Hindawi Publishing Corporation
Journal of Inequalities and Applications
Volume 2009, Article ID 865093, 17 pages
doi:10.1155/2009/865093

Research Article

General Nonlinear Random Equations with Random Multivalued Operator in Banach Spaces

Heng-You Lan,[1] Yeol Je Cho,[2] and Wei Xie[1]

[1] *Department of Mathematics, Sichuan University of Science & Engineering, Zigong, Sichuan 643000, China*
[2] *Department of Mathematics Education and the RINS, Gyeongsang National University, Chinju 660-701, South Korea*

Correspondence should be addressed to Yeol Je Cho, yjcho@gsnu.ac.kr

Received 16 December 2008; Accepted 27 February 2009

Recommended by Jewgeni Dshalalow

We introduce and study a new class of general nonlinear random multivalued operator equations involving generalized m-accretive mappings in Banach spaces. By using the Chang's lemma and the resolvent operator technique for generalized m-accretive mapping due to Huang and Fang (2001), we also prove the existence theorems of the solution and convergence theorems of the generalized random iterative procedures with errors for this nonlinear random multivalued operator equations in q-uniformly smooth Banach spaces. The results presented in this paper improve and generalize some known corresponding results in iterature.

1. Introduction and Preliminaries

The variational principle has been one of the major branches of mathematical sciences for more than two centuries. It is a tool of great power that can be applied to a wide variety of problems in pure and applied sciences. It can be used to interpret the basic principles of mathematical and physical sciences in the form of simplicity and elegance. During this period, the variational principles have played an important and significant part as a unifying influence in pure and applied sciences and as a guide in the mathematical interpretation of many physical phenomena. The variational principles have played a fundamental role in the development of the general theory of relativity, gauge field theory in modern particle physics and soliton theory. In recent years, these principles have been enriched by the discovery of the variational inequality theory, which is mainly due to Hartman and Stampacchia [1]. Variational inequality theory constituted a significant extension of the variational principles and describes a broad spectrum of very interesting developments involving a link among various fields of mathematics, physics, economics, regional, and engineering sciences. The ideas and techniques are being applied in a variety of diverse areas of sciences and prove to be productive and innovative. In fact, many researchers have shown that this theory provides the most natural, direct, simple, unified, and efficient framework for a general treatment of a wide class of unrelated linear and nonlinear problems.

Variational inclusion is an important generalization of variational inequality, which has been studied extensively by many authors (see, e.g., [2–14] and the references therein). In 2001, Huang and Fang [15] introduced the concept of a generalized m-accretive mapping, which is a generalization of an m-accretive mapping, and gave the definition of the resolvent operator for the generalized m-accretive mapping in Banach spaces. Recently, Huang et al. [6, 7], Huang [8], Jin and Liu [9] and Lan et al. [11] introduced and studied some new classes of nonlinear variational inclusions involving generalized m-accretive mappings in Banach spaces. By using the resolvent operator technique in [6], they constructed some new iterative algorithms for solving the nonlinear variational inclusions involving generalized m-accretive mappings. Further, they also proved the existence of solutions for nonlinear variational inclusions involving generalized m-accretive mappings and convergence of sequences generated by the algorithms.

On the other hand, It is well known that the study of the random equations involving the random operators in view of their need in dealing with probabilistic models in applied sciences is very important. Motivated and inspired by the recent research works in these fascinating areas, the random variational inequality problems, random quasi-variational inequality problems, random variational inclusion problems and random quasi-complementarity problems have been introduced and studied by Ahmad and Bazán [16], Chang [17], Chang and Huang [18], Cho et al. [19], Ganguly and Wadhwa [20], Huang [21], Huang and Cho [22], Huang et al. [23], and Noor and Elsanousi [24].

Inspired and motivated by recent works in these fields (see [3, 11, 12, 16, 25–28]), in this paper, we introduce and study a new class of general nonlinear random multivalued operator equations involving generalized m-accretive mappings in Banach spaces. By using the Chang's lemma and the resolvent operator technique for generalized m-accretive mapping due to Huang and Fang [15], we also prove the existence theorems of the solution and convergence theorems of the generalized random iterative procedures with errors for this nonlinear random multivalued operator equations in q-uniformly smooth Banach spaces. The results presented in this paper improve and generalize some known corresponding results in literature.

Throughout this paper, we suppose that (Ω, A, μ) is a complete σ-finite measure space and E is a separable real Banach space endowed with dual space E^*, the norm $\|\cdot\|$ and the dual pair $\langle\cdot,\cdot\rangle$ between E and E^*. We denote by $\mathcal{B}(E)$ the class of Borel σ-fields in E. Let 2^E and $CB(E)$ denote the family of all the nonempty subsets of E, the family of all the nonempty bounded closed sets of E, respectively. The generalized duality mapping $J_q : E \to 2^{E^*}$ is defined by

$$J_q(x) = \left\{ f^* \in E^* : \langle x, f^* \rangle = \|x\|^q,\ \left\|f^*\right\| = \|x\|^{q-1} \right\} \tag{1.1}$$

for all $x \in E$, where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is well known that, in general, $J_q(x) = \|x\|^{q-2} J_2(x)$ for all $x \neq 0$ and J_q is single-valued if E^* is strictly convex (see, e.g., [28]). If $E = H$ is a Hilbert space, then J_2 becomes the identity mapping of H. In what follows we will denote the single-valued generalized duality mapping by j_q.

Suppose that $A : \Omega \times E \times E \to 2^E$ is a random multivalued operator such that for each fixed $t \in \Omega$ and $s \in E$, $A(t, \cdot, s) : E \to 2^E$ is a generalized m-accretive mapping and $\text{Range}(p) \bigcap \text{dom} A(t, \cdot, s) \neq \emptyset$. Let $S, p : \Omega \times E \to E$, $\eta : \Omega \times E \times E \to E$ and $N : \Omega \times E \times E \times E \to E$ be single-valued operators, and let $M, T, G : \Omega \times E \to 2^E$ be three multivalued operators.

Now, we consider the following problem.

Find $x, v, w : \Omega \to E$ such that $v(t) \in T(t, x(t))$, $w(t) \in G(t, x(t))$, and

$$0 \in N(t, S(t, x(t)), u(t), v(t)) + A\big(t, p(t, x(t)), w(t)\big) \tag{1.2}$$

for all $t \in \Omega$ and $u \in M(t, x(t))$. The problem (1.2) is called the *general nonlinear random equation* with multivalued operator involving generalized m-accretive mapping in Banach spaces.

Some special cases of the problem (1.2) are as follows.

(1) If G is a single-valued operator, $p \equiv I$, the identity mapping and $N(t, x, y, z) = f(t, z) + g(t, x, y)$ for all $t \in \Omega$ and $x, y, z \in E$, then problem (1.2) is equivalent to finding $x, v : \Omega \to E$ such that $v(t) \in T(t, x(t))$ and

$$0 \in f(t, v(t)) + g(t, S(t, x(t)), u(t)) + A(t, x(t), G(t, x(t))) \tag{1.3}$$

for all $t \in \Omega$ and $u \in M(t, x(t))$. The determinate form of the problem (1.3) was considered and studied by Agarwal et al. [2] when $G \equiv I$.

(2) If $A(t, x, s) = A(t, x)$ for all $t \in \Omega$, $x, s \in E$ and, for all $t \in \Omega$, $A(t, \cdot) : E \to 2^E$ is a generalized m-accretive mapping, then the problem (1.2) reduces to the following generalized nonlinear random multivalued operator equation involving generalized m-accretive mapping in Banach spaces.

Find $x, v : \Omega \to E$ such that $v(t) \in \mathrm{T}(t, x(t))$ and

$$0 \in N(t, S(t, x(t)), u(t), v(t)) + A\big(t, p(t, x(t))\big) \tag{1.4}$$

for all $t \in \Omega$ and $u \in M(t, x(t))$.

(3) If $E = E^* = H$ is a Hilbert space and $A(t, \cdot) = \partial\phi(t, \cdot)$ for all $t \in \Omega$, where $\partial\phi(t, \cdot)$ denotes the subdifferential of a lower semicontinuous and η-subdifferetiable function $\phi : \Omega \times H \to R \cup \{+\infty\}$, then the problem (1.4) becomes the following problem.

Find $x, v : \Omega \to H$ such that $v(t) \in T(t, x(t))$ and

$$\big\langle N(t, S(t, x(t)), u(t), v(t)), \eta\big(t, z, p(t, x(t))\big) \big\rangle \geq \phi\big(t, p(t, x(t))\big) - \phi(t, z) \tag{1.5}$$

for all $t \in \Omega$, $u \in M(t, x(t))$, and $z \in H$, which is called the *generalized nonlinear random variational inclusions* for random multivalued operators in Hilbert spaces. The determinate form of the problem (1.5) was studied by Agarwal et al. [3] when $N(S(x), u, v) = p(x) - B(u, v)$ for all $x, u, v \in H$, where $B : H \times H \to H$ is a single-valued operator.

(4) If $\eta(t, u(t), v(t)) = u(t) - v(t)$ for all $t \in \Omega$, $u(t), v(t) \in H$, then the problem (1.5) reduces to the following nonlinear random variational inequalities.

Find $x, v, w : \Omega \to H$ such that $v(t) \in T(t, x(t))$, $u \in M(t, x(t))$, and

$$\langle N(t, S(t, x(t)), u(t), v(t)), z - p(t, x(t)) \rangle \geq \phi(t, p(t, x(t))) - \phi(t, z) \tag{1.6}$$

for all $t \in \Omega$ and $z \in H$, whose determinate form is a generalization of the problem considered in [4, 5, 29].

(5) If, in the problem (1.6), ϕ is the *indictor function* of a nonempty closed convex set K in H defined in the form

$$\phi(y) = \begin{cases} 0 & \text{if } y \in K, \\ \infty & \text{otherwise,} \end{cases} \tag{1.7}$$

then (1.6) becomes the following problem.

Find $x, u, v : \Omega \to H$ such that $v(t) \in T(t, x(t))$, $u \in M(t, x(t))$, and

$$\langle N(t, S(t, x(t)), u(t), v(t)), z - p(t, x(t)) \rangle \geq 0 \tag{1.8}$$

for all $t \in \Omega$ and $z \in K$. The problem (1.8) has been studied by Cho et al. [19] when $N(t, x, u(t), v(t)) = u(t) - v(t)$ for all $t \in \Omega$, $x(t)$, $u(t)$, $\mathrm{v}(t) \in H$.

Remark 1.1. For appropriate and suitable choices of S, p, N, η, M, G, T, A and for the space E, a number of known classes of random variational inequality, random quasi-variational inequality, random complementarity, and random quasi-complementarity problems were studied previously by many authors (see, e.g., [17–20, 22–24] and the references therein).

In this paper, we will use the following definitions and lemmas.

Definition 1.2. An operator $x : \Omega \to E$ is said to be *measurable* if, for any $B \in \mathcal{B}(E)$, $\{t \in \Omega : x(t) \in B\} \in \mathcal{A}$.

Definition 1.3. An operator $F : \Omega \times E \to E$ is called a *random operator* if for any $x \in E$, $F(t, x) = y(t)$ is measurable. A random operator F is said to be *continuous* (resp., *linear, bounded*) if, for any $t \in \Omega$, the operator $F(t, \cdot) : E \to E$ is continuous (resp., linear, bounded).

Similarly, we can define a random operator $a : \Omega \times E \times E \to E$. We will write $F_t(x) = F(t, x(t))$ and $a_t(x, y) = a(t, x(t), y(t))$ for all $t \in \Omega$ and $x(t)$, $y(t) \in E$.

It is well known that a measurable operator is necessarily a random operator.

Definition 1.4. A multivalued operator $G : \Omega \to 2^E$ is said to be *measurable* if, for any $B \in \mathcal{B}(E)$, $G^{-1}(B) = \{t \in \Omega : G(t) \cap B \neq \emptyset\} \in \mathcal{A}$.

Definition 1.5. An operator $u : \Omega \to E$ is called a *measurable selection* of a multivalued measurable operator $\Gamma : \Omega \to 2^E$ if u is measurable and for any $t \in \Omega$, $u(t) \in \Gamma(t)$.

Definition 1.6. A multivalued operator $F : \Omega \times E \to 2^E$ is called a *random multivalued operator* if, for any $x \in E$, $F(\cdot, x)$ is measurable. A random multivalued operator $F : \Omega \times E \to CB(E)$

is said to be *H-continuous* if, for any $t \in \Omega$, $F(t, \cdot)$ is continuous in $H(\cdot, \cdot)$, where $H(\cdot, \cdot)$ is the Hausdorff metric on $CB(E)$ defined as follows: for any given $A, B \in CB(E)$,

$$H(A, B) = \max\left\{\sup_{x\in A}\inf_{y\in B} d(x, y), \sup_{y\in B}\inf_{x\in A} d(x, y)\right\}. \tag{1.9}$$

Definition 1.7. A random operator $g : \Omega \times E \to E$ is said to be

(a) *α-strongly accretive* if there exists $j_2(x(t) - y(t)) \in J_2(x(t) - y(t))$ such that

$$\langle g_t(x) - g_t(y), j_2(x(t) - y(t))\rangle \geq \alpha(t)\|x(t) - y(t)\|^2 \tag{1.10}$$

for all $x(t),\ y(t) \in E$ and $t \in \Omega$, where $\alpha(t) > 0$ is a real-valued random variable;

(b) *β-Lipschitz continuous* if there exists a real-valued random variable $\beta(t) > 0$ such that

$$\|g_t(x) - g_t(y)\| \leq \beta(t)\|x(t) - y(t)\| \tag{1.11}$$

for all $x(t),\ y(t) \in E$ and $t \in \Omega$.

Definition 1.8. Let $S : \Omega \times E \to E$ be a random operator. An operator $N : \Omega \times E \times E \times E \to E$ is said to be

(a) *ϱ-strongly accretive* with respect to S in the first argument if there exists $j_2(x(t) - y(t)) \in J_2(x(t) - y(t))$ such that

$$\langle N_t(S_t(x), \cdot, \cdot) - N_t(S_t(y), \cdot, \cdot), j_2(x(t) - y(t))\rangle \geq \varrho(t)\|x(t) - y(t)\|^2 \tag{1.12}$$

for all $x(t),\ y(t) \in E$, and $t \in \Omega$, where $\varrho(t) > 0$ is a real-valued random variable;

(b) *ϵ-Lipschitz continuous* in the first argument if there exists a real-valued random variable $\varepsilon(t) > 0$ such that

$$\|N_t(x, \cdot, \cdot) - N_t(y, \cdot, \cdot)\| \leq \epsilon(t)\|x(t) - y(t)\| \tag{1.13}$$

for all $x(t),\ y(t) \in E$ and $t \in \Omega$.

Similarly, we can define the Lipschitz continuity in the second argument and third argument of $N(\cdot, \cdot, \cdot)$.

Definition 1.9. Let $\eta : \Omega \times \mathrm{E} \times E \to E^*$ be a random operator and $M : \Omega \times E \to 2^E$ be a random multivalued operator. Then M is said to be

(a) *η-accretive* if

$$\langle u(t) - v(t), \eta_t(x, y)\rangle \geq 0 \tag{1.14}$$

for all $x(t),\ y(t) \in E$, $u(t) \in M_t(x)$, $v(t) \in M_t(y)$, and $t \in \Omega$, where $M_t(z) = M(t, z(t))$;

(b) *strictlyη-accretive* if

$$\langle u(t) - v(t), \eta_t(x, y) \rangle \geq 0 \tag{1.15}$$

for all $x(t),\ y(t) \in E$, $u(t) \in M_t(x)$, $v(t) \in M_t(y)$, and $t \in \Omega$ and the equality holds if and only if $u(t) = v(t)$ for all $t \in \Omega$;

(c) *stronglyη-accretive* if there exists a real-valued random variable $r(t) > 0$ such that

$$\langle u(t) - v(t), \eta_t(x, y) \rangle \geq r(t) \| x(t) - y(t) \|^2 \tag{1.16}$$

for all $x(t),\ y(t) \in E$, $u(t) \in M_t(x)$, $v(t) \in M_t(y)$, and $t \in \Omega$;

(d) *generalized$m$-accretive* if M is η-accretive and $(I + \lambda(t) M(t, \cdot))(E) = E$ for all $t \in \Omega$ and (equivalently, for some) $\lambda(t) > 0$.

Remark 1.10. If $E = E^* = H$ is a Hilbert space, then (a)–(d) of Definition 1.9 reduce to the definition of η-monotonicity, strict η-monotonicity, strong η-monotonicity, and maximal η-monotonicity, respectively; if E is uniformly smooth and $\eta(x, y) = j_2(x - y) \in J_2(x - y)$, then (a)–(d) of Definition 1.9reduces to the definitions of accretive, strictly accretive, strongly accretive, and m-accretive operators in uniformly smooth Banach spaces, respectively.

Definition 1.11. The operator $\eta : \ \Omega \times E \times E \ \rightarrow \ E^*$ is said to be

(a) *monotone* if

$$\langle x(t) - y(t), \eta_t(x, y) \rangle \geq 0 \tag{1.17}$$

for all $x(t),\ y(t) \in E$ and $t \in \Omega$;

(b) *strictly monotone* if

$$\langle x(t) - y(t), \eta_t(x, y) \rangle \geq 0 \tag{1.18}$$

for all $x(t),\ y(t) \in E$, and $t \in \Omega$ and the equality holds if and only if $x(t) = y(t)$ for all $t \in \Omega$;

(c) *δ-strongly monotone* if there exists a measurable function $\delta : \ \Omega \ \rightarrow \ (0, \infty)$ such that

$$\langle x(t) - y(t), \eta_t(x, y) \rangle \geq \delta(t) \| x(t) - y(t) \|^2 \tag{1.19}$$

for all $x(t),\ y(t) \in E$ and $t \in \Omega$;

(d) *τ-Lipschitz continuous* if there exists a real-valued random variable $\tau(t) > 0$ such that

$$\| \eta_t(x, y) \| \leq \tau(t) \| x(t) - y(t) \| \tag{1.20}$$

for all $x(t),\ y(t) \in E$, and $t \in \Omega$.

Definition 1.12. A multivalued measurable operator $T : \Omega \times E \to CB(E)$ is said to be *γ-H-Lipschitz continuous* if there exists a measurable function $\gamma : \Omega \to (0, +\infty)$ such that, for any $t \in \Omega$,

$$H(T_t(x), T_t(y)) \leq \gamma(t) \|x(t) - y(t)\| \tag{1.21}$$

for all $x(t),\ y(t) \in E$.

The *modules of smoothness* of E is the function $\rho_E : [0, \infty) \to [0, \infty)$ defined by

$$\rho_E(t) = \sup\left\{ \frac{1}{2}\|x + y\| + \|x - y\| - 1 : \|x\| \leq 1,\ \|y\| \leq t \right\}. \tag{1.22}$$

A Banach space E is called *uniformly smooth* if $\lim_{t \to 0}(\rho_E(t)/t) = 0$ and E is called *q-uniformly smooth* if there exists a constant $c > 0$ such that $\rho_{E(t)} \leq ct^q$, where $q > 1$ is a real number.

It is well known that Hilbert spaces, L_p(or l_p) spaces, $1 < p < \infty$ and the Sobolev spaces $W^{m,p},\ 1 < p < \infty$, are all q-uniformly smooth.

In the study of characteristic inequalities in q-uniformly smooth Banach spaces, Xu [30] proved the following result.

Lemma 1.13. *Let $q > 1$ be a given real number and let E be a real uniformly smooth Banach space. Then E is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that, for all $x,\ y \in E$ and $j_q(x) \in J_q(x)$, the following inequality holds:*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, j_q(x)\rangle + c_q\|y\|^q. \tag{1.23}$$

Definition 1.14. Let $A : \Omega \times E \to 2^E$ be a generalized m-accretive mapping. Then the *resolvent operator* $J_A^{\rho(t)}$ for A is defined as follows:

$$J_A^{\rho(t)}(z) = (I + \rho(t)A)^{-1}(z) \tag{1.24}$$

for all $t \in \Omega$ and $z \in E$, where $\rho : \Omega \to (0, \infty)$ is a measurable function and $\eta : \Omega \times E \times E \to E^*$ is a strictly monotone mapping.

From Huang et al. [6, 15], we can obtain the following lemma.

Lemma 1.15. *Let $\eta : \Omega \times E \times E \to E^*$ be δ-strongly monotone and τ-Lipschitz continuous. Let $A : \Omega \times E \to 2^E$ be a generalized m-accretive mapping. Then the resolvent operator $J_A^{\rho(t)}$ for A is Lipschitz continuous with constant $\tau(t)/\delta(t)$, that is,*

$$\left\| J_A^{\rho(t)}(x) - J_A^{\rho(t)}(y) \right\| \leq \frac{\tau(t)}{\delta(t)}\|x - y\| \tag{1.25}$$

for all $t \in \Omega$ and $x,\ y \in E$.

2. Random Iterative Algorithms

In this section, we suggest and analyze a new class of iterative methods and construct some new random iterative algorithms with errors for solving the problems (1.2)–(1.4), respectively.

Lemma 2.1 ([31]). *Let $M : \Omega \times E \to CB(E)$ be an H-continuous random multivalued operator. Then, for any measurable operator $x : \Omega \to E$, the multivalued operator $M(\cdot, x(\cdot)) : \Omega \to CB(E)$ is measurable.*

Lemma 2.2 ([31]). *Let $M, V : \Omega \times E \to CB(E)$ be two measurable multivalued operators, let $\epsilon > 0$ be a constant, and let $x : \Omega \to E$ be a measurable selection of M. Then there exists a measurable selection $y : \Omega \to E$ of V such that, for any $t \in \Omega$,*

$$\|x(t) - y(t)\| \leq (1 + \epsilon) H(M(t), V(t)). \tag{2.1}$$

Lemma 2.3. *Measurable operators $x, u, v, w : \Omega \to E$ are a solution of the problem* (1.2) *if and only if*

$$p_t(x) = J^{\rho(t)}_{A_t(\cdot, w)}\left(p_t(x) - \rho(t) N_t(S_t(x), u, v)\right), \tag{2.2}$$

where $J^{\rho(t)}_{A_t(\cdot,w)} = (I + \rho(t) A_t(\cdot, w))^{-1}$ and $\rho(t) > 0$ is a real-valued random variable.

Proof. The proof directly follows from the definition of $J^{\rho(t)}_{A_t(\cdot,\mathrm{w})}$ and so it is omitted. □

Based on Lemma 2.3, we can develop a new iterative algorithm for solving the general nonlinear random equation (1.2) as follows.

Algorithm 2.4. Let $A : \Omega \times E \times E \to 2^E$ be a random multivalued operator such that for each fixed $t \in \Omega$ and $s \in E$, $A(t, \cdot, s) : E \to 2^E$ is a generalized m-accretive mapping, and Range$(p) \bigcap \mathrm{dom}\, A(t, \cdot, s) \neq \emptyset$. Let $S, p : \Omega \times E \to E$, $\eta : \Omega \times E \times E \to E$ and $N : \Omega \times E \times E \times E \to E$ be single-valued operators, and let $M, T, G : \Omega \times E \to 2^E$ be three multivalued operators, and let $\lambda : \Omega \to (0, 1]$ be a measurable step size function. Then, by Lemma 2.1 and Himmelberg [32], it is known that, for given $x_0(\cdot) \in E$, the multivalued operators $M(\cdot, x_0(\cdot)), T(\cdot, x_0(\cdot))$, and $G(\cdot, x_0(\cdot))$ are measurable and there exist measurable selections $u_0(\cdot) \in M(\cdot, x_0(\cdot)), v_0(\cdot) \in T(\cdot, x_0(\cdot))$, and $w_0(\cdot) \in G(\cdot, x_0(\cdot))$. Set

$$x_1(t) = x_0(t) - \lambda(t)\left\{p_t(x_0) - J^{\rho(t)}_{A_t(\cdot, w_0)}\left[p_t(x_0) - \rho(t) N_t(S_t(x_0), u_0, v_0)\right]\right\} + \lambda(t) e_0(t), \tag{2.3}$$

where ρ and A are the same as in Lemma 2.3 and $e_0 : \Omega \to E$ is a measurable function. Then it is easy to know that $x_1 : \Omega \to E$ is measurable. Since $u_0(t) \in M_t(x_0) \in CB(E), v_0(t) \in$

$T_t(x_0) \in CB(E)$, and $w_0(t) \in G_t(x_0) \in CB(E)$, by Lemma 2.2, there exist measurable selections $u_1(t) \in M_t(x_1), v_1(t) \in T_t(x_1)$, and $w_1(t) \in G_t(x_1)$ such that, for all $t \in \Omega$,

$$\begin{aligned}
\|u_0(t) - u_1(t)\| &\le \left(1 + \frac{1}{1}\right) H(M_t(x_0), M_t(x_1)),\\
\|v_0(t) - v_1(t)\| &\le \left(1 + \frac{1}{1}\right) H(T_t(x_0), T_t(x_1)),\\
\|w_0(t) - w_1(t)\| &\le \left(1 + \frac{1}{1}\right) H(G_t(x_0), G_t(x_1)).
\end{aligned} \tag{2.4}$$

By induction, one can define sequences $\{x_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$, and $\{w_n(t)\}$ inductively satisfying

$$\begin{aligned}
&x_{n+1}(t) = x_n(t) - \lambda(t)\left\{p_t(x_n) - J^{\rho(t)}_{A_t(\cdot,w_n)}\left[p_t(x_n) - \rho(t)N_t(S_t(x_n), u_n, v_n)\right]\right\} + \lambda(t)e_n(t),\\
&u_n(t) \in M_t(x_n), \quad \|u_n(t) - u_{n+1}(t)\| \le \left(1 + \frac{1}{n+1}\right) H(M_t(x_n), M_t(x_{n+1})),\\
&v_n(t) \in T_t(x_n), \quad \|v_n(t) - v_{n+1}(t)\| \le \left(1 + \frac{1}{n+1}\right) H(T_t(x_n), T_t(x_{n+1})),\\
&w_n(t) \in G_t(x_n), \quad \|w_n(t) - w_{n+1}(t)\| \le \left(1 + \frac{1}{n+1}\right) H(G_t(x_n), G_t(x_{n+1})),
\end{aligned} \tag{2.5}$$

where $e_n(t)$ is an error to take into account a possible inexact computation of the resolvent operator point, which satisfies the following conditions:

$$\lim_{n \to \infty} \|e_n(t)\| = 0, \qquad \sum_{n=1}^{\infty} \|e_n(t) - e_{n-1}(t)\| < \infty \tag{2.6}$$

for all $t \in \Omega$.

From Algorithm 2.4, we can get the following algorithms.

Algorithm 2.5. Suppose that E, A, η, S, M, T and λ are the same as in Algorithm 2.4. Let $G : \Omega \times E \to E$ be a random single-valued operator, $p \equiv I$ and $N(t, x, y, z) = f(t, z) + g(t, x, y)$ for all $t \in \Omega$ and $x, y, z \in E$. Then, for given measurable $x_0 : \ \Omega \to E$, one has

$$\begin{aligned}
&x_{n+1}(t) = (1 - \lambda(t))x_n(t) + \lambda(t)J^{\rho(t)}_{A_t(\cdot,G_t(x_n))}\left\{x_n(t) - \rho(t)\left[f_t(v_n) + g_t(S_t(x_n), u_n)\right]\right\} + \lambda(t)e_n(t),\\
&u_n(t) \in M_t(x_n), \quad \|u_n(t) - u_{n+1}(t)\| \le \left(1 + \frac{1}{n+1}\right) H(M_t(x_n), M_t(x_{n+1})),\\
&v_n(t) \in T_t(x_n), \quad \|v_n(t) - v_{n+1}(t)\| \le \left(1 + \frac{1}{n+1}\right) H(T_t(x_n), T_t(x_{n+1})),
\end{aligned} \tag{2.7}$$

where $e_n(t)$ is the same as in Algorithm 2.4.

Algorithm 2.6. Let $A:\Omega\times E\longrightarrow 2^E$ be a random multivalued operator such that for each fixed $t\in\Omega$, $A(t,\cdot):E\longrightarrow 2^E$ is a generalized m-accretive mapping, and $\mathrm{Range}(p)\bigcap\mathrm{dom}A(t,\cdot)\neq\emptyset$. If S, p, η, N, M, T, and λ are the same as in Algorithm 2.4, then, for given measurable $x_0:\Omega\longrightarrow E$, we have

$$\begin{gathered}
x_{n+1}(t)=x_n(t)-\lambda(t)\left\{p_t(x_n)-J^{\rho(t)}_{A_t(\cdot)}\left[p_t(x_n)-\rho(t)N_t(S_t(x_n),u_n,v_n)\right]\right\}+\lambda(t)e_n(t),\\
u_n(t)\in M_t(x_n),\quad \|u_n(t)-u_{n+1}(t)\|\leq\left(1+\frac{1}{n+1}\right)H(M_t(x_n),M_t(x_{n+1})),\\
v_n(t)\in T_t(x_n),\quad \|v_n(t)-v_{n+1}(t)\|\leq\left(1+\frac{1}{n+1}\right)H(T_t(x_n),T_t(x_{n+1})),
\end{gathered}\tag{2.8}$$

where $e_n(t)$ is the same as in Algorithm 2.4.

Remark 2.7. Algorithms 2.4–2.6 include several known algorithms of [2, 4–9, 12, 17–23, 25, 26, 29] as special cases.

3. Existence and Convergence Theorems

In this section, we will prove the convergence of the iterative sequences generated by the algorithms in Banach spaces.

Theorem 3.1. *Suppose that E is a q-uniformly smooth and separable real Banach space, $p:\Omega\times E\longrightarrow E$ is α-strongly accretive and β-Lipschitz continuous, and $A:\Omega\times E\times E\longrightarrow 2^E$ is a random multivalued operator such that for each fixed $t\in\Omega$ and $s\in E$, $A(t,\cdot,s):E\longrightarrow 2^E$ is a generalized m-accretive mapping and* $\mathrm{Range}(p)\bigcap$*dom* $A(t,\cdot,s)\neq\emptyset$. *Let $\eta:\Omega\times E\times E\longrightarrow E$ be δ-strongly monotone and τ-Lipschitz continuous, and let $S:\Omega\times E\longrightarrow E$ be a σ-Lipschitz continuous random operator, and let $N:\Omega\times E\times E\times E\longrightarrow E$ be ϱ-strongly accretive with respect to S and ϵ-Lipschitz continuous in the first argument, and μ-Lipschitz continuous in the second argument, ν-Lipschitz continuous in the third argument, respectively. Let multivalued operators $M,T,G:\Omega\times E\longrightarrow CB(E)$ be γ-H-Lipschitz continuous, ξ-H-Lipschitz continuous, ζ-H-Lipschitz continuous, respectively. If there exist real-valued random variables $\rho(t)>0$ and $\pi(t)>0$ such that, for any $t\in\Omega$, $x,y,z\in E$,*

$$\left\|J^{\rho(t)}_{A_t(\cdot,x)}(z)-J^{\rho(t)}_{A_t(\cdot,y)}(z)\right\|\leq\pi(t)\|x-y\|,\tag{3.1}$$

$$\begin{gathered}
k(t)=\pi(t)\zeta(t)+\left(1+\tau(t)\delta(t)^{-1}\right)\left(1-q\alpha(t)+c_q\beta(t)^q\right)^{1/q}<1,\\
\rho(t)\left(\mu(t)\gamma(t)+\nu(t)\xi(t)\right)+\left(1-q\rho(t)\varrho(t)+c_q\rho(t)^q\epsilon(t)^q\sigma(t)^q\right)^{1/q}<\frac{\delta(t)(1-k(t))}{\tau(t)},
\end{gathered}\tag{3.2}$$

where c_q is the same as in Lemma 1.13, then, for any $t\in\Omega$, there exist $x^*(t)\in E$, $u^*(t)\in M_t(x^*)$, $v^*(t)\in T_t(x^*)$, and $w^*(t)\in G_t(x^*)$ such that $(x^*(t),u^*(t),v^*(t),w^*(t))$ is a solution of the problem (1.2) *and*

$$x_n(t)\longrightarrow x^*(t),\ u_n(t)\longrightarrow u^*(t),\ \mathrm{v}_n(t)\longrightarrow v^*(t), w_n(t)\longrightarrow w^*(t)\tag{3.3}$$

as $n \to \infty$, where $\{x_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$ and $\{w_n(t)\}$ are iterative sequences generated by Algorithm 2.4.

Proof. It follows from (2.5), Lemma 1.15 and (3.1) that

$$
\begin{aligned}
&\|x_{n+1}(t) - x_n(t)\| \\
&\quad= \Big\| x_n(t) - \lambda(t)\Big\{ p_t(x_n) - J^{\rho(t)}_{A_t(\cdot,w_n)}\left[p_t(x_n) - \rho(t)N_t(S_t(x_n),u_n,v_n)\right]\Big\} + \lambda(t)e_n(t) \\
&\qquad - x_{n-1}(t) + \lambda(t)\Big\{ p_t(x_{n-1}) - J^{\rho(t)}_{A_t(\cdot,w_{n-1})}\left[p_t(x_{n-1}) - \rho(t)N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\right]\Big\} \\
&\qquad -\lambda(t)e_{n-1}(t)\Big\| \\
&\quad\le \left\| x_n(t) - x_{n-1}(t) - \lambda(t)\left(p_t(x_n) - p_t(x_{n-1})\right)\right\| \\
&\qquad + \lambda(t)\Big\| J^{\rho(t)}_{A_t(\cdot,w_n)}\left[p_t(x_n) - \rho(t)N_t(S_t(x_n),u_n,v_n)\right] \\
&\qquad\qquad -J^{\rho(t)}_{A_t(\cdot,w_{n-1})}\left[p_t(x_{n-1}) - \rho(t)N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\right]\Big\| + \lambda(t)\|e_n(t) - e_{n-1}(t)\| \\
&\quad\le \left\| x_n(t) - x_{n-1}(t) - \lambda(t)\left(p_t(x_n) - p_t(x_{n-1})\right)\right\| \\
&\qquad + \lambda(t)\Big\| J^{\rho(t)}_{A_t(\cdot,w_n)}\left[p_t(x_n) - \rho(t)N_t(S_t(x_n),u_n,v_n)\right] \\
&\qquad\qquad -J^{\rho(t)}_{A_t(\cdot,w_n)}\left[p_t(x_{n-1}) - \rho(t)N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\right]\Big\| \\
&\qquad + \lambda(t)\Big\| J^{\rho(t)}_{A_t(\cdot,w_n)}\left[p_t(x_{n-1}) - \rho(t)N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\right] \\
&\qquad\qquad -J^{\rho(t)}_{A_t(\cdot,w_{n-1})}\left[p_t(x_{n-1}) - \rho(t)N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\right]\Big\| + \lambda(t)\|e_n(t) - e_{n-1}(t)\| \\
&\quad\le \left\| x_n(t) - x_{n-1}(t) - \lambda(t)\left(p_t(x_n) - p_t(x_{n-1})\right)\right\| \\
&\qquad + \lambda(t)\cdot\frac{\tau(t)}{\delta(t)}\big\| p_t(x_n) - \rho(t)N_t(S_t(x_n),u_n,v_n) \\
&\qquad\qquad\qquad - \left[p_t(x_{n-1}) - \rho(t)N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\right]\big\| \\
&\qquad + \lambda(t)\pi(t)\|w_n - w_{n-1}\| + \lambda(t)\|e_n(t) - e_{n-1}(t)\| \\
&\quad\le \left\| x_n(t) - x_{n-1}(t) - \lambda(t)\left(p_t(x_n) - p_t(x_{n-1})\right)\right\| \\
&\qquad + \frac{\lambda(t)\tau(t)}{\delta(t)}\Big\{ \left\| x_n(t) - x_{n-1}(t) - \left(p_t(x_n) - p_t(x_{n-1})\right)\right\| \\
&\qquad + \left\| x_n(t) - x_{n-1}(t) - \rho(t)\left(N_t(S_t(x_n),u_n,v_n) - N_t(S_t(x_{n-1}),u_n,v_n)\right)\right\| \\
&\qquad + \rho(t)\|N_t(S_t(x_{n-1}),u_n,v_n) - N_t(S_t(x_{n-1}),u_{n-1},v_n)\| \\
&\qquad +\rho(t)\|N_t(S_t(x_{n-1}),u_{n-1},v_n) - N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\|\Big\} \\
&\qquad + \lambda(t)\pi(t)\|w_n - w_{n-1}\| + \lambda(t)\|e_n(t) - e_{n-1}(t)\|
\end{aligned}
$$

$$\begin{aligned}
&\leq (1-\lambda(t))\|x_n(t)-x_{n-1}(t)\| \\
&\quad + \lambda(t)\left(1+\frac{\tau(t)}{\delta(t)}\right)\left\|x_n(t)-x_{n-1}(t)-\left(p_t(x_n)-p_t(x_{n-1})\right)\right\| \\
&\quad + \frac{\lambda(t)\tau(t)}{\delta(t)}\left\{\left\|x_n(t)-x_{n-1}(t)-\rho(t)(N_t(S_t(x_n),u_n,v_n)-N_t(S_t(x_{n-1}),u_n,v_n))\right\|\right. \\
&\quad + \rho(t)\|N_t(S_t(x_{n-1}),u_n,v_n)-N_t(S_t(x_{n-1}),u_{n-1},v_n)\| \\
&\quad \left. +\rho(t)\|N_t(S_t(x_{n-1}),u_{n-1},v_n)-N_t(S_t(x_{n-1}),u_{n-1},v_{n-1})\|\right\} \\
&\quad + \lambda(t)\pi(t)\|w_n-w_{n-1}\|+\lambda(t)\|e_n(t)-e_{n-1}(t)\|.
\end{aligned} \tag{3.4}$$

Since p is strongly accretive and Lipschitz continuous,

$$\begin{aligned}
&\left\|x_n(t)-x_{n-1}(t)-(p_t(x_n)-p_t(x_{n-1}))\right\|^q \\
&\quad \leq \|x_n(t)-x_{n-1}(t)\|^q - q\langle p_t(x_n)-p_t(x_{n-1}), j_q(x_n(t)-x_{n-1}(t))\rangle \\
&\qquad + c_q\left\|p_t(x_n)-p_t(x_{n-1})\right\|^q \\
&\quad \leq \left(1-q\alpha(t)+c_q\beta(t)^q\right)\|x_n(t)-x_{n-1}(t)\|^q,
\end{aligned} \tag{3.5}$$

that is,

$$\begin{aligned}
&\left\|x_n(t)-x_{n-1}(t)-\left(p_t(x_n)-p_t(x_{n-1})\right)\right\| \\
&\quad \leq \left(1-q\alpha(t)+c_q\beta(t)^q\right)^{1/q}\|x_n(t)-x_{n-1}(t)\|,
\end{aligned} \tag{3.6}$$

where c_q is the same as in Lemma 1.13. Also from the strongly accretivity of N with respect to S and the Lipschitz continuity of N in the first argument, we have

$$\begin{aligned}
&\left\|x_n(t)-x_{n-1}(t)-\rho(t)(N_t(S_t(x_n),u_n,v_n)-N_t(S_t(x_{n-1}),u_n,v_n))\right\| \\
&\quad \leq \left(1-q\rho(t)\varrho(t)+c_q\rho(t)^q\epsilon(t)^q\sigma(t)^q\right)^{1/q}\|x_n(t)-x_{n-1}(t)\|.
\end{aligned} \tag{3.7}$$

By Lipschitz continuity of N in the second and third argument, and H-Lipschitz continuity of $T,\ M,\ G$, we obtain

$$\begin{aligned}
&\|N_t(S_t(x_{n-1}),u_n,v_n)-N_t(S_t(x_{n-1}),u_{n-1},v_n)\| \\
&\quad \leq \mu(t)\|u_n-u_{n-1}\| \leq \mu(t)\left(1+\frac{1}{n}\right)H(M_t(x_{n-1})-M_t(x_n)) \\
&\quad \leq \left(1+\frac{1}{n}\right)\mu(t)\gamma(t)\|x_n(t)-x_{n-1}(t)\|,
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
&\|N_t(S_t(x_{n-1}), u_{n-1}, v_n) - N_t(S_t(x_{n-1}), u_{n-1}, v_{n-1})\| \\
&\quad \le \nu(t)\|v_n - v_{n-1}\| \le \nu(t)\left(1 + \frac{1}{n}\right) H(T_t(x_{n-1}) - T_t(x_n)) \\
&\quad \le \left(1 + \frac{1}{n}\right)\nu(t)\xi(t)\|x_n(t) - x_{n-1}(t)\|,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\|w_n - w_{n-1}\| &\le \left(1 + \frac{1}{n}\right) H(G_t(x_{n-1}) - G_t(x_n)) \\
&\le \left(1 + \frac{1}{n}\right)\zeta(t)\|x_n(t) - x_{n-1}(t)\|.
\end{aligned} \tag{3.10}$$

Using (3.6)–(3.10) in (3.4), we have, for all $t \in \Omega$,

$$\|x_{n+1}(t) - x_n(t)\| \le \theta(t,n)\|x_n(t) - x_{n-1}(t)\| + \lambda(t)\|e_n(t) - e_{n-1}(t)\|, \tag{3.11}$$

where

$$\begin{aligned}
\theta(t,n) &= 1 - \lambda(t) + \lambda(t)\kappa(t,n), \\
\kappa(t,n) &= \left(1 + \tau(t)\delta(t)^{-1}\right)\left(1 - q\alpha(t) + c_q\beta(t)^q\right)^{1/q} \\
&\quad + \tau(t)\delta(t)^{-1}\Big[\left(1 - q\rho(t)\varrho(t) + c_q\rho(t)^q\epsilon(t)^q\sigma(t)^q\right)^{1/q} \\
&\qquad + \left(1 + \frac{1}{n}\right)\rho(t)\left(\mu(t)\gamma(t) + \nu(t)\xi(t)\right)\Big] + \left(1 + \frac{1}{n}\right)\pi(t)\varsigma(t).
\end{aligned} \tag{3.12}$$

Let

$$\begin{aligned}
\kappa(t) &= \pi(t)\zeta(t) + \left(1 + \tau(t)\delta(t)^{-1}\right)\left(1 - q\alpha(t) + c_q\beta(t)^q\right)^{1/q} \\
&\quad + \tau(t)\delta(t)^{-1}\Big[\rho(t)\left(\mu(t)\gamma(t) + \nu(t)\xi(t)\right) + \left(1 - q\rho(t)\varrho(t) + c_q\rho(t)^q\varepsilon(t)^q\sigma(t)^q\right)^{1/q}\Big], \\
\theta(t) &= 1 - \lambda(t) + \lambda(t)\kappa(t).
\end{aligned} \tag{3.13}$$

Then $\kappa(t,n) \to \kappa(t)$, $\theta(t,n) \to \theta(t)$ as $n \to \infty$. From the condition (3.2), we know that $0 < \theta(t) < 1$ for all $t \in \Omega$ and so there exists a positive measurable function $\widehat{\theta}(t) \in (\theta(t), 1)$

such that $\theta(t,n) \leq \widehat{\theta}(t)$ for all $n \geq n_0$ and $t \in \Omega$. Therefore, for all $n > n_0$, by (3.11), we now know that, for all $t \in \Omega$,

$$\begin{aligned}
&\|x_{n+1}(t) - x_n(t)\| \\
&\quad\leq \widehat{\theta}(t)\|x_n(t) - x_{n-1}(t)\| + \lambda(t)\|e_n(t) - e_{n-1}(t)\| \\
&\quad\leq \widehat{\theta}(t)\Big[\widehat{\theta}(t)\|x_{n-1}(t) - x_{n-2}(t)\| + \lambda(t)\|e_{n-1}(t) - e_{n-2}(t)\|\Big] + \lambda(t)\|e_n(t) - e_{n-1}(t)\| \\
&\quad= \widehat{\theta}(t)^2\|x_{n-1} - x_{n-2}\| + \lambda(t)\Big[\widehat{\theta}(t)\|e_{n-1}(t) - e_{n-2}(t)\| + \|e_n(t) - e_{n-1}(t)\|\Big] \\
&\quad\leq \cdots \\
&\quad\leq \widehat{\theta}(t)^{n-n_0}\|x_{n_0+1} - x_{n_0}\| + \lambda(t)\sum_{i=1}^{n-n_0}\widehat{\theta}(t)^{i-1}\big\|e_{n-(i-1)} - e_{n-i}\big\|,
\end{aligned} \tag{3.14}$$

which implies that, for any $m \geq n > n_0$,

$$\begin{aligned}
\|x_m(t) - x_n(t)\| &\leq \sum_{j=n}^{m-1}\big\|x_{j+1}(t) - x_j(t)\big\| \\
&\leq \sum_{j=n}^{m-1}\widehat{\theta}(t)^{j-n_0}\|x_{n_0+1}(t) - x_{n_0}(t)\| \\
&\quad + \lambda(t)\sum_{j=n}^{m-1}\sum_{i=1}^{j-n_0}\widehat{\theta}(t)^{i-1}\big\|e_{n-(i-1)}(t) - e_{n-i}(t)\big\|.
\end{aligned} \tag{3.15}$$

Since $0 < \lambda(t) \leq 1$ and $\widehat{\theta}(t) < 1$ for all $t \in \Omega$, it follows from (2.6) and (3.15) that $\lim_{n\to\infty}\|x_m(t) - x_n(t)\| = 0$ and so $\{x_n(t)\}$ is a Cauchy sequence. Setting $x_n(t) \to x^*(t)$ as $n \to \infty$ for all $t \in \Omega$. From (3.8)–(3.10), we know that $\{u_n(t)\}$, $\{v_n(t)\}$, $\{w_n(t)\}$ are also Cauchy sequences. Hence there exist $u^*(t),\ v^*(t),\ w^*(t) \in E$ such that $u_n(t) \to u^*(t)$, $v_n(t) \to v^*(t)$, $w_n(t) \to w^*(t)$ as $n \to \infty$.

Now, we show that $u^*(t) \in M_t(x^*)$. In fact, we have

$$\begin{aligned}
d(u^*(t), M_t(x^*)) &= \inf\big\{\big\|u^*(t) - y\big\| : y \in M_t(x^*)\big\} \\
&\leq \|u^*(t) - u_n(t)\| + d(u_n(t), M_t(x^*)) \\
&\leq \|u^*(t) - u_n(t)\| + H(M_t(x_n), M_t(x^*)) \\
&\leq \|u^*(t) - u_n(t)\| + \gamma(t)\|x_n(t) - x^*(t)\| \longrightarrow 0.
\end{aligned} \tag{3.16}$$

This implies that $u^*(t) \in M_t(x^*)$. Similarly, we have $v^*(t) \in T_t(x^*)$ and $w^*(t) \in G_t(x^*)$. Therefore, from (2.5), (2.6) and the continuity of $J^{\rho}_{A_t(\cdot,w)}(t)$, p, N, and S, we have

$$p_t(x^*) = J^{\rho(t)}_{A_t(\cdot,w^*)}\big(p_t(x^*) - \rho(t)N_t(S_t(x^*), u^*, v^*)\big). \tag{3.17}$$

By Lemma 2.3, now we know that $(x^*(t), u^*(t), v^*(t), w^*(t))$ is a solution of the problem (1.2). This completes the proof. □

Remark 3.2. If E is a 2-uniformly smooth Banach space and there exists a measurable function $\rho : \Omega \to (0, \infty)$ such that

$$\begin{aligned}
&k(t) = \pi(t)\zeta(t) + \left(1 + \tau(t)\delta(t)^{-1}\right)\sqrt{1 - 2\alpha(t) + C_2\beta(t)^2} < 1,\\
&h(t) = \mu(t)\gamma(t) + \nu(t)\xi(t) < \sqrt{C_2}\epsilon(t)\sigma(t),\\
&\rho(t) < \frac{\delta(t)(1 - k(t))}{\tau(t)h(t)},\\
&\left|\rho(t) - \frac{\varrho(t)\tau(t) - h(t)\delta(t)(1 - k(t))}{\tau(t)\left(C_2\epsilon(t)^2\sigma(t)^2 - h(t)^2\right)}\right|\\
&\qquad < \frac{\sqrt{\left[\varrho(t)\tau(t) - h(t)\delta(t)(1 - k(t))\right]^2 - \left(C_2\epsilon(t)^2\sigma(t)^2 - h(t)^2\right)\left[\tau(t)^2 - \delta(t)^2(1 - k(t))^2\right]}}{\tau(t)\left(C_2\epsilon(t)^2\sigma(t)^2 - h(t)^2\right)},\\
&\varrho(t)\tau(t) > h(t)\delta(t)(1 - k(t)) + \sqrt{\left(C_2\epsilon(t)^2\sigma(t)^2 - h(t)^2\right)\left[\tau(t)^2 - \delta(t)^2(1 - k(t))^2\right]},
\end{aligned} \tag{3.18}$$

then (3.2) holds. We note that Hilbert spaces and L_p(or l_p) spaces, $2 \le p < \infty$, are 2-uniformly smooth.

From Theorem 3.1, we can get the following theorems.

Theorem 3.3. *Let E, η, S, M, T, and λ be the same as in Theorem 3.1. Assume that $A : \Omega \times E \times E \to 2^E$ is a random multivalued operator such that, for each fixed $t \in \Omega$ and $s \in E$, $A(t, \cdot, s) : E \to 2^E$ is a generalized m-accretive mapping. Let $f : \Omega \times E \to E$ be ν-Lipschitz continuous, let $S : \Omega \times E \to E$ be a σ-Lipschitz continuous random operator, let $G : \Omega \times E \to E$ be ζ-Lipschitz continuous, and let $g : \Omega \times E \times E \to E$ be ϱ-strongly accretive with respect to S and ϵ-Lipschitz continuous in the first argument and μ-Lipschitz continuous in the second argument, respectively. If there exist real-valued random variables $\rho(t) > 0$ and $\pi(t) > 0$ such that* (3.1) *holds and*

$$\rho(t)\left(\mu(t)\gamma(t) + \nu(t)\xi(t)\right) + \left(1 - q\rho(t)\varrho(t) + c_q\rho(t)^q\epsilon(t)^q\sigma(t)^q\right)^{1/q} < \frac{\delta(t)(1 - \pi(t)\zeta(t))}{\tau(t)} \tag{3.19}$$

for all $t \in \Omega$, where c_q is the same as in Lemma 1.13, then, for any $t \in \Omega$, the iterative sequences $\{x_n(t)\}$, $\{u_n(t)\}$, and $\{v_n(t)\}$ defined by Algorithm 2.5 converge strongly to the solution $(x^*(t), u^*(t), v^*(t))$ of the problem (1.3).

Theorem 3.4. *Suppose that E, p, S, η, N, M, T, and λ are the same as in Algorithm 2.4. Let $A : \Omega \times E \to 2^E$ be a random multivalued operator such that, for each fixed $t \in \Omega$, $A(t, \cdot) : E \to 2^E$*

is a generalized m-accretive mapping and $\mathrm{Range}(p)\bigcap \mathrm{dom}A(t,\cdot)\neq\emptyset$*. If there exists a real-valued random variable* $\rho(t)>0$ *such that, for any* $t\in\Omega$,

$$\begin{aligned} k(t) &= \left(1+\tau(t)\delta(t)^{-1}\right)\left(1-q\alpha(t)+c_q\beta(t)^q\right)^{1/q}<1,\\ \rho(t)\left(\mu(t)\gamma(t)+\nu(t)\xi(t)\right) &+ \left(1-q\rho(t)\varrho(t)+c_q\rho(t)^q\epsilon(t)^q\sigma(t)^q\right)^{1/q}<\delta(t)\tau(t)^{-1}(1-k(t)), \end{aligned} \tag{3.20}$$

where c_q *is the same as in Lemma 1.13, then, for any* $t\in\Omega$*, the iterative sequences* $\{x_{\mathrm{n}}(t)\}$, $\{u_n(t)\}$*, and* $\{v_n(t)\}$ *defined by Algorithm 2.6 converge strongly to the solution* $(x^*(t),u^*(t),v^*(t))$ *of the problem* (1.4).

Remark 3.5. For an appropriate choice of the mappings S, p, A, M, T, G, N, η and the space E, Theorems 3.1–3.4 include many known results of generalized variational inclusions as special cases (see [2, 4–9, 12, 17–23, 25, 26, 29] and the references therein).

Acknowledgment

This work was supported by the Scientific Research Fund of Sichuan Provincial Education Department (2006A106) and the Sichuan Youth Science and Technology Foundation (08ZQ026-008). This work was supported by the Korea Research Foundation Grant funded by the Korean Government (KRF-2008-313-C00050).

References

[1] P. Hartman and G. Stampacchia, "On some non-linear elliptic differential-functional equations," *Acta Mathematica*, vol. 115, no. 1, pp. 271–310, 1966.

[2] R. P. Agarwal, Y. J. Cho, and N.-J. Huang, "Generalized nonlinear variational inclusions involving maximal η-monotone mappings," in *Nonlinear Analysis and Applications: to V. Lakshmikantham on His 80th Birthday. Vol. 1, 2*, pp. 59–73, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.

[3] R. P. Agarwal, M. F. Khan, D. O'Regan, and Salahuddin, "On generalized multivalued nonlinear variational-like inclusions with fuzzy mappings," *Advances in Nonlinear Variational Inequalities*, vol. 8, no. 1, pp. 41–55, 2005.

[4] X. P. Ding, "Generalized quasi-variational-like inclusions with nonconvex functionals," *Applied Mathematics and Computation*, vol. 122, no. 3, pp. 267–282, 2001.

[5] N.-J. Huang, "Generalized nonlinear variational inclusions with noncompact valued mappings," *Applied Mathematics Letters*, vol. 9, no. 3, pp. 25–29, 1996.

[6] N.-J. Huang and Y.-P. Fang, "A new class of general variational inclusions involving maximal η-monotone mappings," *Publicationes Mathematicae Debrecen*, vol. 62, no. 1-2, pp. 83–98, 2003.

[7] N. J. Huang, Y. P. Fang, and C. X. Deng, "Nonlinear variational inclusions involving generalized m-accretive mappings," in *Proceedings of the 9th Bellman Continuum International Workshop on Uncertain Systems and Soft Computing*, pp. 323–327, Beijing, China, July 2002.

[8] N.-J. Huang, "Nonlinear implicit quasi-variational inclusions involving generalized m-accretive mappings," *Archives of Inequalities and Applications*, vol. 2, no. 4, pp. 413–425, 2004.

[9] M.-M. Jin and Q.-K. Liu, "Nonlinear quasi-variational inclusions involving generalized m-accretive mappings," *Nonlinear Functional Analysis and Applications*, vol. 9, no. 3, pp. 485–494, 2004.

[10] H.-Y. Lan, J. K. Kim, and N. J. Huang, "On the generalized nonlinear quasi-variational inclusions involving non-monotone set-valued mappings," *Nonlinear Functional Analysis and Applications*, vol. 9, no. 3, pp. 451–465, 2004.

[11] H.-Y. Lan, Q.-K. Liu, and J. Li, "Iterative approximation for a system of nonlinear variational inclusions involving generalized m-accretive mappings," *Nonlinear Analysis Forum*, vol. 9, no. 1, pp. 33–42, 2004.

[12] L.-W. Liu and Y.-Q. Li, "On generalized set-valued variational inclusions," *Journal of Mathematical Analysis and Applications*, vol. 261, no. 1, pp. 231–240, 2001.

[13] R. U. Verma, M. F. Khan, and Salahuddin, "Generalized setvalued nonlinear mixed quasivariational-like inclusions with fuzzy mappings," *Advances in Nonlinear Variational Inequalities*, vol. 8, no. 2, pp. 11–37, 2005.

[14] H.-Y. Lan, Z.-Q. He, and J. Li, "Generalized nonlinear fuzzy quasi-variational-like inclusions involving maximal η-monotone mappings," *Nonlinear Analysis Forum*, vol. 8, no. 1, pp. 43–54, 2003.

[15] N. J. Huang and Y. P. Fang, "Generalized m-accretive mappings in Banach spaces," *Journal of Sichuan University*, vol. 38, no. 4, pp. 591–592, 2001.

[16] R. Ahmad and F. F. Bazán, "An iterative algorithm for random generalized nonlinear mixed variational inclusions for random fuzzy mappings," *Applied Mathematics and Computation*, vol. 167, no. 2, pp. 1400–1411, 2005.

[17] S. S. Chang, *Variational Inequality and Complementarity Problem Theory with Applications*, Shanghai Scientific and Technological Literature, Shanghai, China, 1991.

[18] S. S. Chang and N. J. Huang, "Generalized random multivalued quasi-complementarity problems," *Indian Journal of Mathematics*, vol. 35, no. 3, pp. 305–320, 1993.

[19] Y. J. Cho, N. J. Huang, and S. M. Kang, "Random generalized set-valued strongly nonlinear implicit quasi-variational inequalities," *Journal of Inequalities and Applications*, vol. 5, no. 5, pp. 515–531, 2000.

[20] A. Ganguly and K. Wadhwa, "On random variational inequalities," *Journal of Mathematical Analysis and Applications*, vol. 206, no. 1, pp. 315–321, 1997.

[21] N.-J. Huang, "Random generalized nonlinear variational inclusions for random fuzzy mappings," *Fuzzy Sets and Systems*, vol. 105, no. 3, pp. 437–444, 1999.

[22] N.-J. Huang and Y. J. Cho, "Random completely generalized set-valued implicit quasi-variational inequalities," *Positivity*, vol. 3, no. 3, pp. 201–213, 1999.

[23] N.-J. Huang, X. Long, and Y. J. Cho, "Random completely generalized nonlinear variational inclusions with non-compact valued random mappings," *Bulletin of the Korean Mathematical Society*, vol. 34, no. 4, pp. 603–615, 1997.

[24] M. A. Noor and S. A. Elsanousi, "Iterative algorithms for random variational inequalities," *Panamerican Mathematical Journal*, vol. 3, no. 1, pp. 39–50, 1993.

[25] Y. J. Cho, S. H. Shim, N. J. Huang, and S. M. Kang, "Generalized strongly nonlinear implicit quasi-variational inequalities for fuzzy mappings," in *Set Valued Mappings with Applications in Nonlinear Analysis*, vol. 4 of *Series in Mathematical Analysis and Applications*, pp. 63–77, Taylor & Francis, London, UK, 2002.

[26] Y. J. Cho and H.-Y. Lan, "Generalized nonlinear random (A, η)-accretive equations with random relaxed cocoercive mappings in Banach spaces," *Computers & Mathematics with Applications*, vol. 55, no. 9, pp. 2173–2182, 2008.

[27] H.-Y. Lan, N.-J. Huang, and Y. J. Cho, "A new method for nonlinear variational inequalities with multi-valued mappings," *Archives of Inequalities and Applications*, vol. 2, no. 1, pp. 73–84, 2004.

[28] R. U. Verma, "A class of projection-contraction methods applied to monotone variational inequalities," *Applied Mathematics Letters*, vol. 13, no. 8, pp. 55–62, 2000.

[29] A. Hassouni and A. Moudafi, "A perturbed algorithm for variational inclusions," *Journal of Mathematical Analysis and Applications*, vol. 185, no. 3, pp. 706–712, 1994.

[30] H. K. Xu, "Inequalities in Banach spaces with applications," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 16, no. 12, pp. 1127–1138, 1991.

[31] S. S. Chang, *Fixed Point Theory with Applications*, Chongqing Publishing, Chongqing, China, 1984.

[32] C. J. Himmelberg, "Measurable relations," *Fundamenta Mathematicae*, vol. 87, pp. 53–72, 1975.