \left|v_n(t_n)\right|\right).$$

Moreover, $(u_n(t_n), v_n(t_n))$ converges strongly to $(u(t_0), v(t_0))$ in $L^2(D) \times L^2(D)$. The proof of Lemma 3.2 is completed.

Lemma 3.3. *Let Ω be a metric space and Φ be a Borel σ -algebra. Assume that a multivalued mapping $G : R_+ \times \Omega \times X \to C(X)$ satisfies the following condition: if $x_n \to x$ weakly in X as $t_n \to t_0 > 0$, $\omega_n \to \omega$ in Ω and $y_n \in G(t_n, \omega_n)x_n$ then $y_n \to y_0 \in G(t_0, \omega_0)x_0$ in X for some subsequence. Then, assumption G_1 holds for the mapping G.*

Proof. The proof is similar to the argument of Lemma 4 in [10], and is omitted here.

Lemma 3.4. *For $(u_0, v_0) \in L^2(D) \times L^2(D)$, there exists an unique solution $(u, v) \in C(0, \infty; L^2(D) \times L^2(D))$ of the equations* (1.3) *satisfying $u \in L^2(0,T;H_0^1(D)) \bigcap L^p(D \times (0,T))$. The mapping defined by*

$$G(t,\omega) : (u_0, v_0) \to (u(t), v(t)) \tag{3.5}$$

is a multivalued random dynamical system generated by equation (1.3).

Proof. By the same proof as that of Property 1.1 in [11] and the classical Galerkin approximates, noting that $k_1(u)\omega(t)$ and $k_2(v)\omega(t)$ are right continuous with left limit with respect to t, we can show that the equation (1.3) admits at least a solution $(u(t, \omega, u_0, v_0), v(t, \omega, u_0, v_0)) \in C(0, \infty; L^2(D) \times L^2(D))$ for every $\omega \in \Omega$. Moreover, $u \in L^2(0,T;H_0^1(D)) \bigcap L^p(D \times (0,T))$. Similar to the Property 4 in [6], it can be verified the family of mappings $\{G(t, \omega)_{t\geq 0}$ generates a MRDS.

Lemma 3.5. *Assume that* (H1)-(H6) *hold, and Levy noise $\omega(t)$ satisfies the following conditions*

(HY0) *If for all $\epsilon > 0$, $\alpha > 0$, there exists $T = T(\epsilon) > 0$ such that Lévy noise $\omega(t)$ satisfies*

$$P\left(\omega(t) : \sup_{t\geq T} \frac{1}{t} \int_{-t}^{0} |\omega(p)| \mathrm{d}p - \frac{m_2}{m_1} \leq -\alpha\right) > 1 - \varepsilon,$$

(HY1) *If for all $\epsilon > 0$, there exists a positive constant $D > 0$ such that Lévy noise $\omega(t)$ satisfies*

$$\sup_{t\geq 0} P\left\{\omega(t) : \int_{-t}^{0} |\omega(s)|\, e^{m_1 \alpha s} \mathrm{d}s \leq D\right\} > 1 - \varepsilon.$$

Then, the MRDS G generated by equation (1.3) *satisfies the condition G_2.*

Proof. It follows from Lemma 3.1 that there exist random variable $t(\omega) \in [T_1, T_2]$ and initial value $x_0(\omega) \in B_r$ such that the solution of equation (1.3) reaches the superium at $t(\omega)$ for all $T_2 > T_1 > 0$ and any $\omega \in \Omega$, that is,

$$\sup_{t\in[T_1,T_2]} \left\|G(t,\theta_{-t}\omega)B_r\right\| = \left\|(u(t(\omega),\theta_{-t(\omega))}\omega), v(t(\omega),\theta_{-t(w))}\omega))(u_0(\omega), v_0(\omega)\right\|.$$

Notes that $\omega \to \sup_{t\in[T_1,T_2]} \left\|G(t,\theta_{-t}\omega)B_r\right\|$ is Φ-measurable, therefore, $(u(t(\omega), \theta_{-t(\omega)}\omega)x_0(\omega)$, and $v(t(\omega), \theta_{-t(\omega))}\omega))$ are also Φ-measurable.

Fixed $\epsilon > 0$, then for any arbitrary $N > 0$, T and for $t \in [T, T + N]$, denote

$$\begin{aligned} L^2 &= \left\{\omega, : \sup \left\|G(t,\theta_{-t}\omega)B_r\right\|^2 > R^2 \right\} \\ &= \{\omega : \left\|(u(t(\omega),\theta_{-t(\omega)}\omega)x_0(\omega), v(t(\omega),\theta_{-t(\omega))}\omega))\right\|^2 > R^2\}. \end{aligned}$$

Define

$$A_1 = \left\{ \omega : \ r^2 e^{m_1 t(\omega)\left(\frac{1}{t(\omega)} \int_{-t(\omega)}^{0} |\omega(p)| \mathrm{d}p - \frac{m_2}{m_1}\right)} \geq 1 \right\}$$
$$\subset \left\{ w : \left(\frac{1}{t(\omega)} \int\limits_{-t(\omega)}^{0} |\omega(p)| \mathrm{d}p - \frac{m_2}{m_1} \right) \geq \frac{1}{m_1 t(\omega)} \ln \left(\frac{1}{r^2} \right) \right\}.$$

Choosing $T = T\,(r)$ such that

$$\frac{1}{m_1 t(\omega)} \ln \left(\frac{1}{r^2} \right) > -\alpha, \quad (\alpha > 0).$$

Then,

$$A_1 \subset \left\{ \frac{1}{t(\omega)} \int\limits_{-t(\omega)}^{0} \left|\omega(p)\right| \mathrm{d}p - \frac{m_2}{m_1} > -\alpha \right\}$$
$$\subset \left\{ \omega : \sup_{t \geq T} \frac{1}{t} \int\limits_{-t}^{0} \left|\omega(p)\right| \mathrm{d}p - \frac{m_2}{m_1} > -\alpha \right\}.$$

It follows from (HY0) that there exists a positive constant random variant $T_1 = T_1(\omega)$ such that for $t \geq T_1(\epsilon)$,

$$P\left(\omega \sup_{t \geq T_1} \frac{1}{t} \int\limits_{-t}^{0} \left|\omega(p)\right| \mathrm{d}p - \frac{m_2}{m_1} \leq -\alpha \right) > 1 - \varepsilon,$$

Thus, there exists a random set $A_2 \subset \Omega$ such that $P(A_2) \leq \frac{\varepsilon}{4}$, and

$$\sup_{t \geq T_1} \frac{1}{t} \int_{-t}^{0} \left|\omega(p)\right| \mathrm{d}p - \frac{m_2}{m_1} \leq -\alpha.$$

for all $\omega \in \Omega \backslash A_2$.
Thus, there exists $T_2 = T_2(\epsilon, r) \geq T_1 + T(r)$ such that

$$P(A_1) < \frac{\varepsilon}{4}$$

for all $t(\omega) \in [T_2, T_2 + N]$.
It follows from (3.1) that

$$L^N \subset \left\{ \omega : r^2 e^{m_1 t(\omega)\left(\frac{1}{t(\omega)} \int_{-t(\omega)}^{0} |\omega(p)| \mathrm{d}p - \frac{m_2}{m_1}\right.} \right.$$
$$\left. + \int\limits_{-t(\omega)}^{0} e^{m_1 s\left(\int_s^0 \frac{1}{s}\omega(p)\mathrm{d}p + \frac{m_2}{m_1}\right)} \left((c_3 \,|D| + \delta_3 + (c_r + c_a) \left|\omega(s)\right| \right) ds > R^2 \right\}.$$

The definitions A_1 and A_2 imply that

$$L^N \subset \left\{\omega : r^2 e^{m_1 t(\omega)\left(\frac{1}{t(\omega)}\int_{-t(\omega)}^0 |\omega(p)|\mathrm{d}p - \frac{m_2}{m_1}\right)} \geq 1\right\}$$

$$\bigcup \left\{\omega : \int_{-t(\omega)}^{0} e^{m_1 s\left(\frac{1}{s}\int_s^0 |\omega(p)|\mathrm{d}p + \frac{m_2}{m_1}\right)}\left(\left(c_3\,|D| + \delta_3 + (c_r + c_a)\,|\omega(s)|\right)\mathrm{d}s > R^2 - 1\right)\right\}$$

$$\subset A_1 \bigcup A_2 \bigcup \left\{\omega : \int_{-T_1}^{0} \left(\left(c_3\,|D| + \delta_3 + (c_r + c_a)\,|\omega(s)|\right) e^{\left\{m_1 s\left(\frac{1}{s}\int_s^0 |\omega(p)|\mathrm{d}p + \frac{m_2}{m_1}\right)\right\}}\mathrm{d}s\right.$$

$$\left. + \int_{-T_2-N}^{0} \left(\left(c_3\,|D| + \delta_3 + (c_r + c_a)\,|\omega(s)|\right) e^{\alpha m_1 s}\mathrm{d}s > R^2 - 1\right\}$$

$$= A_1 \bigcup A_2 \bigcup \left\{\omega : f_\varepsilon(\omega) + \int_{-T_2+N}^{0} |\,\omega(s) e^{m_1 \alpha s}\mathrm{d}s > \frac{R^2 - A}{B}\right\}.$$

where A and B are some positive constants, and $f_\epsilon : \omega \to R$ is a measurable and P -almost everywhere bounded function. Hence there exists a real number $R_1 = R_1(\epsilon)$ and a random set $A_3 \subset \Omega$ such that

$$f_\varepsilon(\omega) > R_1, \quad P(A_3) < \frac{\varepsilon}{4}$$

for all $\omega \in A_3$. Therefore,

$$L^N \subset A_1 \bigcup A_2 \bigcup A_3 \bigcup \left\{\omega \;:\; \int_{-T-N}^{0} |\omega(s)| e^{m_1 \alpha s}\mathrm{d}s > \frac{R^2 - A}{B} - R_1(\varepsilon)\right\}.$$

It follows from (HY1) that there exists a positive constant $D = D(\epsilon)$ such that

$$P\left\{\omega : \int_{-t}^{0} |\omega(s)| e^{m_1 \alpha s} ds > D\right\} < \frac{\varepsilon}{4}.$$

for all $t > 0$. Choose $R = R(\epsilon)$ such that

$$\frac{R^2 - A}{B} - R_1(\varepsilon) > D.$$

Then, for all $\epsilon > 0$, there exists $R = R(\epsilon) > 0$ for which, whatever B_r is a ball with radius r, there exists $T = T(\epsilon, R, r)$ such that

$$P(L^N) = P\left\{\omega \;:\; \sup_{t\in[T, T+N]} \left\|G(t,\ \theta_{-t}\omega)B_r\right\|^2 > R^2\right\} < \varepsilon,\ \forall N \geq 1.$$

Notice that $L^N \subset L^{N+1}$, and let $L = \bigcup_{i=1}^N L^N$, then $P(L) < \epsilon$, and

$$\left\{\omega \;:\; \sup_{t\geq T} \left\|G(t,\ \theta_{-t}\omega B_r\right\|^2 > R^2\right\} = L,$$

which implies that $G2$ holds. Thus, the proof of Lemma (3.5) is completed.

Theorem 3.1. *Assume that conditions* (H1)-(H6) *hold, and the Lévy noise satisfy the (HY0)- (HY1). Then, the MRDS G generated by the systems* (1.3) *admits a random attractors.*

Proof. It follows from Lemma 3.4 that the equations (1.3) generates a MRDS G. Lemma 3.3 and 3.5 imply that the MRDS G satisfies the conditions G_1 and G_2. Then, the existence random attractor is established by theorem (2.1). Thus, the proof is completed.

In the sequel, we are going to show the existence of random attractor for the equation (1.3) with weaker restrictions on the functions $k_1(u)$ and $k_2(v)$ instead of the stronger restrictions imposed on it, that is to say, $k_1(u)$ and $k_2(v)$ are assumed to be the functions

$$\left|k_1(u)\right| \le a_3|u|^{\gamma} + b_3, \quad \left|k_2(v)\right| \le a_2\,|v| + b_2. \tag{3.6}$$

for $\gamma < p - 1$.

Lemma 3.6. *Assume that* (H1)-(H5) *and* (3.6) *hold. Then there exists the some positive constants m_2, a_5 and b_5 such that the solution* $(u(t), v(t))$ *of the equations* (1.3) *satisfy the following estimates*

$$\begin{aligned}\left|u(t)\right|^2 + \left|v(t)\right|^2 \le\ & (\left|u(0)\right|^2 + \left|v(0)\right|^2)e^{\int_0^t (2a_2+\alpha)|\omega(p)|-m_2)\mathrm{d}p} \\ & + \int_0^t \left(b_5 + a_5\left|\omega(s)\right|^{\frac{p}{p-r-1}}\right)e^{\int_0^t (2a_2+\alpha)|\omega(p)|-m_2)\mathrm{d}p}\mathrm{d}s.\end{aligned}$$

Proof. By the Young inequality and (3.6), we have

$$2(k_1(u)\omega(t),\ u) \le 2a_3\,|\omega|\cdot\|u\|_{L^{r+1}}^{r+1} + 2b_3\,|\omega|\int_D |u|dx \le a_4|\omega|^{\frac{p}{p-r-1}} + \frac{3}{2}c_1\,\|u\|_{L^p}^p + b_4.$$

Multiplying the first equation of (1.3) by u and the second one by v, and integrate over D, we obtain

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(|u|^2 + |v|^2) + 2d\|u\|^2 + \delta|v|^2 \\ &\le c_3\,|D| + \delta_3 + b_4 + a_4|\omega|^{\frac{p}{p-r-1}} + (2a_2+\alpha)\,|\omega|\cdot|v|^2 + c_\alpha\,|\omega| \\ &\le b_5 + a_5|\omega|^{\frac{p}{p-r-1}} + (2a_2+\alpha)\,|\omega|\cdot|v|^2 \\ &\le b_5 + a_5|\omega|^{\frac{p}{p-r-1}} + (2a_2+\alpha)\,|\omega|\cdot(|u|^2 + |v|^2).\end{aligned}$$

Let $m_2 = \min\{\delta, 2d\lambda_1\}$, where $\lambda_1 > 0$ is the first eigenvalue of $-\Delta$ in $H_0^1(D)$. Then

$$\frac{\mathrm{d}}{\mathrm{d}t}(|u|^2 + |v|^2) \le b_5 + a_5|\omega|^{\frac{p}{p-r-1}} + ((2a_2+\alpha)\,|\omega| - m_2)\cdot(|u|^2 + |v|^2). \tag{3.7}$$

Integrating (3.7) on the interval $[s, t](t \ge s \ge 0)$, we get

$$\begin{aligned}\left|u(t)\right|^2 + \left|v(t)\right|^2 \le\ & \left|u(s)\right|^2 + \left|v(s)\right|^2 + b_5(t-s) + a_5\int_s^t \left|\omega(s)\right|^{\frac{p}{p-r-1}}\mathrm{d}s \\ & + \int_s^t ((2a_2+\alpha)\left|\omega(p)\right| - m_2)\cdot(\left|u(p)\right|^2 + \left|v(p)\right|^2)\mathrm{d}p).\end{aligned} \tag{3.8}$$

Applying the Gronwall's Lemma to (3.8) gives

$$\begin{aligned}|u(t)|^2 + |v(t)|^2 \quad &\leq \quad (|u(0)|^2 + |v(0)|^2)e^{\int_0^t (2a_2+\alpha)|\omega(p)|-m_2)\mathrm{d}p} \\ &+ \int_0^t \left(b_5 + a_5|\omega(s)|^{\frac{p}{p-r-1}}\right)e^{\int_0^t (2a_2+\alpha)|\omega(p)|-m_2)\mathrm{d}p}\mathrm{d}s.\end{aligned} \tag{3.9}$$

for all $t > 0$. Thus, we complete the proof of Lemma 3.6.

Lemma 3.7. *Assume that* (H1)-(H5) *and* (3.6) *hold, and Lévy noise* $\omega(t)$ *satisfies the following conditions*

(HY2) *If for all* $\epsilon > 0$, $\alpha > 0$, *there exists* $T = T(\epsilon) > 0$ *such that Levy noise* $\omega(t)$ *satisfies*

$$P\left((\omega(t) : \sup_{t\geq T} \frac{1}{t}\int_{-t}^{0} |\omega(p)|\,\mathrm{d}p - \frac{m_2}{2a_2+\alpha} \leq -\gamma\right) > 1-\varepsilon,$$

(HY3) *If for all* $\epsilon > 0$, *there exists a positive constant* $D > 0$ *such that Levy noise* $\omega(t)$ *satisfies*

$$\sup_{t\geq 0} P\left\{\omega(t) : \int_{-t}^{0}\left(a_5|\omega(s)|^{\frac{p}{p-r-1}} + b_5\right)e^{\beta(2a_2+\alpha)s} \leq D\right\} > 1-\varepsilon.$$

Then, the MRDS G generated by equation (1.3) *satisfies the condition* G_2.

Proof. The proof is similar to the one of Lemma 3.5, and is omitted here.

Theorem 3.2. *Assume that conditions* (H1)-(H5) *and* (3.6) *hold, and the Lévy noise satisfy the (HY2)-(HY3). Then, the MRDS G generated by the systems* (1.3) *admits a random attractors.*

Proof. The proof is similar to the one of Theorem 3.1, and is omitted here.

Remark 3.1. *From the proof of Theorem 3.1 and Theorem 3.2, the Theorem 3.1 and Theorem 3.2 hold for more general càdlàg process.*

Acknowledgements
This article was supported by the NSF of China (No. 10971225) NSF of Hunan (No. 11JJ3004) and sponsored by SRF for ROCS, SEM, China.

Competing interests
The authors declare that they have no competing interests.

Received: 24 February 2012 Accepted: 11 June 2012 Published: 11 June 2012

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doi:10.1186/1029-242X-2012-132
Cite this article as: Huang: Random attractor of stochastic partly dissipative systems perturbed by Lévy noise. *Journal of Inequalities and Applications* 2012 **2012**:132.