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Some identities on the weighted q-Euler numbers and *q*-Bernstein polynomials

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Abstract

Recently, Ryoo introduced the weighted *q*-Euler numbers and polynomials which are a slightly different Kim's weighted *q*-Euler numbers and polynomials(see C. S. Ryoo, A note on the weighted q-Euler numbers and polynomials, 2011]). In this paper, we give some interesting new identities on the weighted *q*-Euler numbers related to the *q*-Bernstein polynomials

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1. Introduction

Let p be a fixed odd prime number. Throughout this paper $\mathbb{Z}_p$, $\mathbb{Q}_p$, $\mathbb{C}$ and $\mathbb{C}_p$ will denote the ring of p-adic integers, the field of p-adic rational numbers, the complex number fields and the completion of algebraic closure of $\mathbb{Q}_p$, respectively. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. Let v_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-v_p(p)} = \frac{1}{p}$. When one talks of q-extension, q is variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, then one normally assumes $|q| <1$, and if $q \in \mathbb{C}_p$, then one normally assumes $|q - 1|_p <1$. In this paper, the q-number is defined by

$$[x]_q = \frac{1-q^x}{1-q},.$$

(see [1-19])

Note that $\lim_{q\to 1}[x]_q = x$ (see [1-19]). Let f be a continuous function on $\mathbb{Z}_p$. For $\alpha \in \mathbb{N}$ and $k, n \in \mathbb{Z}_+$, the weighted p-adic q-Bernstein operator of order n for f is defined by Kim as follows:

$$\begin{aligned}\mathbb{B}_{n,q}^{(\alpha)}(f|x) &= \sum_{k=0}^{n}\binom{n}{k} f\left(\frac{k}{n}\right)[x]_{q^\alpha}^{k}[1-x]_{q^{-\alpha}}^{n-k}\\ &= \sum_{k=0}^{n} f\left(\frac{k}{n}\right) B_{k,n}^{(\alpha)}(x,q),.\end{aligned} \tag{1}$$

see [4,9,19].

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Here $B_{k,n}^{(\alpha)}(x,q)=\binom{n}{k}[x]_{q^\alpha}^{k}[1-x]_{q^{-\alpha}}^{n-k}$ are called the q-Bernstein polynomials of degree n with weighted α.

Let $C(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f \in C(\mathbb{Z}_p)$, the fermionic q-integral on $\mathbb{Z}_p$ is defined by

$$I_q(f)=\int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x)=\lim_{N\to\infty}\frac{1+q}{1+q^{p^N}}\sum_{x=0}^{p^N-1} f(x)(-q)^x, \tag{2}$$

see [5-19].

For $n \in \mathbb{N}$, by (2), we get

$$q^n\int_{\mathbb{Z}_p} f(x+n)d\mu_{-q}(x)=(-1)^n\int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x)+[2]_q\sum_{l=0}^{n-1}(-1)^{n-1-l}q^l f(l), \tag{3}$$

see [6,7].

Recently, by (2) and (3), Ryoo considered the weighted q-Euler polynomials which are a slightly different Kim's weighted q-Euler polynomials as follows:

$$\int_{\mathbb{Z}_p}[x+y]_{q^\alpha}^n d\mu_{-q}(y)=E_{n,q}^{(\alpha)}(x), \quad \text{for } n\in\mathbb{Z}_+ \text{ and } \alpha\in\mathbb{Z}, \tag{4}$$

see [17].

In the special case, $x = 0$, $E_{n,q}^{(\alpha)}(0)=E_{n,q}^{(\alpha)}$ are called the n-th q-Euler numbers with weight α (see [14]).

From (4), we note that

$$E_{n,q}^{(\alpha)}(x)=\frac{[2]_q}{(1-q^\alpha)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l\frac{q^{\alpha l x}}{1+q^{\alpha l+1}}, \tag{5}$$

see [17].

and

$$E_{n,q}^{(\alpha)}(x)=\sum_{l=0}^{n}\binom{n}{l}[x]_{q^\alpha}^{n-l}q^{\alpha l x}E_{l,q}^{(\alpha)}, \tag{6}$$

see [17].

That is, (6) can be written as

$$E_{n,q}^{(\alpha)}(x)=(q^{\alpha x}E_q^{(\alpha)}+[x]_{q^\alpha})^n, n\in\mathbb{Z}_+. \tag{7}$$

with usual convention about replacing $(E_q^{(\alpha)})^n$ by $E_{n,q}^{(\alpha)}$.

In this paper we study the weighted q-Bernstein polynomials to express the fermionic q-integral on $\mathbb{Z}_p$ and investigate some new identities on the weighted q-Euler numbers related to the weighted q-Bernstein polynomials.

2. q-Euler numbers with weight α

In this section we assume that $\alpha \in \mathbb{N}$ and $q \in \mathbb{C}$ with $|q| < 1$.

Let $F_q(t, x)$ be the generating function of q-Euler polynomials with weight α as followings:

$$F_q(t,x)=\sum_{n=0}^{\infty}E_{n,q}^{(\alpha)}(x)\frac{t^n}{n!}. \tag{8}$$

By (5) and (8), we get

$$\begin{aligned}F_q(t,x)&=\sum_{n=0}^{\infty}\left(\frac{[2]_q}{(1-q^\alpha)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l\frac{q^{\alpha lx}}{1+q^{\alpha l+1}}\right)\frac{t^n}{n!}\\&=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m e^{[x+m]_{q^\alpha}t}.\end{aligned} \tag{9}$$

In the special case, $x = 0$, let $F_q(t, 0) = F_q(t)$. Then we obtain the following difference equation.

$$qF_q(t,1)+F_q(t)=[2]_q. \tag{10}$$

Therefore, by (8) and (10), we obtain the following proposition.

Proposition 1. For $n \in \mathbb{Z}_+$, we have

$$E_{0,q}^{(\alpha)}=1,\quad \text{and}\quad qE_{n,q}^{(\alpha)}(1)+E_{n,q}^{(\alpha)}=0 \text{ if } n>0.$$

By (6), we easily get the following corollary.

Corollary 2. For $n \in \mathbb{Z}_+$, we have

$$E_{0,q}^{(\alpha)}=1,\quad \text{and } q(q^\alpha E_q^{(\alpha)}+1)^n+E_{n,q}^{(\alpha)}=0 \text{ if } n>0,$$

with usual convention about replacing $(E_q^{(\alpha)})^n$ by $E_{n,q}^{(\alpha)}$.

From (9), we note that

$$F_{q^{-1}}(t,1-x)=F_q(-q^\alpha t,x). \tag{11}$$

Therefore, by (11), we obtain the following lemma.

Lemma 3. Let $n \in \mathbb{Z}_+$. Then we have

$$E_{n,q^{-1}}^{(\alpha)}(1-x)=(-1)^n q^{\alpha n}E_{n,q}^{(\alpha)}(x).$$

By Corollary 2, we get

$$\begin{aligned}q^2E_{n,q}^{(\alpha)}(2)-q^2-q&=q^2\sum_{l=0}^{n}\binom{n}{l}q^{\alpha l}(q^\alpha E_q^{(\alpha)}+1)^l-q^2-q\\&=-q\sum_{l=1}^{n}\binom{n}{l}q^{\alpha l}E_{l,q}^{(\alpha)}-q\\&=-q\sum_{l=0}^{n}\binom{n}{l}q^{\alpha l}E_{l,q}^{(\alpha)}\\&=-qE_{n,q}^{(\alpha)}(1)=E_{n,q}^{(\alpha)} \text{ if } n>0.\end{aligned} \tag{12}$$

Therefore, by (12), we obtain the following theorem.

Theorem 4. For $n \in \mathbb{N}$, we have

$$E_{n,q}^{(\alpha)}(2)=\frac{1}{q^2}E_{n,q}^{(\alpha)}+\frac{1}{q}+1.$$

Theorem 4 is important to study the relations between q-Bernstein polynomials and the weighted q-Euler number in the next section.

3. Weighted q-Euler numbers concerning q-Bernstein polynomials

In this section we assume that $\alpha \in \mathbb{Z}_p$ and $q \in \mathbb{C}_p$ with $|1 - q|_p < 1$.

From (2), (3) and (4), we note that

$$q\int_{\mathbb{Z}_p}[1-x]_{q^{-\alpha}}^n d\mu_{-q}(x) = (-1)^n q^{\alpha n+1}\int_{\mathbb{Z}_p}[x-1]_{q^{\alpha}}^n d\mu_{-q}(x) = q\sum_{l=0}^{n}\binom{n}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_{q^{\alpha}}^l d\mu_{-q}(x). \tag{13}$$

Therefore, by (13) and Lemma 3, we obtain the following theorem.

Theorem 5. For $n \in \mathbb{Z}_+$, we get

$$q\int_{\mathbb{Z}_p}[1-x]_{q^{-\alpha}}^n d\mu_{-q}(x) = (-1)^n q^{\alpha n+1}E_{n,q}^{(\alpha)}(-1) = qE_{n,q^{-1}}^{(\alpha)}(2) = q\sum_{l=0}^{n}\binom{n}{l}(-1)^l E_{l,q}^{(\alpha)}.$$

Let $n \in \mathbb{N}$. Then, by Theorem 4, we obtain the following corollary.

Corollary 6. For $n \in \mathbb{N}$, we have

$$\int_{\mathbb{Z}_p}[1-x]_{q^{-\alpha}}^n d\mu_{-q}(x) = E_{n,q^{-1}}^{(\alpha)}(2) = q^2E_{n,q^{-1}}^{(\alpha)} + [2]_q.$$

For $x \in \mathbb{Z}_p$, the p-adic q-Bernstein polynomials with weight α of degree n are given by

$$B_{k,n}^{(\alpha)}(x,q) = \binom{n}{k}[x]_{q^{\alpha}}^k[1-x]_{q^{-\alpha}}^{n-k}, \text{ where } n,k \in \mathbb{Z}_+, \tag{14}$$

see [9].

From (14), we can easily derive the following symmetric property for q-Bernstein polynomials:

$$B_{k,n}^{(\alpha)}(x,q) = B_{n-k,n}^{(\alpha)}(1-x,q^{-1}), \tag{15}$$

see [11]

By (15), we get

$$\int_{\mathbb{Z}_p}B_{k,n}^{(\alpha)}(x,q)d\mu_{-q}(x) = \int_{\mathbb{Z}_p}B_{n-k,n}^{(\alpha)}(1-x,q^{-1})d\mu_{-q}(x) = \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\int_{\mathbb{Z}_p}[1-x]_{q^{-\alpha}}^{n-l}d\mu_{-q}(x). \tag{16}$$

Let $n, k \in \mathbb{Z}_+$ with $n > k$. Then, by (16) and Corollary 6, we have

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n}^{(\alpha)}(x,q)d\mu_{-q}(x) \\
&= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\left(q^2E_{n-l,q^{-1}}^{(\alpha)}+[2]_q\right) \\
&= \begin{cases} q^2E_{n,q^{-1}}^{(\alpha)}+[2]_q, & \text{if } k=0, \\ q^2\binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}E_{n-l,q^{-1}}^{(\alpha)}, & \text{if } k>0. \end{cases}
\end{aligned} \tag{17}$$

Taking the fermionic q-integral on $\mathbb{Z}_p$ for one weighted q-Bernstein polynomials in (14), we have

$$\begin{aligned}
\int_{\mathbb{Z}_p} B_{k,n}^{(\alpha)}(x,q)d\mu_{-q}(x) &= \binom{n}{k}\int_{\mathbb{Z}_p}[x]_{q^\alpha}^{k}[1-x]_{q^{-\alpha}}^{n-k}d\mu_{-q}(x) \\
&= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_{q^\alpha}^{k+l}d\mu_{-q}(x) \\
&= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l E_{l+k,q}^{(\alpha)}.
\end{aligned} \tag{18}$$

Therefore, by comparing the coefficients on the both sides of (17) and (18), we obtain the following theorem.

Theorem 7. For $n, k \in \mathbb{Z}_+$ with $n > k$, we have

$$\sum_{l=0}^{n-k}(-1)^l\binom{n-k}{l}E_{l+k,q}^{(\alpha)} = \begin{cases} q^2E_{n,q^{-1}}^{(\alpha)}+[2]_q, & \text{if } k=0, \\ q^2\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}E_{n-l,q^{-1}}^{(\alpha)}, & \text{if } k>0. \end{cases}$$

Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$. Then we see that

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n_1}^{(\alpha)}(x,q)B_{k,n_2}^{(\alpha)}(x,q)d\mu_{-q}(x) \\
&= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{l+2k}\int_{\mathbb{Z}_p}[1-x]_{q^{-\alpha}}^{n_1+n_2-l}d\mu_{-q}(x) \\
&= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{l+2k}\left(q^2E_{n_1+n_2-l,q^{-1}}^{(\alpha)}+[2]_q\right).
\end{aligned} \tag{19}$$

By the binomial theorem and definition of q-Bernstein polynomials, we get

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n_1}^{(\alpha)}(x,q)B_{k,n_2}^{(\alpha)}(x,q)d\mu_{-q}(x) \\
&= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{n_1+n_2-2k}(-1)^l\binom{n_1+n_2-2k}{l}\int_{\mathbb{Z}_p}[x]_{q^\alpha}^{2k+l}d\mu_{-q}(x) \\
&= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{n_1+n_2-2k}(-1)^l\binom{n_1+n_2-2k}{l}E_{2k+l,q}^{(\alpha)}.
\end{aligned} \tag{20}$$

By comparing the coefficients on the both sides of (19) and (20), we obtain the following theorem.

Theorem 8. Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$. Then we have

$$\sum_{l=0}^{n_1+n_2-2k} (-1)^l \binom{n_1+n_2-2k}{l} E_{2k+l,q}^{(\alpha)} = \begin{cases} q^2 E_{n_1+n_2,q^{-1}}^{(\alpha)} + [2]_q, & \text{if } k = 0, \\ q^2 \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} E_{n_1+n_2-l,q^{-1}}^{(\alpha)}, & \text{if } k > 0. \end{cases}$$

Let $s \in \mathbb{N}$ with $s \geq 2$. For $n_1, n_2, ..., n_s, k \in \mathbb{Z}_+$ with $n_1 + ... + n_s > sk$, we have

$$\begin{aligned} &\int_{\mathbb{Z}_p} \underbrace{B_{k,n_1}^{(\alpha)}(x,q) \cdots B_{k,n_s}^{(\alpha)}(x,q)}_{s-times} d\mu_{-q}(x) \\ &= \binom{n_1}{k} \cdots \binom{n_s}{k} \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{sk} [1-x]_{q^{-\alpha}}^{n_1+\cdots+n_s-sk} d\mu_{-q}(x) \\ &= \binom{n_1}{k} \cdots \binom{n_s}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} \int_{\mathbb{Z}_p} [1-x]_{q^{-\alpha}}^{n_1+\cdots+n_s-l} d\mu_{-q}(x) \\ &= \binom{n_1}{k} \cdots \binom{n_s}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} \left(q^2 E_{n_1+\cdots+n_s-l,q^{-1}}^{(\alpha)} + [2]_q \right). \end{aligned} \tag{21}$$

From the binomial theorem and the definition of q-Bernstein polynomials, we note that

$$\begin{aligned} &\int_{\mathbb{Z}_p} \underbrace{B_{k,n_1}^{(\alpha)}(x,q) \cdots B_{k,n_s}^{(\alpha)}(x,q)}_{s-\text{times}} d\mu_{-q}(x) \\ &= \binom{n_1}{k} \cdots \binom{n_s}{k} \sum_{l=0}^{n_1+\cdots+n_s-sk} (-1)^l \binom{n_1+\cdots+n_s-sk}{l} \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{sk+l} d\mu_{-q}(x) \\ &= \binom{n_1}{k} \cdots \binom{n_s}{k} \sum_{l=0}^{n_1+\cdots+n_s-sk} (-1)^l \binom{n_1+\cdots+n_s-sk}{l} E_{sk+l,q}^{(\alpha)}. \end{aligned} \tag{22}$$

Therefore, by (21) and (22), we obtain the following theorem.

Theorem 9. Let $s \in \mathbb{N}$ with $s \geq 2$. For $n_1, n_2, ..., n_s, k \in \mathbb{Z}_+$ with $n_1 + ... + n_s > sk$, we have

$$\sum_{l=0}^{n_1+\cdots+n_s-sk} (-1)^l \binom{n_1+\cdots+n_s-sk}{l} E_{sk+l,q}^{(\alpha)} = \begin{cases} q^2 E_{n_1+\cdots+n_s,q^{-1}}^{(\alpha)} + [2]_q, & \text{if } k = 0, \\ q^2 \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} E_{n_1+\cdots+n_s-l,q^{-1}}^{(\alpha)}, & \text{if } k > 0. \end{cases}$$

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All authors contributed equally to the manuscript and read and approved the finial manuscript.

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