Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

Local stability of the Pexiderized Cauchy and Jensen's equations in fuzzy spaces

Abbas Najati[1], Jung Im Kang[2*] and Yeol Je Cho[3]

* Correspondence: jikang@nims.re.kr
[2]National Institute for Mathematical Sciences, KT Daeduk 2 Research Center, 463-1 Jeonmin-dong, Yuseong-gu, Daejeon 305-811, Korea
Full list of author information is available at the end of the article

Abstract

Lex X be a normed space and Y be a Banach fuzzy space. Let $D = \{(x, y) \in X \times X : \|x\| + \|y\| \geq d\}$ where $d > 0$. We prove that the Pexiderized Jensen functional equation is stable in the fuzzy norm for functions defined on D and taking values in Y. We consider also the Pexiderized Cauchy functional equation.
2000 Mathematics Subject Classification: 39B22; 39B82; 46S10.

Keywords: Pexiderized Cauchy functional equation, generalized Hyers-Ulam stability, Jensen functional equation, non-Archimedean space

1. Introduction

The functional equation (ξ) is *stable* if any function g satisfying the equation (ξ) approximately is near to the true solution of (ξ).

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms:

Let G_1 be a group and let G_2 be a metric group with the metric $d(\cdot,\cdot)$. Given $\varepsilon > 0$, does there exist $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \varepsilon$ for all $x \in G_1$?

In other words, we are looking for situations when the homomorphisms are stable, i.e., if a mapping is almost a homomorphism, then there exists a true homomorphism near it. If we turn our attention to the case of functional equations, then we can ask the question: When the solutions of an equation differing slightly from a given one must be close to the true solution of the given equation.

In 1941, Hyers [2] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1950, Aoki [3] provided a generalization of the Hyers' theorem for additive mappings, and in 1978, Th.M. Rassias [4] succeeded in extending the result of Hyers for linear mappings by allowing the Cauchy difference to be unbounded (see also [5]). The stability phenomenon that was introduced and proved by Th.M. Rassias is called the *generalized Hyers-Ulam stability*. Forti [6] and Găvruta [7] have generalized the result of Th.M. Rassias, which permitted the Cauchy difference to become arbitrary unbounded. The stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem. A large list of references can be found, for example, in [8-29].

Following [30], we give the following notion of a fuzzy norm.

Definition 1.1. [30] Let X be a real vector space. A function $N : X \times \mathbb{R} \to [0, 1]$ is called a *fuzzy norm* on X if, for all $x, y \in X$ and $s, t \in \mathbb{R}$,

(N_1) $N(x, t) = 0$ for all $t \leq 0$;
(N_2) $x = 0$ if and only if $N(x, t) = 1$ for all $t > 0$;
(N_3) $N(cx, t) = N(x, \frac{t}{|c|})$ if $c \neq 0$;
(N_4) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$;
(N_5) $N(x,\cdot)$ is a nondecreasing function on $\mathbb{R}$ and $\lim_{t\to\infty} N(x, t) = 1$;
(N_6) for $x \neq 0$, $N(x,\cdot)$ is continuous on $\mathbb{R}$.

The pair (X, N) is called a *fuzzy normed vector space*.

Example 1.2. Let $(X, ||\cdot||)$ be a normed linear space and let $\alpha, \beta > 0$. Then,

$$N(x,t) = \begin{cases} \dfrac{\alpha t}{\alpha t + \beta\|x\|}, & t > 0,\ x \in X, \\ 0, & t \leq 0,\ x \in X \end{cases}$$

is a fuzzy norm on X.

Example 1.3. Let $(X, ||\cdot||)$ be a normed linear space and let $\beta > \alpha > 0$. Then,

$$N(x,t) = \begin{cases} 0, & t \leq \alpha\|x\|, \\ \dfrac{t}{t + (\beta-\alpha)\|x\|}, & \alpha\|x\| < t \leq \beta\|x\|; \\ 1, & t > \beta\|x\| \end{cases}$$

is a fuzzy norm on X.

Definition 1.4. Let (X, N) be a fuzzy normed space. A sequence $\{x_n\}$ in X is said to *be convergent* if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case, x is called the *limit* of the sequence $\{x_n\}$, and we denote it by N - $\lim x_n = x$.

The limit of the convergent sequence $\{x_n\}$ in (X, N) is unique. Since if N - $\lim x_n = x$ and N-$\lim x_n = y$ for some $x, y \in X$, it follows from (N_4) that

$$N(x-y,t) \geq \min\left\{N\left(x-x_n, \frac{t}{2}\right), N\left(x_n - y, \frac{t}{2}\right)\right\}$$

for all $t > 0$ and $n \in \mathbb{N}$. So, $N(x - y, t) = 1$ for all $t > 0$. Hence, (N_2) implies that $x = y$.

Definition 1.5. Let (X, N) be a fuzzy normed space. A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for any $\varepsilon > 0$ and $t > 0$, there exists $M \in \mathbb{N}$ such that, for all $n \geq M$ and $p > 0$,

$$N(x_{n+p} - x_n, t) > 1 - \varepsilon.$$

It follows from (N_4) that every convergent sequence in a fuzzy normed space is a Cauchy sequence. If, in a fuzzy normed space, every Cauchy sequence is convergent, then the fuzzy norm is said to be *complete*, and the fuzzy normed space is called a *fuzzy Banach space*.

Example 1.6. [21] Let $N : \mathbb{R} \times \mathbb{R} \to [0, 1]$ be a fuzzy norm on $\mathbb{R}$ defined by

$$N(x,t) = \begin{cases} \dfrac{t}{t+|x|}, & t > 0, \\ 0, & t \le 0. \end{cases}$$

Then, $(\mathbb{R}, N)$ is a fuzzy Banach space.

Recently, several various fuzzy stability results concerning a Cauchy sequence, Jensen and quadratic functional equations were investigated in [17-20].

2. A local Hyers-Ulam stability of Jensen's equation

In 1998, Jung [16] investigated the Hyers-Ulam stability for Jensen's equation on a restricted domain. In this section, we prove a local Hyers-Ulam stability of the Pexiderized Jensen functional equation in fuzzy normed spaces.

Theorem 2.1. *Let X be a normed space, (Y, N) be a fuzzy Banach space, and $f, g, h : X \to Y$ be mappings with $f(0) = 0$. Suppose that $\delta > 0$ is a positive real number, and z_0 is a fixed vector of a fuzzy normed space (Z, N') such that*

$$N\left(2f\left(\frac{x+y}{2}\right) - g(x) - h(y), t+s\right) \ge \min\{N'(\delta z_0, t), N'(\delta z_0, s)\} \tag{2.1}$$

for all $x, y \in X$ with $||x|| + ||y|| \ge d$ and positive real numbers t, s. Then, there exists a unique additive mapping $T : X \to Y$ such that

$$N(f(x) - T(x), t) \ge N'(40\delta z_0, t), \tag{2.2}$$

$$N(T(x) - g(x) + g(0), t) \ge N'(30\delta z_0, t), \tag{2.3}$$

$$N(T(x) - h(x) + h(0), t) \ge N'(30\delta z_0, t) \tag{2.4}$$

for all $x \in X$ and $t > 0$.

Proof. Suppose that $||x|| + ||y|| < d$ holds. If $||x|| + ||y|| = 0$, let $z \in X$ with $||z|| = d$. Otherwise,

$$z := \begin{cases} (d + \|x\|)\dfrac{x}{\|x\|}, & \text{if } \|x\| \ge \|y\|, \\ (d + \|y\|)\dfrac{y}{\|y\|}, & \text{if } \|x\| < \|y\|. \end{cases}$$

It is easy to verify that

$$\begin{aligned} &\|x - z\| + \|y + z\| \ge d, \quad \|2z\| + \|x - z\| \ge d, \quad \|y\| + \|2z\| \ge d, \\ &\|y + z\| + \|z\| \ge d, \quad \|x\| + \|z\| \ge d. \end{aligned} \tag{2.5}$$

It follows from (N_4), (2.1) and (2.5) that

$$\begin{aligned}
&N\Big(2f\left(\frac{x+y}{2}\right)-g(x)-h(y),t+s\Big)\\
&\quad\geq \min\Big\{N\Big(2f\left(\frac{x+y}{2}\right)-g(y+z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{x+z}{2}\right)-g(2z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{y+2z}{2}\right)-g(2z)-h(y),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{y+2z}{2}\right)-g(y+z)-h(z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{x+z}{2}\right)-g(x)-h(z),\frac{t+s}{5}\Big)\Big\}\\
&\quad\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned}$$

for all $x, y \in X$ with $||x|| + ||y|| < d$ and positive real numbers t, s. Hence, we have

$$N\Big(2f\left(\frac{x+y}{2}\right)-g(x)-h(y),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\} \tag{2.6}$$

for all $x, y \in X$ and positive real numbers t, s. Letting $x = 0$ ($y = 0$) in (2.6), we get

$$\begin{aligned}
&N\Big(2f\left(\frac{y}{2}\right)-g(0)-h(y),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\},\\
&N\Big(2f\left(\frac{x}{2}\right)-g(x)-h(0),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned} \tag{2.7}$$

for all $x, y \in X$ and positive real numbers t, s. It follows from (2.6) and (2.7) that

$$\begin{aligned}
&N\Big(2f\left(\frac{x+y}{2}\right)-2f\left(\frac{x}{2}\right)-2f\left(\frac{y}{2}\right),t+s\Big)\\
&\quad\geq \min\Big\{N\Big(2f\left(\frac{x+y}{2}\right)-g(x)-h(y),\frac{t+s}{4}\Big),\\
&\qquad N\Big(2f\left(\frac{x}{2}\right)-g(x)-h(0),\frac{t+s}{4}\Big),\\
&\qquad N\Big(2f\left(\frac{y}{2}\right)-g(0)-h(y),\frac{t+s}{4}\Big),N\Big(g(0)+h(0),\frac{t+s}{4}\Big\}\\
&\quad\geq \min\{N'(20\delta z_0,t),N'(20\delta z_0,s)\}
\end{aligned}$$

for all $x, y \in X$ and positive real numbers t, s. Hence,

$$N\big(f(x+y)-f(x)-f(y),t+s\big)\geq \min\{N'(10\delta z_0,t),N'(10\delta z_0,s)\} \tag{2.8}$$

for all $x, y \in X$ and positive real numbers t, s. Letting $y = x$ and $t = s$ in (2.8), we infer that

$$N\Big(\frac{f(2x)}{2}-f(x),t\Big)\geq N'(10\delta z_0,t) \tag{2.9}$$

for all $x \in X$ and positive real number t. replacing x by $2^n x$ in (2.9), we get

$$N\left(\frac{f(2^{n+1}x)}{2^{n+1}}-\frac{f(2^n x)}{2^n},\frac{t}{2^n}\right)\geq N'(10\delta z_0,t) \tag{2.10}$$

for all $x \in X$, $n \geq 0$ and positive real number t. It follows from (2.10) that

$$\begin{aligned} N\Big(\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m}, \sum_{k=m}^{n-1} \frac{t}{2^k}\Big) &\geq \min \bigcup_{k=m}^{n-1} \Big\{ N\Big(\frac{f(2^{k+1}x)}{2^{k+1}} - \frac{f(2^k x)}{2^k}, \frac{t}{2^k}\Big) \\ &\geq N'(10\delta z_0, t) \end{aligned} \tag{2.11}$$

for all $x \in X$, $t > 0$ and integers $n \geq m \geq 0$. For any s, $\varepsilon > 0$, there exist an integer $l > 0$ and $t_0 > 0$ such that $N'(10\delta z_0, t_0) > 1 - \varepsilon$ and $\sum_{k=m}^{n-1} \frac{t_0}{2^k} > s$ for all $n \geq m \geq l$. Hence, it follows from (2.11) that

$$N\Big(\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m}, s\Big) > 1 - \varepsilon$$

for all $n \geq m \geq l$. So $\{\frac{f(2^n x)}{2^n}\}$ is a Cauchy sequence in Y for all $x \in X$. Since (Y, N) is complete, $\{\frac{f(2^n x)}{2^n}\}$ converges to a point $T(x) \in Y$. Thus, we can define a mapping $T : X \to Y$ by $T(x) := N - \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$. Moreover, if we put $m = 0$ in (2.11), then we observe that

$$N\Big(\frac{f(2^n x)}{2^n} - f(x), \sum_{k=0}^{n-1} \frac{t}{2^k}\Big) \geq N'(10\delta z_0, t).$$

Therefore, it follows that

$$N\Big(\frac{f(2^n x)}{2^n} - f(x), t\Big) \geq N'\Big(10\delta z_0, \frac{t}{\sum_{k=0}^{n-1} 2^{-k}}\Big) \tag{2.12}$$

for all $x \in X$ and positive real number t.

Next, we show that T is additive. Let $x, y \in X$ and $t > 0$. Then, we have

$$\begin{aligned} &N\big(T(x+y) - T(x) - T(y), t\big) \\ &\quad \geq \min \Big\{ N'\Big(T(x+y) - \frac{f(2^n(x+y))}{2^n}, \frac{t}{4}\Big), \\ &\qquad N'\Big(\frac{f(2^n x)}{2^n} - T(x), \frac{t}{4}\Big), N'\Big(\frac{f(2^n y)}{2^n} - T(y), \frac{t}{4}\Big), \\ &\qquad N'\Big(\frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n}, \frac{t}{4}\Big)\Big\}. \end{aligned} \tag{2.13}$$

Since, by (2.8),

$$N'\Big(\frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n}, \frac{t}{4}\Big) \geq N'(40\delta z_0, 2^n t),$$

we get

$$\lim_{n\to\infty} N'\Big(\frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n}, \frac{t}{4}\Big) = 1.$$

By the definition of T, the first three terms on the right hand side of the inequality (2.13) tend to 1 as $n \to \infty$. Therefore, by tending $n \to \infty$ in (2.13), we observe that T is additive.

Next, we approximate the difference between f and T in a fuzzy sense. For all $x \in X$ and $t > 0$, we have

$$N(T(x) - f(x), t) \geq \min\left\{N\left(T(x) - \frac{f(2^n x)}{2^n}, \frac{t}{2}\right), N\left(\frac{f(2^n x)}{2^n} - f(x), \frac{t}{2}\right)\right\}.$$

Since $T(x) := N - \lim_{n\to\infty} \dfrac{f(2^n x)}{2^n}$, letting $n \to \infty$ in the above inequality and using (N) and (2.12), we get (2.2). It follows from the additivity of T and (2.7) that

$$\begin{aligned} N(T(x) - g(x) + g(0), t) \geq \min\Big\{&N\Big(2T\big(\frac{x}{2}\big) - 2f\big(\frac{x}{2}\big), \frac{t}{3}\Big), \\ &N\Big(2f\big(\frac{x}{2}\big) - g(x) - h(0), \frac{t}{3}\Big), \\ &N\Big(g(0) + h(0), \frac{t}{3}\Big)\Big\} \\ \geq N'&(30\delta z_0, t) \end{aligned}$$

for all $x \in X$ and $t > 0$. So, we get (2.3). Similarly, we can obtain (2.4).

To prove the uniqueness of T, let $S : X \to Y$ be another additive mapping satisfying the required inequalities. Then, for any $x \in X$ and $t > 0$, we have

$$\begin{aligned} N(T(x) - S(x), t) &\geq \min\left\{N\left(T(x) - f(x), \frac{t}{2}\right), N\left(f(x) - S(x), \frac{t}{2}\right)\right. \\ &\geq N'(80\delta z_0, t). \end{aligned}$$

Therefore, by the additivity of T and S, it follows that

$$N(T(x) - S(x), t) = N(T(nx) - S(nx), nt) \geq N'(80\delta z_0, nt)$$

for all $x \in X$, $t > 0$ and $n \geq 1$. Hence, the right hand side of the above inequality tends to 1 as $n \to \infty$. Therefore, $T(x) = S(x)$ for all $x \in X$. This completes the proof. □

The following is a local Hyers-Ulam stability of the Pexiderized Cauchy functional equation in fuzzy normed spaces.

Theorem 2.2. *Let X be a normed space, (Y, N) be a fuzzy Banach space, and $f, g, h : X \to Y$ be mappings with $f(0) = 0$. Suppose that $\delta > 0$ is a positive real number, and z_0 is a fixed vector of a fuzzy normed space (Z, N') such that*

$$N(f(x + y) - g(x) - h(y), t + s) \geq \min\{N'(\delta z_0, t), N'(\delta z_0, s)\} \tag{2.14}$$

for all $x, y \in X$ with $||x|| + ||y|| \geq d$ and positive real numbers t, s. Then, there exists a unique additive mapping $T : X \to Y$ such that

$$\begin{aligned} N(f(x) - T(x), t) &\geq N'(80\delta z_0, t), \\ N(T(x) - g(x) + g(0), t) &\geq N'(60\delta z_0, t), \\ N(T(x) - h(x) + h(0), t) &\geq N'(60\delta z_0, t) \end{aligned}$$

for all $x \in X$ and $t > 0$.

Proof. For the case $||x|| + ||y|| < d$, let z be an element of X which is defined in the proof of Theorem 2.1. It follows from (N_4), (2.5) and (2.14) that

$$\begin{aligned}
&N(f(x+y)-g(x)-h(y),t+s)\\
&\quad\geq \min\Big\{N\Big(f(x+y)-g(y+z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(x+z)-g(2z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(y+2z)-g(2z)-h(y),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(y+2z)-g(y+z)-h(z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(x+z)-g(x)-h(z),\frac{t+s}{5}\Big)\Big\}\\
&\quad\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned}$$

for all $x, y \in X$ with $||x|| + ||y|| < d$ and positive real numbers t, s. Hence, we have

$$N\Big(f(x+y)-g(x)-h(y),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\} \tag{2.15}$$

for all $x, y \in X$ and positive real numbers t, s. Letting $x = 0$ ($y = 0$) in (2.15), we get

$$\begin{aligned}
&N(f(y)-g(0)-h(y),t+s)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\},\\
&N(f(x)-g(x)-h(0),t+s)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned} \tag{2.16}$$

for all $x, y \in X$ and positive real numbers t, s. It follows from (2.15) and (2.16) that

$$\begin{aligned}
&N(f(x+y)-f(x)-f(y),t+s)\\
&\quad\geq \min\Big\{N\Big(f(x+y)-g(x)-h(y),\frac{t+s}{4}\Big),\\
&\qquad N\Big(f(x)-g(x)-h(0),\frac{t+s}{4}\Big),\\
&\qquad N\Big(f(y)-g(0)-h(y),\frac{t+s}{4}\Big),\\
&\qquad N(g(0)+h(0),\frac{t+s}{4})\Big\}\\
&\quad\geq \min\{N'(20\delta z_0,t),N'(20\delta z_0,s)\}
\end{aligned}$$

for all $x, y \in X$ and positive real numbers t, s. The rest of the proof is similar to the proof of Theorem 2.1, and we omit the details. □

Acknowledgements
This work was supported by the Korea Research Foundation (KRF) grant funded by the Korea government (MEST) (no. 2009-0075850).

Author details
[1]Department of Mathematics, Faculty of Sciences, University of Mohaghegh Ardabili, 56199-11367 Ardabil, Iran
[2]National Institute for Mathematical Sciences, KT Daeduk 2 Research Center, 463-1 Jeonmin-dong, Yuseong-gu, Daejeon 305-811, Korea [3]Department of Mathematics Education and the RINS, Gyeongsang National University, Jinju 660-701, Korea

Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 16 May 2011 Accepted: 6 October 2011 Published: 6 October 2011

References
1. Ulam, SM: Problems in Modern Mathematics. New York:Wiley, Science**chap. VI** (1964)

2. Hyers, DH: On the stability of the linear functional equation. Proc Nat Acad Sci USA. **27**, 222–224 (1941). doi:10.1073/pnas.27.4.222
3. Aoki, T: On the stability of the linear transformationin Banach spaces. J Math Soc Japan. **2**, 64–66 (1950). doi:10.2969/jmsj/00210064
4. Rassias, ThM: On the stability of the linear mapping in Banach spaces. Proc Am Math Soc. **72**, 297–300 (1978). doi:10.1090/S0002-9939-1978-0507327-1
5. Bourgin, DG: Classes of transformations and bordering transformations. Bull Am Math Soc. **57**, 223–237 (1951). doi:10.1090/S0002-9904-1951-09511-7
6. Forti, GL: An existence and stability theorem for a class of functional equations. Stochastica. **4**, 23–30 (1980). doi:10.1080/17442508008833155
7. Găvruta, P: A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings. J Math Anal Appl. **184**, 431–436 (1994). doi:10.1006/jmaa.1994.1211
8. Bae, J, Jun, K, Lee, Y: On the Hyers-Ulam-Rassias stability of an *n*-dimensional Pexiderized quadratic equation. Math Inequal Appl. **7**, 63–77 (2004)
9. Faizev, VA, Rassias, ThM, Sahoo, PK: The space of (ψ, φ)-additive mappings on semigroups. Trans Am Math Soc. **354**, 4455–4472 (2002). doi:10.1090/S0002-9947-02-03036-2
10. Forti, GL: Hyers-Ulam stability of functional equations in several variables. Aequ Math. **50**, 143–190 (1995). doi:10.1007/BF01831117
11. Forti, GL: Comments on the core of the direct method for proving Hyers-Ulam stability of functional equations. J Math Anal Appl. **295**, 127–133 (2004). doi:10.1016/j.jmaa.2004.03.011
12. Haruki, H, Rassias, ThM: A new functional equation of Pexider type related to the complex exponential function. Trans Am Math Soc. **347**, 3111–3119 (1995). doi:10.2307/2154775
13. Hyers, DH, Isac, G, Rassias, ThM: Stability of Functional Equations in Several Variables. Birkhäuser, Basel (1998)
14. Hyers, DH, Isac, G, Rassias, ThM., *et al*: On the asymptoticity aspect of Hyers-Ulam stability of mappings. Proc Am Math Soc. **126**, 425–430 (1998). doi:10.1090/S0002-9939-98-04060-X
15. Jun, K, Lee, Y: On the Hyers-Ulam-Rassias stability of a Pexiderized quadratic inequality. Math Inequal Appl. **4**, 93–118 (2001)
16. Jung, S-M: Hyers-Ulam-Rassias stability of Jensen's equation and its application. Proc Am Math Soc. **126**, 3137–3143 (1998). doi:10.1090/S0002-9939-98-04680-2
17. Miheţ, D: The fixed point method for fuzzy stability of the Jensen functional equation. Fuzzy Sets Syst. **160**, 1663–1667 (2009). doi:10.1016/j.fss.2008.06.014
18. Miheţ, D, Radu, V: On the stability of the additive Cauchy functional equation in random normed spaces. J Math Anal Appl. **343**, 567–572 (2008)
19. Mirmostafaee, M, Mirzavaziri, M, Moslehian, MS: Fuzzy stability of the Jensen functional equation. Fuzzy Sets Syst. **159**, 730–738 (2008). doi:10.1016/j.fss.2007.07.011
20. Mirmostafee, AK, Moslehian, MS: Fuzzy versions of Hyers-Ulam-Rassias theorem. Fuzzy Sets Syst. **159**, 720–729 (2008). doi:10.1016/j.fss.2007.09.016
21. Najati, A: Fuzzy stability of a generalized quadratic functional equation. Commun Korean Math Soc. **25**, 405–417 (2010). doi:10.4134/CKMS.2010.25.3.405
22. Najati, A, Moghimi, MB: Stability of a functional equation deriving from quadratic and additive functions in quasi-Banach spaces. J Math Anal Appl. **337**, 399–415 (2008). doi:10.1016/j.jmaa.2007.03.104
23. Najati, A, Park, C: Hyers-Ulam-Rassias stability of homomorphisms in quasi-Banach algebras associated to the Pexiderized Cauchy functional equation. J Math Anal Appl. **335**, 763–778 (2007). doi:10.1016/j.jmaa.2007.02.009
24. Rassias, ThM, Tabor, J, (eds.): Stability of Mappings of Hyers-Ulam Type. Hadronic Press Inc. Florida (1994)
25. Rassias, ThM: On the stability of the quadratic functional equation and its applications. Studia Univ Babes Bolyai Math. **43**, 89–124 (1998)
26. Rassias, ThM: On the stability of functional equations and a problem of Ulam. Acta Appl Math. **62**, 23–130 (2000). doi:10.1023/A:1006499223572
27. Rassias, ThM, (ed.): Functional Equations and Inequalities. Kluwer Academic Publishers, Dordrecht (2000)
28. Rassias, ThM: On the stability of functional equations in Banach spaces. J Math Anal Appl. **251**, 264–284 (2000). doi:10.1006/jmaa.2000.7046
29. Šemrl, P: On quadratic functionals. Bull Aust Math Soc. **37**, 27–28 (1987)
30. Bag, T, Samanta, SK: Finite dimensional fuzzy normed linear spaces. J Fuzzy Math. **11**, 687–705 (2003)

doi:10.1186/1029-242X-2011-78
Cite this article as: Najati *et al.*: Local stability of the Pexiderized Cauchy and Jensen's equations in fuzzy spaces. *Journal of Inequalities and Applications* 2011 **2011**:78.