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Approximate lie brackets: a fixed point approach

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Abstract
The aim of this article is to investigate the stability and superstability of Lie brackets on Banach spaces by using fixed point methods.
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1. Introduction

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms. Hyers [2] gave a first affirmative answer to the question of Ulam for Banach spaces. Rassias [3] considered the stability problem with unbounded Cauchy differences. The stability problems of several functional equations have extensively been investigated by a number of authors and there are many interesting results concerning this problem (see [4-18]).

In 2003, Cădariu and Radu applied the fixed point method and they could present a short and simple proof (different from the "*direct method*", initiated by Hyers in 1941) for the generalized Hyers-Ulam stability of Jensen functional equation.

In this article, by using the fixed point method, we prove that, if there exists an approximately Lie bracket $f : A \times A \to A$ on Banach spaces A, then there exists a Lie bracket $T : A \times A \to A$ which is near to f. Moreover, under some conditions on f, the Banach space A has a Lie algebra structure with Lie bracket T.

We recall a Lie algebra consists of a (finite dimensional) vector space A over a field $\mathbb{F}$ and a multiplication in A (usually, the product of $x, y \in A$ is denoted by $[x, y]$ and called a Lie bracket or commutator) with the following two properties:

(1) *Anti-commutativity*: $[x, x] = 0$ for any $x \in A$;

(2) *Jacobi identity*: $[z, [x, y]] = [[z, x], y] + [x, [z, y]]$ for any $x, y, z \in A$.

For more details about Lie algebras, the readers are referred to [19-22]. Throughout this article, we assume that $n_0 \in \mathbb{N}$ is a positive integer,

$$\mathbb{T}^1 := \{z \in \mathbb{C} : |z| = 1\}, \quad \mathbb{T}^1_{\frac{1}{n_o}} := \{e^{i\theta} : 0 \le \theta \le \frac{2\pi}{n_0}\}.$$

It is easy to see that $\mathbb{T}^1 = \mathbb{T}^1_{\frac{1}{1}}$. Moreover, we suppose that A is a complex Banach space. For any mapping $f : A \times A \to A$, we define

$$\begin{aligned}D_\mu f(x,y,z,t) : &= 4\mu f\left(\frac{x+y}{2},\frac{z+t}{2}\right) + 4\mu f\left(\frac{x-y}{2},\frac{z+t}{2}\right)\\ &+ 4\mu f\left(\frac{x+y}{2}+\frac{z-t}{2}\right) + 4\mu f\left(\frac{x-y}{2},\frac{z-t}{2}\right)\\ &- 4f(\mu x,z)\end{aligned}$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_o}}$ and $x, y, z, t \in A$.

2. Main results

We need the following theorem to prove the main result of this article.

Theorem 2.1. (The alternative of fixed point theorem [23,24]) *Suppose that* (Ω, d) *is a complete generalized metric space and* $T : \Omega \to \Omega$ *is a strictly contractive mapping with Lipschitz constant L. Then, for any* $x \in \Omega$, *either* $d(T^m x, T^{m+1}x) = \infty$ *for all* $m \geq 0$ *or there exists a natural number* m_0 *such that*

(1) $d(T^m x, T^{m+1}x) < 1$ *for all* $m \geq m_0$;

(2) *the sequence* $\{T^m x\}$ *is convergent to a fixed point* y^* *of T;*

(3) y^* *is the unique fixed point of T in the set* $\Lambda = \{y \in \Omega : d(T^{m_0}x, y) < \infty$;

(4) $d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$*for all* $y \in \Lambda$.

Now, we give our main results by using. Theorem 2.1.

Theorem 2.2. *Let* $f : A \times A \to A$ *be a continuous mapping and let* $\varphi : A^4 = A \times A \times A \times A \to [0, \infty)$ *be a mapping such that*

$$||D_\mu f(x,y,z,t)|| \leq \phi(x,y,z,t), \tag{2.1}$$

$$\begin{aligned}&\lim_{n\to\infty} \left\|4^{-n}f(2^n z, 2^n f(x,y)) - f(4^{-n}f(2^n z, 2^n x), y) - f(x, 4^{-n}f(2^n z, 2^n y))\right\|\\ &\leq \phi(x,y,0,0),\end{aligned} \tag{2.2}$$

$$\lim_{n\to\infty} 4^{-n}f(2^n x, 2^n x) = 0 \tag{2.3}$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_o}}$ *and* $x, y, z, t \in A$. *If there exists* $L < 1$ *such that* $\phi(x,y,z,t) \leq 4L\phi(\frac{x}{2},\frac{y}{2},\frac{z}{2},\frac{t}{2})$*for all* $x, y, z, t \in A$, *then there exists a unique bilinear mapping* $T : A \times A \to A$ *such that*

$$\left\|f(x,z) - T(x,z)\right\| \leq \frac{L}{1-L}\phi(x,0,z,0) \tag{2.4}$$

for all $x, z \in M$. *Moreover, for any sequence* $\{a_m\}$ *in A, if*

$$\lim_{m\to\infty}\lim_{n\to\infty} 4^{-n}f(2^n x, 2^n a_m) = \lim_{n\to\infty}\lim_{m\to\infty} 4^{-n}f(2^n x, 2^n a_m) \tag{2.5}$$

for all $x \in A$, *then A is a Lie algebra with Lie bracket* $[x, y] = T(x, y)$ *for all* $x, y \in A$.

Proof. Putting $\mu = 1$ and $y = t = 0$ in (2.1), we get

$$\left\|4f(\frac{x}{2},\frac{z}{2}) - f(x,z)\right\| \leq \phi(x,0,z,0)$$

for all $x, z \in A$ and so

$$\left\| \frac{1}{4} f(2x, 2z) - f(x, z) \right\| \leq \frac{1}{4} \phi(2x, 0, 2z, 0) \leq L\phi(x, 0, z, 0) \tag{2.6}$$

for all $x, z \in A$. Consider the set $X := \{g : g : A \times A \to A\}$ and introduce the generalized metric on X by:

$$d(h, g) := \inf\{C \in \mathbb{R}^+ : \ ||g(x, z) - h(x, z)|| \ \leq C\phi(x, 0, z, 0) \text{ for all } x, z \in A\}.$$

It is easy to show that (X, d) is a complete generalized metric space. Now, we define the mapping $J : X \to X$ by

$$J(h)(x, z) = \frac{1}{4} h(2x, 2z)$$

for all $x, z \in A$. For any $g, h \in X$, we have

$$\begin{aligned} d(g, h) < C \Rightarrow & \|g(x, z) - h(x, z)\| \leq C\phi(x, 0, z, 0) \\ \Rightarrow & \left\| \frac{1}{4} g(2x, 2z) - \frac{1}{4} h(2x, 2z) \right\| \leq \frac{1}{4} C\phi(2x, 0, 2z, 0) \\ \Rightarrow & \left\| \frac{1}{4} g(2x, 2z) - \frac{1}{4} h(2x, 2z) \right\| \leq LC\phi(x, 0, z, 0) \\ \Rightarrow & \ d(J(g), J(h)) \leq LC \end{aligned}$$

for all $x, z \in A$, which means that

$$d(J(g), J(h)) \leq Ld(g, h)$$

for all $g, h \in X$. It follows from (2.6) that

$$d(f, J(f)) \leq L.$$

From Theorem 2.1, it follows that J has a unique fixed point in the set $X_1 := \{I \in X : d(f, T) < \infty\}$. Let T be the fixed point of J. Then we have $\lim_{n\to\infty} d(J^n(f), T) = 0$ and

$$\lim_{n\to\infty} \frac{1}{4^n} f(2^n x, 2^n z) = T(x, z) \tag{2.7}$$

for all $x, z \in A$. By the inequality $d(f, J(f)) \leq L$ and $J(T) = T$, we have

$$d(f, T) \leq d(f, J(f)) + d(J(f), J(T)) \leq L + Ld(f, T)$$

and so

$$d(f, T) \leq \frac{L}{1 - L}.$$

This implies the inequality (2.4). From $\phi(x, y, z, t) \leq 4L\phi\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}, \frac{t}{2}\right)$, we have

$$\lim_{j\to\infty} 4^{-j} \phi(2^j x, 2^j y, 2^j z, 2^j t) = 0 \tag{2.8}$$

for all $x, y, z \in A$. Thus it follows from (2.1), (2.7) and (2.8) that

$$\begin{aligned}
&\left\| 4T\left(\frac{x+y}{2}, \frac{z+t}{2}\right) + 4T\left(\frac{x-y}{2}, \frac{z+t}{2}\right) \right. \\
&\quad \left. -4T\left(\frac{x+y}{2}, \frac{z-t}{2}\right) + 4T\left(\frac{x-y}{2}, \frac{z-t}{2}\right) - 4T(x, z) \right\| \\
&= \lim_{n\to\infty} \frac{1}{4^n} \left\| 4f\left(\frac{2^n x + 2^n y}{2}, \frac{2^n z + 2^n t}{2}\right) + 4f\left(\frac{2^n x - 2^n y}{2} - \frac{2^n z + 2^n t}{2}\right) \right. \\
&\quad \left. -4f\left(\frac{2^n x + 2^n y}{2}, \frac{2^n z - 2^n t}{2}\right) - 4f\left(\frac{2^n x - 2^n y}{2}, \frac{2^n z - 2^n t}{2}\right) - 4f(2^n x, 2^n z) \right\| \\
&\leq \lim_{n\to\infty} \frac{1}{4^n} \phi(2^n x, 2^n y, 2^n z, 2^n t) = 0
\end{aligned}$$

for all $x, y, z \in M$ and so

$$\begin{aligned}
&T\left(\frac{x+y}{2}, \frac{z+t}{2}\right) + T\left(\frac{x-y}{2}, \frac{z+t}{2}\right) - T\left(\frac{x+y}{2}, \frac{z-t}{2}\right) + T\left(\frac{x-y}{2}, \frac{z-t}{2}\right) \\
&= T(x, z)
\end{aligned}$$

for all $x, y, z, t \in A$. This shows that

$$T(x+y, z+t) = T(x, z) + T(y, z) + T(x, t) + T(y, t)$$

for all $x, y, z, t \in A$. Hence, T is Cauchy additive with respect to the first and second variables. By putting $y := x$ and $t := z$ in (2.1), we have

$$\left\| 4\mu f(x, z) - 4f(\mu x, z) \right\| \leq \phi(x, x, z, z) \| \tag{2.9}$$

for all $x, z \in A$. and $\mu \in \mathbb{T}^1_{\frac{1}{n_o}}$ and so

$$\begin{aligned}
\left\| 4\mu T(x, z) - 4T(\mu x, z) \right\| &= \lim_{n\to\infty} \frac{1}{4^n} \left\| 4\mu f(2^n x, 2^n z) - 4f(2^n \mu x, 2^n z) \right. \\
&\leq \lim_{n\to\infty} \frac{1}{4^n} \phi(2^n x, 2^n x, 2^n z, 2^n z) = 0
\end{aligned}$$

for all $x, z \in A$ and $\mu \in \mathbb{T}^1_{\frac{1}{n_o}}$, that is,

$$T(\mu x, z) = \mu T(x, z) \tag{2.10}$$

for all $x, z \in A$.

If λ belongs to $\mathbb{T}^1$, then there exists $\theta \in [0, 2\pi]$ such that $\lambda = e^{i\theta}$. If we set $\lambda_1 = e^{\frac{i\theta}{n_o}}$, then λ_1 belongs to $\mathbb{T}^1_{\frac{1}{n_o}}$. By using (2.10), we have

$$T(\lambda x, z) = T(\lambda_1^{n_0} x, z) = \lambda_1^{n_0} T(x, z) = \lambda T(x, z)$$

for all $x, z \in M$.

If λ belongs to $n\mathbb{T}^1 = \{nz : z \in \mathbb{T}^1\}$ for some $n \in \mathbb{N}$, then, by (2.9), we have

$$\begin{aligned} T(\lambda x, z) = T(n\lambda_1 x, z) &= T(\lambda_1(nx), z) \\ &= \lambda_1 T(nx, z) \\ &= \lambda_1 n T(x, z) \\ &= \lambda T(x, z) \end{aligned}$$

for all $x, z \in A$. Let $s \in (0, \infty)$. Then, by Archimedean property of $\mathbb{C}$, there exists a positive real number n such that the point $(s, 0) \in \mathbb{R}^2$ lies in the interior of circle with center at origin and radius n in $\mathbb{R}^2$. Putting $s_1 := s + \sqrt{n^2 - s^2}i$ and $s_2 := t - \sqrt{n^2 - s^2}i$, we have $s = \frac{s_1 + s_2}{2}$ and $s_1, s_2 \in n\mathbb{T}^1$. Thus, by (2.9), we have

$$\begin{aligned} T(sx, z) = T\left(\frac{s_1 + s_2}{2}x, z\right) &= T\left(s_1\frac{x}{2}, z\right) + T\left(s_2\frac{x}{2}, z\right) \\ &= s_1 T\left(\frac{x}{2}, z\right) + s_2 T(\frac{x}{2}, z) \\ &= 4\left(\frac{s_1 + s_2}{2}\right) T\left(\frac{x}{2}, \frac{z}{2}\right) \\ &= sT(x, z) \end{aligned}$$

for all $x, z \in s$. Moreover, there exists $\theta \in [0, 2\pi]$ such that $\lambda = |\lambda|\, e^{i\theta}$.
Therefore, we have

$$T(\lambda x, z) = T(|\lambda|\, e^{i\theta}x, z) = |\lambda|\, T(e^{i\theta}x, z) = |\lambda|\, e^{i\theta}T(x, z) = \lambda T(x, z) \tag{2.11}$$

for all $x, z \in A$ and so $T : A \times A \to A$ is homogeneous with respect to the first variable. It follows from (2.9) and (2.11) that T is $\mathbb{C}$-Linear with respect to the first variable.

Moreover, by (2.3), $T(x, x) = 0$ for all $x \in A$, whence

$$0 = T(x + y, x + y) = T(x, x) + T(x, y) + T(y, x) + T(y, y) = T(x, y) + T(y, x)$$

for all $x, y \in A$ and so

$$T(x, y) = -T(y, x)$$

for all $x, y \in A$, that is, T is skew symmetric. Let $z \in A$ and define a mapping $ad(z)$: $A \to A$ by

$$ad(z)(x) = T(z, x)$$

for all $x \in A$. It is clear that $ad(z)$ is a linear and continuous mapping at zero. In fact, if $\{a_m\}$ is a sequence in A such that $\lim_{n\to\infty} a_m = 0$, then, by (2.5), we have

$$\begin{aligned} \lim_{m\to\infty} ad(z)\,(a_m) &= \lim_{m\to\infty}\lim_{n\to\infty} 4^{-n} f(2^n z, 2^n a_m) \\ &= \lim_{n\to\infty}\lim_{m\to\infty} 4^{-n} f(2^n z, 2^n a_m) \\ &= \lim_{n\to\infty} 4^{-n} f(2^n z, 0) = ad(z)\,(0) = 0. \end{aligned}$$

Thus, for all $z \in A$, $ad(z)$ is continuous at zero and so $ad(z)$ is a continuous and linear mapping. Substituting x with $2^m x$ and y with $2^m y$ in (2.2) and multiplying by 4^{-m} both sides of the inequality, we have

$$\begin{aligned}\lim_{n\to\infty} 4^{-m}\,\|4^{-n}f(2^n z,\ 2^n f(2^m x,\ 2^m y)) - f(4^{-n}f(2^n z,\ 2^{n+m}x),\ 2^m y)\\ -f(2^m x,\ 4^{-n}f(2^n z,\ 2^{n+m}y))\|\\ \le 4^{-m}\phi(2^m x,\ 2^m y,\ 0,0)\end{aligned}$$

for all $x, y, z \in A$ and $m \in \mathbb{N}$. Since f is continuous, we have

$$\begin{aligned}4^{-m}\,\|ad(z)\,(f(2^m x,\ 2^m y)) - f(ad(z)\,(2^m x),\ 2^m y) - f(2^m x,\ ad(z)2^m y)\|\\ \le 4^{-m}\phi(2^m x,\ 2^m y,\ 0,0)\end{aligned}$$

for all $x, y, z \in A$. Since, for all $z \in A$, $ad(z)$ is a linear and continuous mapping, we get

$$ad(z)T(x,\ y) - T(ad(z)(x),\ y) - T(x,\ ad(z)(y)) = 0$$

for all $x, y, z \in A$. Since T is skew symmetric, it is easy to show that T is satisfies in the Jacobi identity condition. Thus T is a Lie bracket satisfies in (2.4) and (A, T) is a Lie algebra.

To prove the uniqueness property of T, let $Q : A \times A \to A$ be another bilinear mapping satisfying (2.7). Then we have

$$\begin{aligned}\|T(x,\ z) - Q(x,\ z)\| &= \lim_{n\to\infty}\left\|\frac{f(2^n x, 2^n z)}{4^n} - \frac{Q(2^n x, 2^n z)}{4^n}\right\|\\ &\le \lim_{n\to\infty}\frac{1}{4^n}\left(\frac{L}{1-L}\right)\phi\,(2^n x,\ 0, 2^n z,\ 0) = 0\end{aligned}$$

for all $x, z \in A$. This means that $T = Q$. This completes the proof. □

Corollary 2.3. *Let $p \in (0, 1)$ and $\theta \in [0, \infty)$ be real numbers. Suppose that $f : A \times A \to A$ is a mapping such that*

$$\|D_\mu f(x,\ y,\ z,\ t)\| \le \theta\left(\|x\|^p + \|y\|^p + \|z\|^p + \|t\|^p\right),$$

$$\begin{aligned}&\lim_{n\to\infty}\|4^{-n}f(2^n z,\ 2^n f(x,\ y)) - f(4^{-n}f(2^n z,\ 2^n x),\ y) - f(x,\ 4^{-n}f(2^n z,\ 2^n y))\|\\ &\le \theta(||x||^p + ||y||^p),\end{aligned}$$

$$\lim_{n\to\infty} 4^{-n}f(2^n x,\ 2^n x) = 0$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_o}}$ and $x, y, z, t \in A$. Then there exists a unique bilinear mapping $T : A \times A \to A$ such that

$$|f(x,\ z) - T(x,\ z)|| \le \frac{4^p\theta}{4-4^p}(||x||^p + ||z||^p)$$

for all $x, z \in A$. Moreover, for any sequence $\{a_m\}$ in A, if

$$\lim_{m\to\infty}\lim_{n\to\infty} 4^{-n}f(2^n x,\ 2^n a_m) = \lim_{n\to\infty}\lim_{m\to\infty} 4^{-n}f(2^n x,\ 2^n a_m)$$

for all $x \in A$, then A is a Lie algebra with Lie bracket $[x, y] = T(x, y)$ for all $x, y \in A$.

Proof. It follows from Theorem 2.2 by putting $\varphi(x, y, z) := \theta(\| x \|^p + \| y \|^p + \| z \|^p + \| t \|^p)$ for all $x, y, z, \in M$ and $L = 4^{p-1}$. □

Finally, we prove the superstability of Lie brackets as follows:

Corollary 2.4. *Let* $p \in \left(0, \frac{1}{4}\right)$ *and* $\theta \in [0, \infty)$ *be real numbers. Suppose that* $f: A \times A \to A$ *is a mapping such that*

$$||D_\mu f(x, y, z, t)|| \le \theta(||x||^p \, ||y||^p \, ||z||^p \, ||t||^p),$$

$$\lim_{n\to\infty} ||4^{-n} f(2^n z, 2^n f(x, y)) - f(4^{-n} f(2^n z, 2^n x), y) - f(x, 4^{-n} f(2^n z, 2^n y))|| \le \theta(||x||^p \, ||y||^p),$$

$$\lim_{n\to\infty} 4^{-n} f(2^n x, 2^n x) = 0$$

for all $\mu \in \mathbb{T}^1_{\frac{1}{n_o}}$ *and* $x, y, z, t \in A$. *Moreover, for any sequence* $\{a_m\}$ *in* A, *if*

$$\lim_{m\to\infty} \lim_{n\to\infty} 4^{-n} f(2^n x, 2^n a_m) = \lim_{n\to\infty} \lim_{m\to\infty} 4^{-n} f(2^n x, 2^n a_m)$$

for all $x \in A$, *then* A *is a Lie algebra with Lie bracket* $[x, y] = f(x, y)$ *for all* $x, y \in A$.

Proof. Putting $\varphi(x, y, z, t) := \theta(||x||^p \, ||y||^p \, ||z||^p \, ||t||^p)$ for all $x, y, z \in M$ and $L = \frac{1}{2}$ in Theorem 2.2, the conclusion follows. □

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Authors' contributions
All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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