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*Research Article*

# New Iterative Schemes for Asymptotically Quasi-Nonexpansive Mappings

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We consider an iterative scheme for approximating the common fixed points of two asymptotically quasi-nonexpansive mappings in the intermediate sense in Banach spaces. The present results improve and extend some recent corresponding results of Lan (2006) and many others.

## 1. Introduction

Let $C$ be a nonempty subset of a real Banach space $E$. Let $T : C \to C$ be a mapping. We use $F(T)$ to denote the set of fixed points of $T$. Recall that a mapping $T : C \to C$ is said to be generalized asymptotically quasi-nonexpansive with respect to $\{\sigma_n\}$ and $\{\delta_n\}$ if there exists the sequences $\{\sigma_n\}$ and $\{\delta_n\} \subset [0,1)$ with $\sigma_n \to 0$ and $\delta_n \to 0$ as $n \to \infty$ such that

$$\left\|T^n(x) - p\right\| \leq (1 + \sigma_n)\left\|x - p\right\| + \delta_n\|x - T^n(x)\| \tag{1.1}$$

for all $x \in C$, $p \in F(T)$ and $n \geq 1$. It is clear that if $F(T)$ is nonempty, then the asymptotically nonexpansive mapping, the asymptotically quasi-nonexpansive mapping, and the generalized quasi-nonexpansive mapping are all the generalized asymptotically quasi-nonexpansive mapping.

Recall also that a mapping $T : C \to C$ is said to be asymptotically quasi-nonexpasnive in the intermediate sense provided that $T$ is uniformly continuous and

$$\limsup_{n\to\infty} \sup_{x\in C, p\in F(T)} \left(\left\|T^n(x) - p\right\| - \left\|x - p\right\|\right) \leq 0. \tag{1.2}$$

*Remark 1.1.* From the above definition, if $F(T)$ is nonempty, it is easy to see that the generalized asymptotically quasi-nonexpansive mapping must be the asymptotically quasi-nonexpasnive mapping in the intermediate sense.

It is well known that the concept of asymptotically nonexpansive mapping, which is closely related to the theory of fixed points in Banach spaces, is introduced by Goebel and Kirk [1]. An early fundamental result due to Goebel and Kirk [1] proved that every asymptotically nonexpansive mapping of a nonempty closed bonded and convex subset of a uniformly convex Banach space has a fixed point. Since 1972, the weak and strong convergence problems of iterative sequences (with errors) for nonexpansive mappings, asymptotically nonexpansive mappings in the setting of Hilbert space or Banach space, have been studied by many authors; please see, for example, [1–29] and the references therein. Recently, Zhou et al. [30] introduced a class of new generalized asymptotically nonexpansive mappings and gave a sufficient and necessary condition for the modified Ishikawa and Mann iterative sequences to converge to fixed points for the class of mappings. Define the Ishikawa iterative process involving the generalized asymptotically nonexpansive mappings in a Banach space $E$ as follows.

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T(y_n), \\ y_n &= (1-\beta_n)x_n + \beta_n T(x_n), \quad n = 1,2,\ldots \end{aligned} \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are two real sequences in $[0,1]$ satisfying some conditions. For details, we can refer to [31–33]. Very recently, Lan [3] introduced a new class of iterative procedures as follows:

Let $T_i : C \to C$ $(i = 1,2)$ be given mappings. Then, for arbitrary $\omega \in C$ and $x_1 \in C$, the sequence $\{x_n\}$ in $C$ defined by

$$\begin{aligned} y_n &= (1-\beta_n)\omega + \beta_n T_2^n(x_n), \\ x_{n+1} &= (1-\alpha_n)\omega + \alpha_n T_1^n(y_n), \quad n = 1,2,\ldots, \end{aligned} \tag{1.4}$$

is called the generalized modified Ishikawa iterative sequence.

Further, Lan [3] remarked that the above iterative processes include many iterative processes as special cases and he gave a sufficient and necessary condition for the iterative sequence to converge to the common fixed points for two generalized asymptotically quasi-nonexpansive mappings.

It is our purpose in this paper that we will extend the above iterative processes to the more general iterative processes and give a sufficient and necessary condition for two asymptotically quasi-nonexpasnive mapping in the intermediate sense. Our result extends the corresponding results of Lan [3], Zhou et al. [30], and many others.

## 2. Preliminaries

Let $C$ be a nonempty closed convex subset of a real Banach space $E$. Let $T_i : C \to C$ $(i = 1, 2)$ be given mappings. For given $x_1 \in C$, the sequence $\{x_n\}$ in $C$ defined iteratively by

$$\begin{aligned} y_n &= (1-\beta_n)\omega_n + \beta_n T_2^n(x_n), \\ x_{n+1} &= (1-\alpha_n)\nu_n + \alpha_n T_1^n(y_n), \quad n = 1, 2, \ldots \end{aligned} \tag{2.1}$$

is called the more general modified Ishikawa iterative sequence, where $\{\omega_n\}$ and $\{\nu_n\}$ are sequences in $C$, and $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,1]$ satisfying some conditions.

If we replace $\omega_n$ and $\nu_n$ in all the iteration steps by $\omega$, then the sequence $\{x_n\}$ defined by (2.1) becomes to (1.4) which is studied by Lan [3].

If we replace $\omega_n$ and $\nu_n$ in all the iteration steps by $x_n$ and $\omega$, respectively, then the sequence $\{x_n\}$ defined by (2.1) becomes to

$$\begin{aligned} y_n &= (1-\beta_n)x_n + \beta_n T_2^n(x_n), \\ x_{n+1} &= (1-\alpha_n)\omega + \alpha_n T_1^n(y_n), \quad n = 1, 2, \ldots. \end{aligned} \tag{2.2}$$

If we replace $\omega_n$ and $\nu_n$ in all the iteration steps by $x_n$, then the sequence $\{x_n\}$ defined by (2.1) becomes to

$$\begin{aligned} y_n &= (1-\beta_n)x_n + \beta_n T_2^n(x_n), \\ x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T_1^n(y_n), \quad n = 1, 2, \ldots. \end{aligned} \tag{2.3}$$

If $\beta_n = v_n = 0$ in (2.1), then the sequence $\{x_n\}$ defined by

$$x_{n+1} = (1-\alpha_n)\nu_n + \alpha_n T_1^n(x_n), \quad n = 1, 2 \tag{2.4}$$

is called the more general modified Mann iterative sequence.

It is clear that the iterative processes (2.1) include many iterative processes as special cases.

In the sequel, we need the following lemmas for the main results in this paper.

**Lemma 2.1** (see [32]). *Let $\{a_n\}$, $\{b_n\}$, and $\{\delta_n\}$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} \le (1+\delta_n)a_n + b_n. \tag{2.5}$$

*If $\sum_{n=1}^{\infty} \delta_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$, then $\lim_{n\to\infty} a_n$ exists. In particular, if $\{a_n\}$ has a subsequence converging to zero, then $\lim_{n\to\infty} a_n = 0$.*

## 3. Main Results

**Theorem 3.1.** *Let $C$ be a nonempty closed convex subset of a real Banach space $E$. Let $T_i : C \to C$ $(i = 1, 2)$ be asymptotically quasi-nonexpansive mappings in the intermediate sense such that $F(T_1) \cap F(T_2) \neq \emptyset$. Let $\omega_n \in C, \nu_n \in C$ be two bounded sequences. For any given $x_1 \in C$, let the sequences $\{x_n\}$ and $\{y_n\}$ be defined by* (2.1)*. Put*

$$G_n = \max\left\{ \sup_{p\in F(T_1)\cap F(T_2), n\geq 1} \left(\|T_2^n(x_n) - p\| - \|x_n - p\|\right) \vee 0, \sup_{p\in F(T_1)\cap F(T_2), n\geq 1} \left(\|T_1^n(y_n) - p\| - \|y_n - p\|\right) \vee 0 \right\}. \tag{3.1}$$

*Assume that $\sum_{n=1}^{\infty} G_n < \infty$, $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$ and $\sum_{n=1}^{\infty}(1-\beta_n) < \infty$.*

*Then the sequence $\{x_n\}$ converges strongly to a common fixed point $p^*$ of $T_1$ and $T_2$ if and only if*

$$\liminf_{n\to\infty} d(x_n, F(T_1) \cap F(T_2)) = 0, \tag{3.2}$$

*where $d(x, F(T_1) \cap F(T_2))$ denotes the distance between $x$ and the set $F(T_1) \cap F(T_2)$.*

*Proof.* The necessity is obvious and so it is omitted.

Now, we prove the sufficiency. For any $p \in F(T_1) \cap F(T_2)$, from (2.1), we have

$$\begin{aligned} \|y_n - p\| &= \|(1-\beta_n)\omega_n + \beta_n T_2^n(x_n) - p\| \\ &\leq (1-\beta_n)\|\omega_n - p\| + \beta_n \|T_2^n(x_n) - p\| \\ &= (1-\beta_n)\|\omega_n - p\| + \beta_n\left(\|T_2^n(x_n) - p\| - \|x_n - p\|\right) + \beta_n\|x_n - p\| \\ &\leq \beta_n\|x_n - p\| + \beta_n G_n + (1-\beta_n)\|\omega_n - p\|, \end{aligned} \tag{3.3}$$

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1-\alpha_n)\nu_n + \alpha_n T_1^n(y_n) - p\| \\ &\leq (1-\alpha_n)\|\nu_n - p\| + \alpha_n\|T_1^n(y_n) - p\| \\ &= (1-\alpha_n)\|\nu_n - p\| + \alpha_n\left(\|T_1^n(y_n) - p\| - \|y_n - p\|\right) + \alpha_n\|y_n - p\| \\ &\leq \alpha_n\|y_n - p\| + \alpha_n G_n + (1-\alpha_n)\|\nu_n - p\|. \end{aligned} \tag{3.4}$$

Substituting (3.3) into (3.4) and simplifying, we have

$$\begin{aligned} \|x_{n+1} - p\| &\leq \alpha_n\beta_n\|x_n - p\| + \alpha_n(1+\beta_n)G_n \\ &\quad + \alpha_n(1-\beta_n)\|\omega_n - p\| + (1-\alpha_n)\|\nu_n - p\| \\ &\leq \|x_n - p\| + b_n, \end{aligned} \tag{3.5}$$

where

$$b_n = \alpha_n(1+\beta_n)G_n + \alpha_n(1-\beta_n)\|\omega_n - p\| + (1-\alpha_n)\|\nu_n - p\|. \tag{3.6}$$

We note that $\sum_{n=1}^{\infty} G_n < \infty$, $\sum_{n=1}^{\infty}(1-\beta_n) < \infty$, $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$ and $\|\omega_n - p\|, \|\nu_n - p\|$ are bounded; therefore, we have $\sum_{n=1}^{\infty} b_n < \infty$. Then, from (3.5), we have

$$d(x_{n+1}, F(T_1)\cap F(T_2)) \le d(x_n, F(T_1)\cap F(T_2)) + b_n. \tag{3.7}$$

By Lemma 2.1, we know that $\lim_{n\to\infty} d(x_n, F(T_1)\cap F(T_2))$ exists. Because $\liminf_{n\to\infty} d(x_n, F(T_1)\cap F(T_2)) = 0$, then

$$\lim_{n\to\infty} d(x_n, F(T_1)\cap F(T_2)) = 0. \tag{3.8}$$

Next we prove that $\{x_n\}$ is a Cauchy sequence in $C$.

It follows from (3.5) that for any $m \ge 1$, for all $n \ge n_0$, for all $p \in F(T_1)\cap F(T_2)$,

$$\begin{aligned}\|x_{n+m} - p\| &\le \|x_{n+m-1} - p\| + b_{n+m-1} \le \|x_{n+m-2} - p\| + (b_{n+m-1} + b_{n+m-2})\\ &\le \cdots \le \|x_n - p\| + \sum_{k=n}^{n+m-1} b_k.\end{aligned} \tag{3.9}$$

So we have

$$\|x_{n+m} - x_n\| \le \|x_{n+m} - p\| + \|x_n - p\| \le 2\|x_n - p\| + \sum_{k=n}^{\infty} b_k. \tag{3.10}$$

Then, we have

$$\|x_{n+m} - x_n\| \le 2d(x_n, F(T_1)\cap F(T_2)) + \sum_{k=n}^{\infty} b_k, \quad \forall n \ge n_0. \tag{3.11}$$

For any given $\epsilon > 0$, there exists a positive integer $n_1 \ge n_0$ such that for any $n \ge n_1$, $d(x_n, F(T_1)\cap F(T_2)) < \epsilon/4$ and $\sum_{k=n}^{\infty} b_k < \epsilon/2$. Thus when $n \ge n_1$, $\|x_{n+m} - x_n\| < \epsilon$. So we have that

$$\lim_{n\to\infty} \|x_{n+m} - x_n\| = 0. \tag{3.12}$$

This implies that $\{x_n\}$ is a Cauchy sequence in $E$. Thus, the completeness of $E$ implies that $\{x_n\}$ must be convergent. Assume that $x_n \to p^*$ as $n \to \infty$.

Now we have to prove that $p^*$ is a common fixed point of $T_1$ and $T_2$. Indeed, we know that the set $F(T_1)\cap F(T_2)$ is closed. From the continuity of $d(x, F(T_1)\cap F(T_2)) = 0$

with $\lim_{n\to\infty} d(x_n, F(T_1)\cap F(T_2)) = 0$ and $\lim_{n\to\infty} x_n = p^*$, we get

$$d\big(p^*, F(T_1)\cap F(T_2)\big) = 0, \tag{3.13}$$

and so $p^* \in F(T_1)\cap F(T_2)$. This completes the proof. □

We can conclude immediately Theorem 3.1 in [3], which can be reviewed as a corollary of Theorem 3.1.

**Corollary 3.2.** *Let $C$ be a nonempty closed convex subset of a real Banach space $E$ and for $i = 1, 2$, let $T_i : C \to C$ be a generalized asymptotically quasi-nonexpansive mapping with respect to $\{\sigma_{in}\}$ and $\{\delta_{in}\}$ such that $F(T_1)\cap F(T_2) \neq \emptyset$ in $C$, and $\sum_{n=1}^{\infty}(\sigma_n + 2\delta_n)/(1-\delta_n) < \infty$, where $\sigma_n = \max\{\sigma_{1n}, \sigma_{2n}\}$ and $\delta_n = \max\{\delta_{1n}, \delta_{2n}\}$. Assume that $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$ and $\sum_{n=1}^{\infty}(1-\beta_n) < \infty$.*

*Then the iterative sequence $\{x_n\}$ defined by (1.4) converges strongly to a common fixed point $p^*$ of $T_1$ and $T_2$ if and only if*

$$\liminf_{n\to\infty} d(x_n, F(T_1)\cap F(T_2)) = 0, \tag{3.14}$$

*where $d(x, F(T_1)\cap F(T_2))$ denotes the distance between $x$ and the set $F(T_1)\cap F(T_2)$.*

*Proof.* We note that condition $\sum_{n=1}^{\infty}(\sigma_n + 2\delta_n)/(1-\delta_n) < \infty$ implies $\sum_{n=1}^{\infty}\sigma_n < \infty$ and $\sum_{n=1}^{\infty}\delta_n < \infty$. From the boundedness of $\{x_n\}$, $\{y_n\}$, and (1.1), we can obtain $\sum_{n=1}^{\infty} G_n < \infty$. It is easy to see that all conditions of Theorem 3.1 are satisfied; it follows from Theorem 3.1; we can conclude our desired result. This completes the proof. □

**Theorem 3.3.** *Let $C$ be a nonempty closed convex subset of a real Banach space $E$. Let $T_i : C \to C (i = 1, 2)$ be asymptotically quasi-nonexpansive mappings in the intermediate sense such that $F(T_1)\cap F(T_2) \neq \emptyset$. For any given $x_1 \in C$, let the sequences $\{x_n\}$ and $\{y_n\}$ be defined by (2.3). Assume that $\sum_{n=1}^{\infty}\alpha_n < \infty$.*

*Then the sequence $\{x_n\}$ converges strongly to a common fixed point $p^*$ of $T_1$ and $T_2$ if and only if*

$$\liminf_{n\to\infty} d(x_n, F(T_1)\cap F(T_2)) = 0. \tag{3.15}$$

*Proof.* The necessity is obvious and so it is omitted.

Now, we prove the sufficiency. For any $p \in F(T_1)\cap F(T_2)$, it follows from (1.2) that for any given $\epsilon > 0$, there exists a positive integer $n_1$ such that for $n \geq n_1$, we have

$$\max\left\{\sup_{p\in F(T_1)\cap F(T_2), n\geq n_1}\left(\left\|T_2^n(x_n)-p\right\|-\left\|x_n-p\right\|\right), \sup_{p\in F(T_1)\cap F(T_2), n\geq n_1}\left(\left\|T_1^n(y_n)-p\right\|-\left\|y_n-p\right\|\right)<\epsilon\right\}. \tag{3.16}$$

From (2.3), we have

$$\begin{aligned}\|y_n - p\| &\le (1-\beta_n)\|x_n - p\| + \beta_n\left(\|T_2^n(x_n) - p\| - \|x_n - p\|\right) + \beta_n\|x_n - p\| \\ &\le \|x_n - p\| + \beta_n\epsilon,\end{aligned} \tag{3.17}$$

$$\begin{aligned}\|x_{n+1} - p\| &\le (1-\alpha_n)\|x_n - p\| + \alpha_n\left(\|T_1^n(y_n) - p\| - \|y_n - p\|\right) + \alpha_n\|y_n - p\| \\ &\le (1-\alpha_n)\|x_n - p\| + \alpha_n\epsilon + \alpha_n\|y_n - p\|.\end{aligned} \tag{3.18}$$

Substituting (3.17) into (3.18), we have

$$\|x_{n+1} - p\| \le \|x_n - p\| + \alpha_n(1+\beta_n)\epsilon = \|x_n - p\| + b_n, \tag{3.19}$$

where $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \alpha_n(1+\beta_n)\epsilon < \infty$. The rest proof follows as those of Theorem 3.1 and therefore is omitted. This completes the proof. □

From Theorem 3.1, we can obtain the following results.

**Theorem 3.4.** *Let $C$ be a nonempty closed convex subset of a real Banach space $E$. Let $T : C \to C$ be asymptotically quasi-nonexpansive mappings in the intermediate sense such that $F(T) \neq \emptyset$. Let $\nu_n \in C$ be bounded sequence. For any given $x_1 \in C$, let the sequence $\{x_n\}$ be defined by* (2.4)*. Put*

$$G_n = \sup_{p\in F(T), n\ge 1}\left(\|T^n(x_n) - p\| - \|x_n - p\|\right) \vee 0. \tag{3.20}$$

*Assume that $\sum_{n=1}^{\infty} G_n < \infty$ and $\sum_{n=1}^{\infty}(1-\alpha_n) < \infty$.*

*Then the sequence $\{x_n\}$ converges strongly to a fixed point $p^*$ of $T$ if and only if*

$$\liminf_{n\to\infty} d(x_n, F(T)) = 0. \tag{3.21}$$

*Remark 3.5.* Constructing iterative algorithms for approximating (common) fixed points of some nonlinear operators has been studied extensively. It is worth mentioning that our iterative scheme (2.1) appears to be a new one, which includes many iterative schemes as special cases. Our results improve and extend the corresponding results of Lan [3], Chang et al. [4], Xu and Noor [7], Zhou et al. [30], and many others.

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