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Comment on "on the stability of quadratic double centralizers and quadratic multipliers: a fixed point approach" [Bodaghi et al., j. inequal. appl. 2011, article id 957541 (2011)]

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Abstract

Bodaghi et al. [On the stability of quadratic double centralizers and quadratic multipliers: a fixed point approach. J. Inequal. Appl. **2011**, Article ID 957541, 9pp. (2011)] proved the Hyers-Ulam stability of quadratic double centralizers and quadratic multipliers on Banach algebras by fixed point method. One can easily show that all the quadratic double centralizers (L, R) in the main results must be (0, 0). The results are trivial. In this article, we correct the results.

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1. Introduction

In 1940, Ulam [1] raised the following question concerning stability of group homomorphisms: *Under what condition does there exist an additive mapping near an approximately additive mapping?* Hyers [2] answered the problem of Ulam for Banach spaces. He showed that for Banach spaces $\mathcal{X}$ and $\mathcal{Y}$, if $\varepsilon > 0$ and $f : \mathcal{X} \to \mathcal{Y}$ such that

$$\| f(x+y) - f(x) - f(y) \| \leq \varepsilon$$

for all $x, y \in \mathcal{X}$, then there exists a unique additive mapping $T : \mathcal{X} \to \mathcal{Y}$ such that

$$\| f(x) - T(x) \| \leq \varepsilon \quad (x \in \mathcal{X}).$$

Consider $f : \mathcal{X} \to \mathcal{Y}$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$ for all $x \in \mathcal{X}$. Assume that there exist constant $\varepsilon \geq 0$ and $p \in [0, 1)$ such that

$$\| f(x+y) - f(x) - f(y) \| \leq \varepsilon(\| x \, |\,|^p + \| y \, |\,|^p) \quad (x \in \mathcal{X}).$$

Rassias [3] showed that there exists a unique $\mathbb{R}$-linear mapping $T : \mathcal{X} \to \mathcal{Y}$ such that

$$\| f(x) - T(x) \| \leq \frac{2\varepsilon}{2-2^p} \| x \, |\,|^p \quad (x \in \mathcal{X}).$$

Găvruta [4] generalized the Rassias' result. A square norm on an inner product space satisfies the important parallelogram equality

$$\| x+y \|^2 + \| x-y \|^2 = 2(\| x \|^2 + \| y \|^2).$$

Recall that the functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.1}$$

is called a *quadratic functional equation*. In particular, every solution of the functional equation (1.1) is said to be a *quadratic mapping*. A Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [5] for mappings $f : \mathcal{X} \to \mathcal{Y}$, where $\mathcal{X}$ is a normed space and $\mathcal{Y}$ is a Banach space. Cholewa [6] noticed that the theorem of Skof is still true if the relevant domain E_1 is replaced by an Abelian group. Indeed, Czerwik [7] proved the Hyers-Ulam stability of the quadratic functional equation. Since then, the stability problems of various functional equation have been extensively investigated by a number of authors [8-20].

2. Stability of quadratic double centralizers

A linear mapping $L : \mathcal{A} \to \mathcal{A}$ is said to be *left centralizer* on $\mathcal{A}$ if $L(ab) = L(a)b$ for all $a,\ b \in \mathcal{A}$. Similarly, a linear mapping $R : \mathcal{A} \to \mathcal{A}$ satisfying that $R(ab) = aR(b)$ for all $a, b \in \mathcal{A}$ is called *right centralizer* on $\mathcal{A}$. A *double centralizer* on $\mathcal{A}$ is a pair (L, R), where L is a left centralizer, R is a right centralizer and $aL(b) = R(a)b$ for all $a, b \in \mathcal{A}$. An operator $T : \mathcal{A} \to \mathcal{A}$ is said to be a *multiplier* if $aT(b) = T(a)b$ for all $a, b \in \mathcal{A}$.

Throughout this article, let $\mathcal{A}$ be a complex Banach algebra. Recall that a mapping $L : \mathcal{A} \to \mathcal{A}$ is a quadratic left centralizer if L is a quadratic homogeneous mapping, that is quadratic and $L(\lambda a) = \lambda^2 L(a)$ for all $a \in \mathcal{A}$ and $\lambda \in \mathbb{C}$ and $L(ab) = L(a)b^2$ for all $a, b \in \mathcal{A}$, and a mapping $R : \mathcal{A} \to \mathcal{A}$ is a quadratic right centralizer if R is a quadratic homogeneous mapping and $R(ab) = a^2R(b)$ for all $a, b \in \mathcal{A}$. Also a quadratic double centralizer of an algebra $\mathcal{A}$ is a pair (L, R), where L is a quadratic left centralizer, R is a quadratic right centralizer and $a^2L(b) = R(a)b^2$ for all $a, b \in \mathcal{A}$ (see [21] for details).

It is proven in [8] that for vector spaces $\mathcal{X}$ and $\mathcal{Y}$ and a fixed positive integer k, a mapping $f : \mathcal{X} \to \mathcal{Y}$ is quadratic if and only if the following equality holds:

$$2f\left(\frac{kx+ky}{2}\right) + 2f\left(\frac{kx-ky}{2}\right) = k^2f(x) + k^2f(y).$$

We thus can show that f is quadratic if and only if for a fixed positive integer k, the following equality holds:

$$f(kx+ky) + f(kx-ky) = 2k^2f(x) + 2k^2f(y).$$

Before proceeding to the main results, we will state the following theorem which is useful to our purpose.

Theorem 2.1. (The alternative of fixed point [22]). *Suppose that we are given a complete generalized metric space (X, d) and a strictly contractive mapping $T: X \to X$ with Lipschitz constant L. Then, for each given $x \in X$, either $d(T^nx, T^{n+1}x) = \infty$ for all $n \geq 0$ or other exists a natural number n_0 such that*

(i) $d(T^n x, T^{n+1} x) < \infty$ *for all* $n \geq n_0$;
(ii) *the sequence* $\{T^n x\}$ *is convergent to a fixed point* y^* *of* T;
(iii) y^* *is the unique fixed point of* T *in the set* $\Lambda = \{y \in X : d(T^{n_0} x, y) < \infty\}$;
(iv) $d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$ *for all* $y \in \Lambda$.

Theorem 2.2. *Let* $f_j : \mathcal{A} \to \mathcal{A}$ *be continuous mappings with* $f_j(0) = 0$ *(j = 0, 1), and let* $\phi : \mathcal{A}^4 \to [0, \infty)$ *be continuous in the first and second variables such that*

$$\| f_j(\lambda a + \lambda b) + f_j(\lambda a - \lambda b) - 2\lambda^2 [f_j(a) + f_j(b)] \| \leq \phi(a, b, 0, 0), \tag{2.1}$$

$$\begin{aligned} &\| f_j(cd) - [(1-j)(f_j(c)d^2)^{1-j} + j(c^2 f_j(d))^j] + u^2 f_0(v) - f_1(u)v^2 \| \\ &\quad \leq \phi(c, d, u, v) \end{aligned} \tag{2.2}$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ *and all* $a, b, c, d, u, v \in \mathcal{A}$, $j = 0, 1$. *If there exists a constant* m, $0 < m < 1$, *such that*

$$\phi(c, d, u, v) \leq 4m\phi\left(\frac{c}{2}, \frac{d}{2}, \frac{u}{2}, \frac{v}{2}\right) \tag{2.3}$$

for all $c, d, u, v \in \mathcal{A}$, *then there exists a unique quadratic double centralizer* (L, R) *on* $\mathcal{A}$ *satisfying*

$$\| f_0(a) - L(a) \| \leq \frac{1}{4(1-m)} \phi(a, a, 0, 0), \tag{2.4}$$

$$\| f_1(a) - R(a) \| \leq \frac{1}{4(1-m)} \phi(a, a, 0, 0) \tag{2.5}$$

for all $a \in \mathcal{A}$.

Proof. From (2.3), it follows that

$$\lim_i 4^{-i} \phi(2^i c, 2^i d, 2^i u, 2^i v) = 0 \tag{2.6}$$

for all $c, d, u, v \in \mathcal{A}$. Putting $j = 0$, $\lambda = 1$, $a = b$ and replacing a by $2a$ in (2.1), we get

$$\| f_0(2a) - 4f_0(a) \| \leq \phi(a, a, 0, 0)$$

for all $a \in \mathcal{A}$. By the above inequality, we have

$$\left\| \frac{1}{4} f_0(2a) - f_0(a) \right\| \leq \frac{1}{4} \phi(a, a, 0, 0) \tag{2.7}$$

for all $a \in \mathcal{A}$. Consider the set $X := \{g \mid g : \mathcal{A} \to \mathcal{A}\}$ and introduce the generalized metric on X:

$$d(h, g) := \inf\{C \in \mathbb{R}^+ : \| g(a) - h(a) \| \leq C\phi(a, a, 0, 0) \text{ for all } a \in \mathcal{A}\}.$$

It is easy to show that (X, d) is complete. Now, we define the mapping $Q : X \to X$ by

$$Q(h)(a) = \frac{1}{4} h(2a) \tag{2.8}$$

for all $a \in \mathcal{A}$. Given $g, h \in X$, let $C \in \mathbb{R}^+$ be an arbitrary constant with $d(g, h) \le C$, that is,

$$\| g(a) - h(a) \| \le C\phi(a, a, 0, 0) \tag{2.9}$$

for all $a \in \mathcal{A}$. Substituting a by $2a$ in the inequality (2.9) and using (2.3) and (2.8), we have

$$\begin{aligned} \| (Qg)(a) - (Qh)(a) \| &= \frac{1}{4} \| g(2a) - h(2a) \| \\ &\le \frac{1}{4} C\phi(2a, 2a, 0, 0) \\ &\le Cm\phi(a, a, 0, 0) \end{aligned}$$

for all $a \in \mathcal{A}$. Hence, $d(Qg, Qh) \le Cm$. Therefore, we conclude that $d(Qg, Qh) \le md(g, h)$ for all $g, h \in X$. It follows from (2.7) that

$$d(Qf_0, f_0) \le \frac{1}{4}. \tag{2.10}$$

By Theorem 2.1, Q has a unique fixed point $L : \mathcal{A} \to \mathcal{A}$ in the set $= \{h \in X, d(f_0, h) < \infty\}$. On the other hand,

$$\lim_{n\to\infty} \frac{f_0(2^n a)}{4^n} = L(a) \tag{2.11}$$

for all $a \in \mathcal{A}$. By Theorem 2.1 and (2.10), we obtain

$$d(f_0, L) \le \frac{1}{1-m} d(Qf_0, L) \le \frac{1}{4(1-m)},$$

i.e., the inequality (2.4) is true for all $a \in \mathcal{A}$. Now, substitute $2^n a$ and $2^n b$ by a and b, respectively, and put $j = 0$ in (2.1). Dividing both sides of the resulting inequality by 2^n, and letting n go to infinity, it follows from (2.6) and (2.11) that

$$L(\lambda a + \lambda b) + L(\lambda a - \lambda b) = 2\lambda^2 L(a) + 2\lambda^2 L(b) \tag{2.12}$$

for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. Putting $\lambda = 1$ in (2.12), we have

$$L(a + b) + L(a - b) = 2L(a) + 2L(b) \tag{2.13}$$

for all $a, b \in \mathcal{A}$. Hence, L is a quadratic mapping.

Letting $b = 0$ in (2.13), we get $L(\lambda a) = \lambda^2 L(a)$ for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. By (2.13), $L(ra) = r^2 L(a)$ for any rational number r. It follows from the continuity f_0 and φ for each $\lambda \in \mathbb{R}$, $L(\lambda a) = \lambda^2 L(a)$. Hence,

$$L(\lambda a) = L\left(\frac{\lambda}{|\lambda|} |\lambda| a\right) = \frac{\lambda^2}{|\lambda|^2} L(|\lambda| a) = \frac{\lambda^2}{|\lambda|^2} |\lambda|^2 L(a)$$

for all $a \in \mathcal{A}$ and $\lambda \in \mathbb{C}$ ($\lambda \neq 0$). Therefore, L is quadratic homogeneous. Putting $j = 0$, $u = v = 0$ in (2.2) and replacing $2^n c$ by c, we obtain

$$\left\| \frac{f_0(2^n cd)}{4^n} - \frac{f_0(2^n c)}{4^n} d \right\| \le \frac{1}{2} 4^{-n} \phi(2^n c, d, 0, 0).$$

By (2.6), the right-hand side of the above inequality tend to zero as $n \to \infty$. It follows from (2.11) that $L(cd) = L(c)\, d^2$ for all $c, d \in \mathcal{A}$. Thus, L is a quadratic left centralizer.

Also, one can show that there exists a unique mapping $R : \mathcal{A} \to \mathcal{A}$ which satisfies

$$\lim_{n\to\infty} \frac{f_1(2^n a)}{4^n} = R(a)$$

for all $a \in \mathcal{A}$. The same manner could be used to show that R is quadratic right centralizer. If we substitute u and v by $2^n u$ and $2^n v$ in (2.2), respectively, and put $c = d = 0$, and divide the both sides of the obtained inequality by 16^n, then we get

$$\left\| u^2 \frac{f_0(2^n v)}{4^n} - \frac{f_1(2^n u)}{4^n} v^2 \right\| \le 16^{-n}\phi(0,0,2^n u, 2^n v) \le 4^{-n}\phi(0,0,2^n u, 2^n v).$$

Passing to the limit as $n \to \infty$, and again from (2.5) we conclude that $u^2\, L(v) = R(u)\, v^2$ for all $u, v \in \mathcal{A}$. Therefore, (L, R) is a quadratic double centralizer on $\mathcal{A}$. This completes the proof of the theorem.

3. Stability of quadratic multipliers

Assume that $\mathcal{A}$ is a complex Banach algebra. Recall that a mapping $T : \mathcal{A} \to \mathcal{A}$ is a *quadratic multiplier* if T is a quadratic homogeneous mapping, and $a^2 T(b) = T(a) b^2$ for all $a, b \in \mathcal{A}$ (see [21]). We investigate the stability of quadratic multipliers.

Theorem 3.1. *Let* $f : \mathcal{A} \to \mathcal{A}$ *be a continuous mapping with* $f(0) = 0$ *and let* $\phi : \mathcal{A}^4 \to [0, \infty)$ *be a continuous in the first and second variables such that*

$$\begin{aligned} &\| f(a+\lambda b) + f(\lambda a - \lambda b) - 2\lambda^2[f(a) + f(b)] + c^2 f(d) - f(c) d^2 \| \\ &\quad \le \phi(a,b,c,d) \end{aligned} \tag{3.1}$$

for all $\lambda \in \mathbb{T}$*and all* $a, b, c, d \in \mathcal{A}$.*If there exists a constant* m, $0 < m < 1$, *such that*

$$\phi(2a, 2b, 2c, 2d) \le 4m\phi(a,b,c,d) \tag{3.2}$$

for all $a, b, c, d \in \mathcal{A}$. *Then, there exists a unique quadratic multiplier* T *on* $\| f(a) - T(a) \| \le \dfrac{1}{4(1-m)}\phi(a,a,0,0)$ *satisfying*

$$\| f(a) - T(a) \| \le \frac{1}{4(1-m)}\phi(a,a,0,0) \tag{3.3}$$

for all $a \in \mathcal{A}$.

Proof. It follows from $\varphi(2a, 2b, 2c, 2d) \le 4m\varphi(a, b, c, d)$ that

$$\lim_{n\to\infty} \frac{\phi(2^n a, 2^n b, 2^n c, 2^n d)}{4^n} = 0 \tag{3.4}$$

for all $a, b, c, d \in \mathcal{A}$. Putting $\lambda = 1$, $a = b$, $c = d$, $d = 0$ in (3.1), we obtain

$$\| f(2a) - 4f(a) \| \le \phi(a,a,0,0)$$

for all $a \in \mathcal{A}$. Hence,

$$\left\| f(a) - \frac{1}{4} f(2a) \right\| \le \frac{1}{4}\phi(a,a,0,0) \tag{3.5}$$

for all $a \in \mathcal{A}$. Consider the set $X := \{h \mid h : \mathcal{A} \to \mathcal{A}\}$ and introduce the generalized metric on X :

$$d(g, h) := \inf\{C \in \mathbb{R}^+ : \| g(a) - h(a) \| \leq C\phi(a, a, 0, 0) \text{ for all } a \in \mathcal{A}\}.$$

It is easy to show that (X, d) is complete. Now, we define a mapping Φ: $X \to X$ by

$$\Phi(h)(a) = \frac{1}{4}h(2a)$$

for all $a \in \mathcal{A}$. By the same reasoning as in the proof of Theorem 2.2, Φ is strictly contractive on X. It follows from (3.5) that $d(\Phi f, f) \leq \frac{1}{4}$. By Theorem 2.1, Φ has a unique fixed point in the set $X_1 = \{h \in X : d(f, h) < \infty\}$. Let T be the fixed point of Φ. Then, T is the unique mapping with $T(2a) = 4T(a)$, for all $a \in \mathcal{A}$ such that there exists $C \in (0, \infty)$ such that

$$\| T(x) - f(x) \| \leq C\phi(a, a, 0, 0)$$

for all $a \in \mathcal{A}$. On the other hand, we have $\lim_{n \to \infty} d(\Phi^n(f), T) = 0$.
Thus,

$$\lim_{n\to\infty} \frac{1}{4^n} f(2^n x) = T(x) \tag{3.6}$$

for all $a \in \mathcal{A}$. Hence,

$$d(f, T) \leq \frac{1}{1-m} d(T, \Phi(f)) \leq \frac{1}{4(1-m)}. \tag{3.7}$$

This implies the inequality (3.3). It follows from (3.1), (3.4) and (3.6) that

$$\begin{aligned} &\| T(\lambda a + \lambda b) + T(\lambda a - \lambda b) - 2\lambda^2 T(a) - 2\lambda^2 T(b) \| \\ &= \lim_{n\to\infty} \frac{1}{4^n} \| T(2^n(\lambda a + \lambda b)) + T(2^n(\lambda a - \lambda b)) - 2\lambda^2 T(2^n a) - 2\lambda^2 T(2^n b) \| \\ &\qquad \leq \lim_{n\to\infty} \frac{1}{4^n} \phi(2^n a, 2^n b, 0, 0) = 0 \end{aligned}$$

for all $a, b \in \mathcal{A}$. Hence,

$$T(\lambda a + \lambda b) + T(\lambda a - \lambda b) = 2\lambda^2 T(a) + 2\lambda^2 T(b) \tag{3.8}$$

for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. Letting $b = 0$ in (3.8), we have $T(\lambda a) = \lambda^2 T(a)$, for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. Now, it follows from the proof of Theorem 2.1 and the continuity f and φ that T is $\mathbb{C}$-linear. If we substitute c and d by $2^n c$ and $2^n d$ in (3.1), respectively, and put $a = b = 0$ and we divide the both sides of the obtained inequality by 16^n, we get

$$\left\| c^2 \frac{f(2^n d)}{4^n} - \frac{f(2^n c)}{4^n} d^2 \right\| \leq \frac{\phi(0, 0, 2^n c, 2^n d)}{16^n} \leq \frac{\phi(0, 0, 2^n c, 2^n d)}{4^n}.$$

Passing to the limit as $n \to \infty$, and from (3.4) we conclude that $c^2 T(d) = T(c) d^2$ for all $c, d \in \mathcal{A}$.

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Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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