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Research Article

Infinitely Many Periodic Solutions for Variable Exponent Systems

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We mainly consider the system $-\Delta_{p(x)}u = f(v) + h(u)$ in $\mathbb{R}$, $-\Delta_{q(x)}v = g(u) + \omega(v)$ in $\mathbb{R}$, where $1 < p(x), q(x) \in C^1(\mathbb{R})$ are periodic functions, and $-\Delta_{p(x)}u = -(|u'|^{p(x)-2}u')'$ is called $p(x)$-Laplacian. We give the existence of infinitely many periodic solutions under some conditions.

1. Introduction

The study of differential equations and variational problems with variable exponent growth conditions has been a new and interesting topic. Many results have been obtained on this kind of problems, for example [1–18]. On the applied background, we refer to [1, 3, 11, 18]. In this paper, we mainly consider the existence of infinitely many periodic solutions for the system

$$(P)\begin{cases} -\Delta_{p(x)}u = f(v) + h(u) & \text{in } \mathbb{R}, \\ -\Delta_{q(x)}v = g(u) + \omega(v) & \text{in } \mathbb{R}, \end{cases} \tag{1.1}$$

where $p(x), q(x) \in C^1(\mathbb{R})$ are functions. The operator $-\Delta_{p(x)}u = -(|u'|^{p(x)-2}u')'$ is called one-dimensional $p(x)$-Laplacian. Especially, if $p(x) \equiv p$ (a constant) and $q(x) \equiv q$ (a constant), then (P) is the well-known constant exponent system.

(u, v) is called a solution of (P), if $u, v \in C^1(\mathbb{R})$, $|u'|^{p(x)-2}u'$ and $|v'|^{p(x)-2}v'$ are absolute continuous and satisfy (P) almost every where.

In [19], the authors consider the existence of positive weak solutions for the following constant exponent problems:

$$(I)\begin{cases} -\Delta_p u = \lambda f(v), & \text{in } \Omega, \\ -\Delta_p v = \lambda g(u), & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

The first eigenfunction is used to construct the subsolution of constant exponent problems successfully. Under the condition that λ is large enough and

$$\lim_{u\to+\infty} \frac{f\left[M(g(u))^{1/(p-1)}\right]}{u^{p-1}} = 0, \quad \text{for every } M > 0, \tag{1.3}$$

the authors give the existence of positive solutions for problem (I).

In [20], the author considers the existence and nonexistence of positive weak solution to the following constant exponent elliptic system:

$$(II)\begin{cases} -\Delta_p u = \lambda u^{\alpha} v^{\gamma}, & \text{in } \Omega, \\ -\Delta_q v = \lambda u^{\delta} v^{\beta}, & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega. \end{cases} \tag{1.4}$$

The first eigenfunction is used to construct the subsolution of constant exponent problems successfully.

Because of the nonhomogeneity of $p(x)$-Laplacian, $p(x)$-Laplacian problems are more complicated than those of p-Laplacian. Maybe the first eigenvalue and the first eigenfunction of $p(x)$-Laplacian do not exist (see [6]). Even if the first eigenfunction of $p(x)$-Laplacian exists, because of the nonhomogeneity of $p(x)$-Laplacian, the first eigenfunction cannot be used to construct the subsolution of $p(x)$-Laplacian problems.

There are many papers on the existence of periodic solutions for p-Laplacian elliptic systems, for example [21–24]. The results on the periodic solutions for variable exponent systems are rare. Through a new method of constructing sub-supersolution, this paper gives the existence of infinitely many periodic solutions for problem (P).

2. Main Results and Proofs

At first, we give an existence of positive solutions for variable exponent systems on bounded domain via sub-super-solution method. The result itself has dependent value.

Denote $\Omega_R = (-R, R)$. Let us consider the existence of positive solutions of the following:

$$(P_1)\begin{cases} -\Delta_{p(x)}u = f(v) + h(u), & \text{in } \Omega_R, \\ -\Delta_{q(x)}v = g(u) + \omega(v), & \text{in } \Omega_R, \\ u = v = 0, & \text{on } \partial\Omega_R. \end{cases} \tag{2.1}$$

Write $z^+ = \sup_{x\in\mathbb{R}} z(x)$, $z^- = \inf_{x\in\mathbb{R}} z(x)$, for any $z \in C(\mathbb{R})$. Assume that

(H_1) $p(x), q(x) \in C^1(\mathbb{R})$ satisfy

$$1 < p^- \le p^+ < \infty, \qquad \sup|p'(x)| < \infty, \qquad 1 < q^- \le q^+ < \infty, \qquad \sup|q'(x)| < \infty. \tag{2.2}$$

(H_2) $f, g, h, \omega : [0, +\infty) \to \mathbb{R}$ are C^1, monotone functions such that

$$\lim_{t\to+\infty} f(t) = \lim_{t\to+\infty} g(t) = \lim_{t\to+\infty} h(t) = \lim_{t\to+\infty} \omega(t) = +\infty. \tag{2.3}$$

(H_3) For any positive constant M, there are $\lim_{t\to+\infty} f[M(g(t))^{1/(q^- -1)}]/t^{p^- -1} = 0$.

(H_4) $\lim_{t\to+\infty} h(t)/t^{p^- -1} = \lim_{t\to+\infty} \omega(t)/t^{q^- -1} = 0$.

(H_5) f, g, h, and ω are odd functions such that $f(0) = g(0) = h(0) = \omega(0) = 0$, $p(x)$ and $q(x)$ are even, and T is a periodic of p and q, namely, $p(x) = p(x+T)$, $q(x) = q(x+T)$, for all $x \in \mathbb{R}$.

Note. In [14], the present author discussed the existence of solutions of (P_1), under the conditions that (P_1) is radial, $p(x) = q(x)$, and $h = \omega \equiv 0$. Because of the nonhomogeneity of variable exponent problems, variable exponent problems are more complicated than constant exponent problems, and many results and methods for constant exponent problems are invalid for variable exponent problems. In many cases, the radial symmetric conditions are effective to deal with variable exponent problems. There are many results about the radial variable exponent problems (see [4, 14, 16]), but the following Theorem 2.1 does not assume any symmetric conditions.

We will establish.

Theorem 2.1. *If (H_1)–(H_4) hold, then (P_1) possesses a positive solution, when R is sufficiently large.*

Proof. If we can construct a positive subsolution (ϕ_1, ϕ_2) and supersolution (z_1, z_2) of (P_1), namely,

$$\begin{aligned} -\Delta_{p(x)}\phi_1 &\le f(\phi_2) + h(\phi_1), \qquad -\Delta_{q(x)}\phi_2 \le g(\phi_1) + \omega(\phi_2), \quad \text{for a.e. } x \in \mathbb{R}, \\ -\Delta_{p(x)}z_1 &\ge f(z_2) + h(z_1), \qquad -\Delta_{q(x)}z_2 \ge g(z_1) + \omega(z_2), \quad \text{for a.e. } x \in \mathbb{R}, \end{aligned} \tag{2.4}$$

which satisfy $\phi_1 \le z_1$ and $\phi_2 \le z_2$, then (P_1) possesses a positive solution (see [5]).

Step 1. We will construct a subsolution of (P_1).

Denfine

$$\phi_1(x)=\begin{cases} e^{-k_2(x-R)}-1, & R-a<x\le R,\\ e^{ak_2}-1+\displaystyle\int_x^{R-a}\left(k_2e^{ak_2}\right)^{(p(R-a)-1)/(p(r)-1)}\left[\sin\left(\varepsilon_2(r-(R-a))+\frac{\pi}{2}\right)\right]^{1/(p(r)-1)}dr, & R-a-\dfrac{\pi}{2\varepsilon_2}<x\le R-a,\\ e^{ak_2}-1+\displaystyle\int_{R-a-\pi/2\varepsilon_2}^{R-a}\left(k_2e^{ak_2}\right)^{(p(R-a)-1)/(p(r)-1)}\left[\sin\left(\varepsilon_2(r-(R-a))+\frac{\pi}{2}\right)\right]^{1/(p(r)-1)}dr, & -R+a+\dfrac{\pi}{2\varepsilon_1}<x\le R-a-\dfrac{\pi}{2\varepsilon_2},\\ e^{ak_1}-1+\displaystyle\int_{-R+a}^{x}\left(k_1e^{ak_1}\right)^{(p(-R+a)-1)/(p(r)-1)}\left[\sin\left(\frac{\pi}{2}-\varepsilon_1(r-(-R+a))\right)\right]^{1/(p(r)-1)}dr, & -R+a\le x\le -R+a+\dfrac{\pi}{2\varepsilon_1},\\ e^{k_1(x+R)}-1, & -R\le x<-R+a. \end{cases}$$

$$\phi_2(x)=\begin{cases} e^{-k_4(x-R)}-1, & R-a<x\le R,\\ e^{ak_4}-1+\displaystyle\int_x^{R-a}\left(k_4e^{ak_4}\right)^{(q(R-a)-1)/(q(r)-1)}\left[\sin\left(\varepsilon_4(r-(R-a))+\frac{\pi}{2}\right)\right]^{1/(q(r)-1)}dr, & R-a-\dfrac{\pi}{2\varepsilon_4}<x\le R-a,\\ e^{ak_4}-1+\displaystyle\int_{R-a-\pi/2\varepsilon_4}^{R-a}\left(k_4e^{ak_4}\right)^{(q(R-a)-1)/(q(r)-1)}\left[\sin\left(\varepsilon_4(r-(R-a))+\frac{\pi}{2}\right)\right]^{1/(q(r)-1)}dr, & -R+a+\dfrac{\pi}{2\varepsilon_3}<x\le R-a-\dfrac{\pi}{2\varepsilon_4},\\ e^{ak_3}-1+\displaystyle\int_{-R+a}^{x}\left(k_3e^{ak_3}\right)^{(q(-R+a)-1)/(q(r)-1)}\left[\sin\left(\frac{\pi}{2}-\varepsilon_3(r-(-R+a))\right)\right]^{1/(q(r)-1)}dr, & -R+a\le x\le -R+a+\dfrac{\pi}{2\varepsilon_3},\\ e^{k_3(x+R)}-1, & -R\le x<-R+a, \end{cases} \tag{2.5}$$

where

$$a=\min\left\{\frac{\inf p(x)-1}{4(\sup|p'(x)|+1)},\frac{\inf q(x)-1}{4(\sup|q'(x)|+1)}\right\},\qquad b=\min\{f(0),g(0),h(0),\omega(0),-1\},$$
$$\varepsilon_i=k_i^{-p^+}e^{-ak_ip^+},\quad i=1,2,\qquad \varepsilon_i=k_i^{-q^+}e^{-ak_iq^+},\quad i=3,4;\qquad R>\frac{\pi}{\varepsilon_i},\quad i=1,2,3,4, \tag{2.6}$$

k_1 and k_2 satisfy

$$\begin{aligned} &e^{ak_2}-1+\int_{R-a-\pi/2\varepsilon_2}^{R-a}\left(k_2e^{ak_2}\right)^{(p(R-a)-1)/(p(r)-1)}\left[\sin\left(\varepsilon_2(r-(R-a))+\frac{\pi}{2}\right)\right]^{1/(p(r)-1)}dr \\ &\quad=e^{ak_1}-1+\int_{-R+a}^{-R+a+\pi/2\varepsilon_1}\left(k_1e^{ak_1}\right)^{(p(-R+a)-1)/(p(r)-1)}\left[\sin\left(\frac{\pi}{2}-\varepsilon_1(r-(-R+a))\right)\right]^{1/(p(r)-1)}dr, \end{aligned} \tag{2.7}$$

k_3 and k_4 satisfy

$$\begin{aligned} &e^{ak_4}-1+\int_{R-a-\pi/2\varepsilon_4}^{R-a}\left(k_4e^{ak_4}\right)^{q(R-a)-1/q(r)-1}\left[\sin\left(\varepsilon_4(r-(R-a))+\frac{\pi}{2}\right)\right]^{1/q(r)-1}dr \\ &\quad=e^{ak_3}-1+\int_{-R+a}^{-R+a+\pi/2\varepsilon_3}\left(k_3e^{ak_3}\right)^{q(-R+a)-1/q(r)-1}\left[\sin\left(\frac{\pi}{2}-\varepsilon_3(r-(-R+a))\right)\right]^{1/q(r)-1}dr, \end{aligned} \tag{2.8}$$

then $\phi_1(x)\in C([-R,R])$, and $\phi_2(x)\in C([-R,R])$. It is easy to see that $\phi_i\geq 0$ and $\phi_i\in C^1([-R,R])$, $i=1,2$. Obviously, $\varepsilon_i=k_i^{-p^+}e^{-ak_ip^+}$ is continuous about k_i.

In the following, we will prove that (ϕ_1,ϕ_2) is a subsolution for (P_1). By computation,

$$-\Delta_{p(x)}\phi_1= \begin{cases} \left(k_2e^{-k_2(x-R)}\right)^{p(x)-1}\left[-k_2(p(x)-1)+p'(x)\ln k_2-k_2p'(x)(x-R)\right], & R-a<x\leq R, \\ \varepsilon_2\left(k_2e^{ak_2}\right)^{(p(R-a)-1)}\cos\left(\varepsilon_2(x-(R-a))+\dfrac{\pi}{2}\right), & R-\dfrac{\pi}{2\varepsilon_2}<x<R-a, \\ 0, & -R+a+\dfrac{\pi}{2\varepsilon_1}<x<R-a-\dfrac{\pi}{2\varepsilon_2}, \\ \varepsilon_1\left(k_1e^{ak_1}\right)^{p(-R+a)-1}\left[\cos\left(\dfrac{\pi}{2}-\varepsilon_1(x-(-R+a))\right)\right], & -R+a<x<-R+a, \\ \left(k_1e^{k_1(x+R)}\right)^{p(x)-1}\left[-k_1(p(x)-1)-p'(x)\ln k_1-k_1p'(x)(x+R)\right], & -R\leq x<-R+a. \end{cases} \tag{2.9}$$

If k_2 is sufficiently large, we have

$$-\Delta_{p(x)}\phi_1\leq -k_2\left[\inf p(x)-1-\sup\left|p'(x)\right|\left(\frac{\ln k_2}{k_2}+R-r\right)\right]\leq -k_2a,\quad \forall x\in(R-a,R). \tag{2.10}$$

As a is a constant and only depends on $p(x)$ and $q(x)$, when k_2 is large enough, we have $-k_2a<2b$. Since $\phi_1(x)\geq 0$ and $f+h$ is monotone, we have

$$-\Delta_{p(x)}\phi_1\leq 2b\leq f(0)+h(0)\leq f(\phi_2)+h(\phi_1),\quad R-a<x\leq R. \tag{2.11}$$

According to (H_2), when k_i are large enough, we have

$$f\left(e^{ak_i}-1\right)\geq 1,\quad g\left(e^{ak_i}-1\right)\geq 1,\quad h\left(e^{ak_i}-1\right)\geq 1,\quad \omega\left(e^{ak_i}-1\right)\geq 1,\quad i=1,2,3,4, \tag{2.12}$$

where k_i are dependent on f, g, h, ω, p and q, and they are independent on R. Since $\varepsilon_2 = k_2^{-p^+}e^{-ak_2p^+}$, when $x\in(R-a-\pi/2\varepsilon, R-a)$, we have

$$-\Delta_{p(x)}\phi_1=\varepsilon_2\left(k_2e^{ak_2}\right)^{(p(R-1)-1)}\cos\left(\varepsilon(x-(R-a))+\frac{\pi}{2}\right)\leq\varepsilon_2k_2^{p^+}e^{ak_2p^+}=1. \tag{2.13}$$

Then we have

$$-\Delta_{p(x)}\phi_1\leq 1\leq f(\phi_2)+h(\phi_1),\qquad R-a-\frac{\pi}{2\varepsilon_2}<x<R-a. \tag{2.14}$$

Obviously

$$-\Delta_{p(x)}\phi_1=0\leq 1\leq f(\phi_2)+h(\phi_1),\qquad -R+a+\frac{\pi}{2\varepsilon_1}<x<R-a-\frac{\pi}{2\varepsilon_2}. \tag{2.15}$$

When k_2 is large enough, from (2.7) we can see that k_1 is large enough. Similar to the discussion of the above, we can conclude

$$-\Delta_{p(x)}\phi_1\leq 1\leq f(\phi_2)+h(\phi_1),\qquad -R+a<x<-R+a+\frac{\pi}{2\varepsilon_1}, \tag{2.16}$$

$$-\Delta_{p(x)}\phi_1\leq f(0)+h(0)\leq f(\phi_2)+h(\phi_1),\qquad -R<x<-R+a. \tag{2.17}$$

Since $\phi_i(x)\in C^1([-R,R])$, combining (2.11), (2.14), (2.15), (2.16) and (2.17), we have

$$-\Delta_{p(x)}\phi_1\leq f(\phi_2)+h(\phi_1),\quad \text{for a.e. } x\in(-R,R). \tag{2.18}$$

Similarly, when k_4 is large enough, we have

$$-\Delta_{q(x)}\phi_2\leq g(\phi_1)+\omega(\phi_2),\quad \text{for a.e. } x\in(-R,R). \tag{2.19}$$

Then (ϕ_1,ϕ_2) is a subsolution of (P_1).

Step 2. We will construct a supersolution of (P_1).

Let z_1 be a solution of

$$-\left(\left|z_1'\right|^{p(x)-2}z_1'\right)'=2\mu, z_1(R)=0=z_1(-R), \tag{2.20}$$

where μ is a positive constant and $\mu>1$.

Obviously, there exists $x_0 \in \Omega_R$ such that $z_1(x) = \int_x^R |(r-x_0)2\mu|^{1/(p(r)-1)-1} 2\mu(r-x_0)dr$. Note that x_0 is dependent on μ. Denote $\beta = \beta(2\mu) = \max_{|x|\le R} z_1(x)$. It is easy to see that

$$\frac{1}{C}\mu^{1/p^+-1} \le \beta(2\mu) \le C\mu^{1/p^--1}, \quad \text{where } C \ge 1 \text{ is a positive constant.} \tag{2.21}$$

Let us consider

$$\begin{gathered} -\triangle_{p(x)} z_1 = 2\mu \quad \text{in } \Omega_R, \\ -\triangle_{q(x)} z_2 = 2g(\beta(2\mu)) \quad \text{in } \Omega_R, \\ z_1 = z_2 = 0 \quad \text{on } \partial\Omega_R. \end{gathered} \tag{2.22}$$

Similarly, we have

$$\frac{1}{C}[g(\beta(2\mu))]^{1/(q^+-1)} \le \max_{|x|\le R} z_2(x) \le C[g(\beta(2\mu))]^{1/(q^--1)}. \tag{2.23}$$

We will prove that (z_1, z_2) is a supersolution for (P_1). From $\lim_{t\to+\infty}\omega(t)/t^{q^--1} = 0$ and (2.23), when μ is large enough, we can easily see that

$$-\Delta_{q(x)} z_2 = 2g(\beta(2\mu)) \ge g(z_1) + \omega(z_2). \tag{2.24}$$

Since $\lim_{t\to+\infty} f[Cg(2t)^{1/(q^--1)}]/t^{p^--1} = 0$ and $\lim_{t\to+\infty} h(t)/t^{p^--1} = 0$, when μ is large enough, according to (2.21) and (2.23), we have

$$2\mu \ge 2\left(\frac{1}{C}\beta(2\mu)\right)^{p^--1} \ge f\left[C(g(\beta(2\mu)))^{1/(q^--1)}\right] + h((\beta(2\mu))). \tag{2.25}$$

This means that

$$-\Delta_{p(x)} z_1 = 2\mu \ge f\left[C(g(\beta(2\mu)))^{1/(q^--1)}\right] + h((\beta(2\mu))) \ge f(z_2) + h(z_1). \tag{2.26}$$

According to (2.24) and (2.26), we can conclude that (z_1, z_2) is a supersolution for (P_1), when μ is large enough.

Step 3. We will prove that $\phi_1 \le z_1$ and $\phi_2 \le z_2$.

Obviously, when μ is large enough, we can easily see that $g(\beta(2\mu))$ is large enough, then

$$\begin{gathered} f(\phi_2) + h(\phi_1) \le \mu, \quad \forall x \in \Omega_R, \\ g(\phi_1) + \omega(\phi_2) \le g(\beta(2\mu)), \quad \forall x \in \Omega_R. \end{gathered} \tag{2.27}$$

Let us consider

$$-\Delta_{p(x)}\varpi = \mu \text{ in } \Omega_R. \tag{2.28}$$

It is easy to see that ϕ_1 is a subsolution of (2.28), when μ is large enough. Obviously, we can see that z_1 is a supersolution of (2.28), and

$$z_1(R) = \phi_1(R) = z_1(-R) = \phi_1(-R) = 0. \tag{2.29}$$

According to the comparison principle (see [12]), we can see that $\phi_1 \le z_1$.

Let us consider

$$-\Delta_{p(x)}\varpi = g\big(\beta(2\mu)\big) \text{ in } \Omega_R. \tag{2.30}$$

It is easy to see that ϕ_2 is a subsolution of (2.30), when μ is large enough. Obviously, we can see that z_2 is a supersolution of (2.30), and

$$z_2(R) = \phi_2(R) = z_2(-R) = \phi_2(-R) = 0. \tag{2.31}$$

According to the comparison principle (see [12]), we can see that $\phi_2 \le z_2$.

Thus, we can conclude that $\phi_1 \le z_1$ and $\phi_2 \le z_2$, when μ is sufficiently large. This completes the proof. □

Theorem 2.2. *If (H_1)–(H_5) hold, then (P) has infinitely many periodic solutions.*

Proof. Let $R = nT$. According to Theorem 2.1, we can conclude that there exists an integer n_0 which is large enough such that (P_1) has a positive solution $(u_n^\#(x), v_n^\#(x))$ for any integer $n \ge n_0$. Since p and q are even, and $f, g, h,$ and ω are odd, then $(-u_n^\#(-x), -v_n^\#(-x))$ is a negative solution of (P_1). We can define a C^1 function $(u_n(x), v_n(x))$ on $[-nT, 3nT]$ as

$$\begin{aligned} u_n(x) &= \begin{cases} u_n^\#(x), & x \in [-nT, nT], \\ -u_n^\#(-(x-2nT)), & x \in (nT, 3nT], \end{cases} \\ v_n(x) &= \begin{cases} v_n^\#(x), & x \in [-nT, nT], \\ -v_n^\#(-(x-2nT)), & x \in (nT, 3nT]. \end{cases} \end{aligned} \tag{2.32}$$

We extend $(u_n(x), v_n(x))$ as $(u_n(x), v_n(x)) = (u_n(x + m4nT), v_n(x + m4nT))$, where m is an integer such that $x + m4nT \in [-nT, 3nT]$. It is easy to see that $u_n, v_n \in C^1(\mathbb{R})$, $(u_n(x), v_n(x))$ is a solution of (P), and the periodic of $(u_n(x), v_n(x))$ is $4nT$. This completes the proof. □

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