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Research Article

Orlicz Sequence Spaces with a Unique Spreading Model

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We study the set of all spreading models generated by weakly null sequences in Orlicz sequence spaces equipped with partial order by domination. A sufficient and necessary condition for the above-mentioned set whose cardinality is equal to one is obtained.

1. Introduction

Let X be a separable infinite dimensional real Banach space. There are three general types of questions we often ask. In general, not much can be said in regard to this question "what can be said about the structure of X itself" and not much more can be said about the question "does X embedded into a nice subspace". The source of the research on spreading models was mainly from the question "finding a nice subspace $Y \subseteq X$" [1]. The spreading models usually have a simpler and better structure than the class of subspaces of X [2, 3]. In this paper, we study the question concerning the set of all spreading models whose cardinality is equal to one.

The notion of a spreading model is one of the application of Ramsey theory. It is a useful tool of digging asymptotic structure of Banach space, and it is a class of asymptotic unconditional basis. In 1974, Brunel and Sucheston [4] introduced the concept of spreading model and gave a result that every normalized weakly null sequence contains an asymptotic unconditional subsequence, they call the subsequence spreading model. It was not until the last ten years that the theory of spreading models was developed, especially in recent five years. In 2005, Androulakis et al. in [2] put forward several questions on spreading models and solved some of them. Afterwards, Sari et al. discussed some problems among them and obtained fruitful results. This paper is mainly motivated by some results obtained by Sari et al. in their papers [3, 5].

2. Preliminaries and Observations

An Orlicz function M is a real-valued continuous nondecreasing and convex function defined for $t \geq 0$ such that $M(0) = 0$ and $\lim_{t\to\infty} M(t) = \infty$. If $M(t) = 0$ for some $t > 0$, M is said to be a degenerate function. $M(u)$ is said to satisfy the Δ_2 condition ($M \in \Delta_2$) if there exist K, $u_0 > 0$ such that $M(2u) \leq KM(u)$ for $0 \leq u \leq u_0$. We denote the modular of a sequence of numbers $x = \{x(i)\}_{i=1}^{\infty}$ by $\rho_M(x) = \sum_{i=1}^{\infty} M(x(i))$. It is well known that the space

$$l_M = \left\{ x = \{x(i)\}_{i=1}^{\infty} : \rho_M(\lambda x) = \sum_{i=1}^{\infty} M(\lambda x(i)) < \infty \text{ for some } \lambda > 0 \right\} \tag{2.1}$$

endowed with the Luxemburg norm

$$\|x\| = \inf\left\{ \lambda > 0 : \rho_M\left(\frac{x}{\lambda}\right) \leq 1 \right\} \tag{2.2}$$

is a Banach sequence space which is called Orlicz sequence space. The space

$$h_M = \left\{ x = \{x(i)\} : \rho_M(\lambda x) = \sum_{i=1}^{\infty} M(\lambda x(i)) < \infty \text{ for each } \lambda > 0 \right\} \tag{2.3}$$

is a closed subspace of l_M. It is easy to verify that the spaces $l^p (1 \leq p < \infty)$ are just Orlicz sequence spaces, and Orlicz sequence spaces are the generalization of the spaces $l^p (1 \leq p < \infty)$. Furthermore, if M is a degenerate Orlicz function, then $l_M \cong l_\infty$ and $h_M \cong c_0$ [6]. In the context, the Orlicz functions considered are nondegenerate. Let

$$E_{M,1} = \overline{\left\{ \frac{M(\lambda t)}{M(\lambda)} : 0 < \lambda < 1 \right\}}, \qquad C_{M,1} = \overline{\text{conv}} E_{M,1}. \tag{2.4}$$

They are nonvoid norm compact subsets of $C(0,1)$ consisting entirely of Orlicz functions which might be degenerate [6, lemma 4.a.6].

Definition 2.1. Let X be a separable infinite dimensional Banach space. For every normalized basic sequence (y_i) in a Banach space and for every $\varepsilon_n \downarrow 0$, there exist a subsequence (x_i) and a normalized basic sequence $(\tilde{x}_i)$ such that for all $n \in N$, $(a_i)_{i=1}^{n} \in [-1,1]^n$ and $n \leq k_1 < \cdots < k_n$,

$$\left| \left\| \sum_{i=1}^{n} a_i x_{k_i} \right\| - \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\| \right| < \varepsilon_n. \tag{2.5}$$

The sequence $(\tilde{x}_i)$ is called the spreading model of (x_i) and it is a suppression-1 unconditional basic sequence if (y_i) is weakly null [4].

The following theorem guarantees the existence of a spreading model of X. We shall give a detailed proof.

Theorem 2.2. *Let (x_n) be a normalized basic sequence in X and let $\varepsilon_n \downarrow 0$. Then there exists a subsequence (y_n) of (x_n) so that for all n, $(a_i)_{i=1}^n \subseteq [-1,1]$ and integers $n \le k_1 < k_2 < \cdots k_n$, $n \le i_1 < i_2 < \cdots i_n$,*

$$\left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| \right| < \varepsilon_n. \tag{2.6}$$

In order to prove Theorem 2.2, we should have to recall the following definitions and theorem.

For $k \in N$, $[N]^k$ is the set of all subsets of N of size k. We may take it as the set of subsequences of length k, $(n_i)_{i=1}^k$ with $n_1 < \cdots < n_k$. $[N]^\omega$ denotes all subsequences of N. Similar definitions apply to $[M]^k$ and $[M]^\omega$ if $M \in [N]^\omega$.

Definition 2.3 (see [1]). Let I_1 and I_2 be two disjoint intervals. For any $(k_1, \ldots, k_n), (i_1, \ldots, i_n) \in [N]^k$ and scalars $(a_i)_{i=1}^n$ if

$$\left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| \in I_i, \quad \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| \in I_i \quad (i = 1 \text{ or } 2), \tag{2.7}$$

then we call I_i $(i = 1$ or $2)$ "color" $(k_1, \ldots, k_n)$ and $(i_1, \ldots, i_n)$. Meanwhile, we say $(k_1, \ldots, k_n)$ has the same "color" as $(i_1, i_2, \ldots, i_n)$, where (y_i) is a sequence of a Banach space. We identify the same "color" subsets of $[N]^k$, saying they are 1-colored.

Definition 2.4 (see [1]). The family of $[N]^k$ $(k \in N)$ is called finitely colored provided that it only contains finite subsets in "color" sense, and each subset is 1-colored.

Theorem 2.5 (see [1]). *Let $k \in N$ and let $[N]^k$ be finitely colored. Then there exists $M \in [N]^\omega$ so that $[M]^k$ is 1-colored.*

Proof of Theorem 2.2. We accomplish the proof in two steps.

Step 1. We shall prove that for any $n \in Z^+$, there exists $(y_i) \subseteq (x_i)$ such that for any $(a_i)_{i=1}^n \subseteq [-1,1], n \le k_1 < k_2 < \cdots k_n$, $n \le i_1 < i_2 < \cdots i_n$,

$$\left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| \right| < \varepsilon_n (*). \tag{2.8}$$

Firstly, for fixed $(a_i)_{i=1}^n \subseteq [-1,1]$, by the Definition 2.4, we can prove that the above inequality holds. In fact, we partition $[0,n]$ into subintervals $(I_j)_{j=1}^m$ of length $< \varepsilon_n$ and "color"

$(k_1, k_2, \ldots k_n)$ by I_l if

$$\left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| \in I_l. \tag{2.9}$$

In the same way, we can also "color" $(i_1, i_2, \cdots i_n)$ by I_l.

We can take $[-1,1]^n$ as the unit ball in finite-dimensional space l_1^n; then $[-1,1]^n$ is sequentially compact; moreover, it is totally bounded and complete. Under l_1^n-metric, take $N = \{z_1^{(n)}, z_2^{(n)}, \ldots z_m^{(n)}\}$ for $(\varepsilon_n/4)$-net of $[-1,1]^n$. For any element of net N, repeat the above process, and let $z_k^{(n)} = (z_{k_j}^{(n)})_{j=1}^n$, $k = 1, 2, \ldots m$. We partition $[0, n]$ into subintervals $(I_l)_{l=1}^m$ of length $< \varepsilon_n/2$ and "color" $(k_1, k_2, \ldots, k_n)$ by I_l if

$$\left\| \sum_{j=1}^{n} z_{k_j}^{(n)} y_{i_j} \right\| \in I_l. \tag{2.10}$$

Since the length of $I_l < \varepsilon_n/2$, we have

$$\left| \left\| \sum_{j=1}^{n} z_{k_j}^{(n)} y_{k_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_j}^{(n)} y_{i_j} \right\| \right| < \frac{\varepsilon_n}{2} \quad (k = 1, 2, \ldots, m). \tag{2.11}$$

Secondly, we shall prove that for any $(a_i)_{i=1}^n \subseteq [-1,1]^n$, $(*)$ holds. Since $N = \{z_1^{(n)}, z_2^{(n)}, \ldots z_m^{(n)}\}$ is the $(\varepsilon_n/4)$-net of $[-1,1]^n$, there exists $z_{k_0}^{(n)} = (z_{k_{0_j}}^{(n)})_{j=1}^n$ such that

$$\left\| (a_i)_{i=1}^n - z_{k_0}^{(n)} \right\| = \sum_{j=1}^{n} \left| a_j - z_{k_{0_j}}^{(n)} \right| < \frac{\varepsilon_n}{4}. \tag{2.12}$$

Therefore, we have

$$\begin{aligned} \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| &\le \left\| \sum_{j=1}^{n} \left(a_j - z_{k_{0_j}}^{(n)} \right) y_{k_j} \right\| + \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| \\ &\le \sum_{j=1}^{n} \left| a_j - z_{k_{0_j}}^{(n)} \right| \cdot \left\| y_{k_j} \right\| + \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| \\ &= \sum_{j=1}^{n} \left| a_j - z_{k_{0_j}}^{(n)} \right| + \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| \\ &< \frac{\varepsilon_n}{4} + \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\|. \end{aligned} \tag{2.13}$$

Hence,

$$\left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| \right| < \frac{\varepsilon_n}{4}. \tag{2.14}$$

Similarly, we obtain

$$\left| \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{i_j} \right\| \right| < \frac{\varepsilon_n}{4}. \tag{2.15}$$

Thus

$$\begin{aligned}
&\left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| \right| \\
&\quad = \left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| + \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| + \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{i_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{i_j} \right\| \right| \\
&\quad \le \left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| \right| + \left| \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| \right| \\
&\qquad + \left| \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{k_j} \right\| - \left\| \sum_{j=1}^{n} z_{k_{0_j}}^{(n)} y_{i_j} \right\| \right| \\
&\quad < \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{2} = \varepsilon_n.
\end{aligned} \tag{2.16}$$

Step 2. We apply diagonal argument to prove that there exists $(y_i) \subseteq (x_i)$ such that for any $n \in Z^+$, $(a_i)_{i=1}^n \subseteq [-1,1], n \le k_1 < k_2 < \cdots k_n$, $n \le i_1 < i_2 < \cdots i_n$,

$$\left| \left\| \sum_{j=1}^{n} a_j y_{k_j} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j} \right\| \right| < \varepsilon_n. \tag{2.17}$$

By Step 1, in view of $n = 1$, there exists $(y_i^{(1)}) \subseteq (x_i)$ such that for any $a \in [-1,1]$, for any $k_1 \in Z^+$, $i_1 \in Z^+$, $n \le k_1$, $n \le i_1$, we have

$$\left| \left\| a y_{k_1}^{(1)} \right\| - \left\| a y_{i_1}^{(1)} \right\| \right| < \varepsilon_1. \tag{2.18}$$

Obviously, $\{y_i^{(1)}\}$ is also a normalized basic sequence. So in view of $n = 2$, there exists $(y_i^{(2)}) \subseteq (y_i^{(1)})$ such that for any $(a_i)_{i=1}^2 \subseteq [-1,1], n \le k_1 < k_2,\ n \le i_1 < i_2$,

$$\left| \left\| \sum_{j=1}^{2} a_j y_{k_j}^{(2)} \right\| - \left\| \sum_{j=1}^{2} a_j y_{i_j}^{(2)} \right\| \right| < \varepsilon_2. \tag{2.19}$$

Repeating the above process, for any n, there exists $(y_i^{(n)}) \subseteq (y_i^{(n-1)})$ such that for any $(a_i)_{i=1}^n \subseteq [-1,1], n \le k_1 < k_2 < \cdots k_n,\ n \le i_1 < i_2 < \cdots i_n$, we have

$$\left| \left\| \sum_{j=1}^{n} a_j y_{k_j}^{(n)} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j}^{(n)} \right\| \right| < \varepsilon_n. \tag{2.20}$$

Finally, we choose the diagonal subsequence $(y_i^{(i)}) \subset (x_i)$; for any $n, (a_i)_{i=1}^n \subseteq [-1,1],\ n \le k_1 < k_2 < \cdots k_n,\ n \le i_1 < i_2 < \cdots i_n$, we obtain that

$$\left| \left\| \sum_{j=1}^{n} a_j y_{k_j}^{(k_j)} \right\| - \left\| \sum_{j=1}^{n} a_j y_{i_j}^{(i_j)} \right\| \right| < \varepsilon_n. \tag{2.21}$$

□

Definition 2.6. Let X be a separable infinite-dimensional Banach space. A normalized basic sequence $(x_i) \subset X$ generates a spreading model $(\tilde{x}_i)$ if for some $\varepsilon_n \downarrow 0$, for all $n \in N,\ n \le k_1 < \cdots < k_n$, and $(a_i)_1^n \subseteq [-1,1]$,

$$(1+\varepsilon_n)^{-1} \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\| \le \left\| \sum_{i=1}^{n} a_i x_{k_i} \right\| \le (1+\varepsilon_n) \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\|. \tag{2.22}$$

Theme 2.7. Definition 2.6 is equivalent to Definition 2.1.

Proof. We can easily conclude Definition 2.1 from Definition 2.6

By the Definition 2.1, we know that $(\tilde{x}_i)$ is a spreading model generated by (x_i). For any fixed $(a_i)_{i=1}^n \subseteq [-1,1]$, we partition $[0,n]$ into some subintervals $(I_j)_{j=1}^m$ of length $< \varepsilon_\rho$ and "color" $(k_1, k_2, \ldots k_n)$ by I_l if

$$\left\| \sum_{i=1}^{n} a_j y_{k_i} \right\| \in I_l \quad (1 \le l \le m). \tag{2.23}$$

Let $\rho \in Z^+,\ \rho \ge n$ and $\rho \le k_1 < \cdots < k_{i_0} < \cdots k_n$; then

$$\left| \left\| \sum_{i=1}^{n} a_i x_{k_i} \right\| - \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\| \right| < \delta_\rho, \tag{2.24}$$

where $\delta_\rho \downarrow 0$, $\delta_\rho > 0$. Using the same procedure of Theorem 2.2, we can get that for any $(a_i)_{i=1}^n \subseteq [-1,1]$, $\varepsilon_n \downarrow 0$,

$$\left| \left\| \sum_{i=1}^{n} \frac{1}{1+\varepsilon_n} a_i x_{k_i} \right\| - \left\| \sum_{i=1}^{n} \frac{1}{1+\varepsilon_n} a_i \tilde{x}_i \right\| \right| < \delta_\rho. \tag{2.25}$$

Thus

$$\left\| \sum_{i=1}^{n} \frac{1}{1+\varepsilon_n} a_i x_{k_i} \right\| < \delta_\rho + \left\| \sum_{i=1}^{n} \frac{1}{1+\varepsilon_n} a_i \tilde{x}_i \right\| = \delta_\rho + \frac{1}{1+\varepsilon_n} \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\| \le \delta_\rho + \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\|. \tag{2.26}$$

Letting $\rho \to \infty$, then

$$\left\| \sum_{i=1}^{n} \frac{1}{1+\varepsilon_n} a_i x_{k_i} \right\| \le \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\|. \tag{2.27}$$

That is,

$$\left\| \sum_{i=1}^{n} a_i x_{k_i} \right\| \le (1+\varepsilon_n) \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\|. \tag{2.28}$$

Similarly,

$$(1+\varepsilon_n)^{-1} \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\| \le \left\| \sum_{i=1}^{n} a_i x_{k_i} \right\|. \tag{2.29}$$

Hence, we obtain that

$$(1+\varepsilon_n)^{-1} \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\| \le \left\| \sum_{i=1}^{n} a_i x_{k_i} \right\| \le (1+\varepsilon_n) \left\| \sum_{i=1}^{n} a_i \tilde{x}_i \right\|. \tag{2.30}$$

Let $SP_w(X)$ be the set of all spreading models $(\tilde{x}_i)$ generated by weakly null sequences (x_i) in X endowed with order relation by domination, that is, $(\tilde{x}_i) \le (\tilde{y}_i)$ if there exists a constant $K \ge 1$ such that $\| \sum a_i \tilde{x}_i \| \le K \| \sum a_i \tilde{y}_i \|$ for scalars (a_i); then $(SP_w(X), \le)$ is a partial order set. If $(\tilde{x}_i) \le (\tilde{y}_i)$ and $(\tilde{y}_i) \le (\tilde{x}_i)$, we call $(\tilde{x}_i)$ equivalent to $(\tilde{y}_i)$, denoted by $(\tilde{x}_i) \sim (\tilde{y}_i)$. We identify $(\tilde{x}_i)$ and $(\tilde{y}_i)$ in $SP_w(X)$ if $(\tilde{x}_i) \sim (\tilde{y}_i)$. □

Lemma 2.8 (see [5]). *If an Orlicz sequence space h_M does not contain an isomorphic copy of l_1, then the sets $SP_w(h_M)$ and $C_{M,1}$ coincide. That is, $SP_w(h_M) = C_{M,1}$.*

3. Orlicz Sequence Spaces with Equivalent Spreading Models

Definition 3.1 (see [7]). Let (x_n) be a normalized Schauder basis of a Banach space X. (x_n) is said to be lower (resp., upper) semihomogeneous if every normalized block basic sequence of the basis dominates (resp., is dominated by) (x_n).

Lemma 3.2 (see [7]). *Let M be an Orlicz function with $M(1) = 1$, $M \in \Delta_2$, and let (e_i) denote the unit vector basis of the space h_M. The basis is*

(a) *lower semi-homogeneous if and only if $CM(st) \geq M(s)M(t)$ for all $s, t \in [0,1]$ and some $C \geq 1$,*

(b) *upper semi-homogeneous if and only if $M(st) \leq CM(s)M(t)$ for s, t, C as above.*

Lemma 3.3 (see [6]). *The space l_p, or c_0 if $p = \infty$, is isomorphic to a subspace of an Orlicz sequence space h_M if and only if $p \in [\alpha_M, \beta_M]$, where*

$$\alpha_M = \sup\left\{ q : \sup_{\substack{0<\lambda,\\ t\leq 1}} \frac{M(\lambda t)}{M(\lambda)t^q} < \infty \right\}, \tag{3.1}$$

$$\beta_M = \inf\left\{ q : \sup_{\substack{0<\lambda,\\ t\leq 1}} \frac{M(\lambda t)}{M(\lambda)t^q} > 0 \right\}. \tag{3.2}$$

Lemma 3.4 (see [5]). *Let $M \in \Delta_2$, l_M be an Orlicz sequence space which is not isomorphic to l_1. Suppose that $SP_w(l_M)$ is countable, up to equivalence. Then*

(i) *the unit vector basis of l_M is the upper bound of $SP_w(l_M)$;*

(ii) *the unit vector basis of l_p is the lower bound of $SP_w(l_M)$, where $p \in [\alpha_M, \beta_M]$.*

Theorem 3.5. *Let $M \in \Delta_2$, and let (e_i) be the unit basis of the space l_M. If (e_i) is lower semi-homogeneous, then $|SP_w(l_M)| = 1$ if and only if l_M is isomorphic to $l_p, p \in [\alpha_M, \beta_M]$.*

Proof. Sufficiency. Since $M \in \Delta_2$, $SP_w(l_M)$ is countable, then by Lemma 3.4, l_M is the upper bound of $SP_w(l_M)$, and $l_p, p \in [\alpha_M, \beta_M]$ is the lower bound of $SP_w(l_M)$. Since l_M is isomorphic to $l_p, p \in [\alpha_M, \beta_M]$, we get $|SP_w(l_M)| = 1$. □

Necessity. If $|SP_w(l_M)| = 1$, then $|C_{M,1}| = 1$ by Lemma 2.8, that is, all the functions in $C_{M,1}$ are equivalent to M.

For $p \in [\alpha_M, \beta_M]$, we define the function $M_n(t)$ [6] as follows:

$$M_n(t) = A_n^{-1} \int_{u_n/\omega_n}^{1} M(ts\omega_n) s^{-p-1} ds, \tag{3.3}$$

where $0 < u_n < v_n < \omega_n \le 1$ with $\omega_n \to 0$, $u_n/v_n \to 0, A_n = \int_{u_n/\omega_n}^{1} M(s\omega_n)s^{-p-1}ds$. Obviously, $M_n(t) \in C_{M,1}$; next we shall prove that $M_n(t)$ is equivalent to M

$$\frac{M_n(t)}{M(t)} = A_n^{-1}\int_{u_n/w_n}^{1} \frac{M(tsw_n)}{M(t)} s^{-p-1}ds. \tag{3.4}$$

Since $s \le 1$, $sw_n \le w_n$, and M is nondecreasing convex function, therefore, $M(tsw_n) \le M(tw_n)$; then

$$\begin{aligned}\frac{M_n(t)}{M(t)} &= A_n^{-1}\int_{u_n/w_n}^{1} \frac{M(tsw_n)}{M(t)} s^{-p-1}ds \\ &\le A_n^{-1}\int_{u_n/w_n}^{1} \frac{M(tw_n)}{M(t)} s^{-p-1}ds \\ &= \frac{1}{p}A_n^{-1}\frac{M(tw_n)}{M(t)}\left(1-\left(\frac{u_n}{w_n}\right)^{-p}\right).\end{aligned} \tag{3.5}$$

Since $tw_n < t$ and $M(tw_n) < M(t)$, we have

$$\frac{M_n(t)}{M(t)} \le A_n^{-1}\frac{M(tw_n)}{M(t)}\left(1-\left(\frac{u_n}{w_n}\right)^{-p}\right) \le \frac{1}{p}A_n^{-1}\left(1-\left(\frac{u_n}{w_n}\right)^{-p}\right). \tag{3.6}$$

Notice that for any fixed n, the right side of the above inequality is a constant; then we obtain $M_n \le M$

$$\frac{M_n(t)}{M(t)} = A_n^{-1}\int_{u_n/w_n}^{1} \frac{M(tsw_n)}{M(t)} s^{-p-1}ds. \tag{3.7}$$

By $u_n/w_n \le s \le 1$, we have $s^{-p-1} \ge (u_n/w_n)^{-p-1}$ and $M(tsw_n) \ge M(tu_n)$; hence

$$\frac{M_n(t)}{M(t)} \ge A_n^{-1}\frac{M(tu_n)}{M(t)}\left(\frac{u_n}{w_n}\right)^{-p-1}\left(1-\frac{u_n}{w_n}\right). \tag{3.8}$$

Since $\varphi(t) = M(t)/t^p$, $n\varphi(w_n) < \varphi(v_n/2)$, and

$$\frac{nM(u_n)}{w_n^p} < \frac{M(v_n/2)}{(v_n/2)^p}. \tag{3.9}$$

Moreover,

$$\frac{w_n^p}{v_n^p} > \frac{n2^{-p}M(w_n)}{M(v_n/2)}. \tag{3.10}$$

We obtain that

$$\frac{M_n(t)}{M(t)} \geq A_n^{-1}\frac{M(tu_n)}{M(t)}\left(\frac{u_n}{w_n}\right)^{-p-1}\left(1-\frac{u_n}{w_n}\right)$$
$$> A_n^{-1}\left(1-\frac{u_n}{w_n}\right)\frac{w_n^p}{v_n^p}\frac{M(tu_n)}{M(t)} \tag{3.11}$$
$$> n\cdot 2^{-p}A_n^{-1}\left(1-\frac{u_n}{w_n}\right)\frac{M(w_n)}{M(v_n/2)}\frac{M(tu_n)}{M(t)}.$$

Since $0 < t,\ u_n \leq 1,\ \{e_i\}$ is lower semihomogeneous; then by Lemma 3.2, we have for some $C \geq 1$

$$CM(tu_n) \geq M(t)M(u_n). \tag{3.12}$$

Therefore,

$$\frac{M_n(t)}{M(t)} > n\cdot 2^{-p}C^{-1}A_n^{-1}\left(1-\frac{u_n}{w_n}\right)\frac{M(w_n)}{M(v_n/2)}M(u_n). \tag{3.13}$$

Thus we get $M_n \geq M$.

So by (3.4) and (3.7), we can know that M_n is equivalent to M. By Lemma 3.3 and its proof ([6], Theorem 4.a.9), we obtain that $M_n(t)$ uniformly converges to t^p on $[0,1/2]$. Since $C_{M,1}$ is the closed subset of $C[0,1/2]$, we have that $t^p \in C_{M,1}$, t^p is equivalent to M, and therefore l_M is isomorphic to l_p.

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