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Research Article

Delay-Dependent Guaranteed Cost H_∞ Control of an Interval System with Interval Time-Varying Delay

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This paper concerns the problem of the delay-dependent robust stability and guaranteed cost H_∞ control for an interval system with time-varying delay. The interval system with matrix factorization is provided and leads to less conservative conclusions than solving a square root. The time-varying delay is assumed to belong to an interval and the derivative of the interval time-varying delay is not a restriction , which allows a fast time-varying delay; also its applicability is broad. Based on the Lyapunov-Ktasovskii approach, a delay-dependent criterion for the existence of a state feedback controller, which guarantees the closed-loop system stability, the upper bound of cost function, and disturbance attenuation lever for all admissible uncertainties as well as out perturbation, is proposed in terms of linear matrix inequalities (LMIs). The criterion is derived by free weighting matrices that can reduce the conservatism. The effectiveness has been verified in a number example and the compute results are presented to validate the proposed design method.

1. Introduction

Recently, stability analysis, and control synthesis of the uncertainty interval systems and the time-delay system have been discussed extensively [1–7]. Literature [4, 5] has proposed a design method for a specific structure of single-input interval system, it has further been developed in [6], it has proposed a solution technique of an interval system stability and control synthesis by using a Riccati equation, but for the parameter matrix, it has constraint conditions of full column rank when the control input matrix is the interval matrix and matrix factorization requires the solving a square root. Time-delay is generally a source of instability in practical engineering systems; considerable attention has been paid to the problem of stability analysis and controller synthesis for time-delay systems. The guaranteed cost-control approach aims at stabilizing the systems while maintaining an adequate level of performance represented by the quadratic cost [8–10]. Fragility is a common dynamic problem and is caused by many factors; reduction in size and cost of digital control hardware results in limitations in available computer memory and word length capabilities of the digital processor [11–15]. Literature [7] studies the guaranteed cost control of the interval system but does not consider the time delay and the fragility or the results presented by the proposed M_- matrix conditions. The existing approaches are all limited and conservative, and the proposed Riccati equation algorithm cannot be guaranteed to be convergent [8]. Very little open literature covering the research guaranteed cost control of an interval system with time delay has been published and the fragility problem has not been considered.

The H_∞ control is an effective way for dealing with the disturbance uncertainty [16]. Since the delay-dependent results are less conservative than the delay-independent ones, especially when the delay time is small, it is necessary to discuss the delay-dependent guaranteed cost H_∞ control for interval time-varying delay systems. Recently, a special type of time delay in practical engineering systems, that is, interval time-varying delay, was identified and investigated [17–20]. A typical example of systems with interval time-varying delay is networked control systems (NCSs). Employing the Lyapunov-Ktasovskii approach, literature [19–21] requires both the upper bound of the time-varying delay and additional information on the derivative of the time-varying delay, while literature [17, 18, 22, 23] has no restriction on the derivative of the time varying delay, which allows a fast time-varying delay. Moreover, the general model transformation and bounding technique may be the sources of conservatism results; to further improve the performance of delay-dependent stability criteria, much effort has been devoted recently to the development of the free weighting matrices method [24], in which neither the bounding technique nor model transformation is employed.

In this paper, we present a new method of dealing with the problem of the delay-dependent stability and guaranteed cost H_∞ control of interval system with interval time-varying delay based on the LMIs [25]. This method employs Lyapunov-Krasovskii functional and free weighting matrices approaches, which are used to reduce the conservative result, guaranteeing that the closed-loop system gives a better dynamic performance.

2. Problem Formulation

Consider uncertainty interval system with time-varying delay

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - h(t)) + Bu(t) + B_1 w(t), \\ Z(t) &= Cx(t) + Du(t), \\ x(t) &= \varphi(t), \quad t \in [-h_M, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$, $u \in R^p$, and $w \in R^w$ denote the state vector, the control vector, and the disturbance input, respectively, and $z(t) \in R^z$ is the controlled output. The time-varying delay $h(t)$ is a time-varying continuous function satisfying

$$0 \le h_m \le h(t) \le h_M, \tag{2.2}$$

where h_m, h_M are known constants. Moreover, $h_M > 0$ and the initial condition $\varphi(t)$ denotes a continuous vector-valued initial function of $t \in [-h_M, 0]$. A is the state matrix. B is the input matrix, B_1 is the disturbance matrix, and C and D are appropriate dimension constant matrices.

Interval matrix $A, A_d \in R^{n\times n}$ and $B \in R^{n\times p}$ vary with the parameters and can be shown as [3, 10]

$$\begin{gathered} A \in \left[A^m, A^M\right] = \left\{ \left[a_{ij}\right], a_{ij}^m < a_{ij} < a_{ij}^M, i, j = 1 \cdots n \right\}, \\ B \in \left[B^m, B^M\right] = \left\{ \left[b_{ij}\right] : b_{ij}^m < b_{ij} < b_{ij}^M, i = 1 \cdots n, j = 1 \cdots p \right\}, \end{gathered} \tag{2.3}$$

where

$$\begin{gathered} A^m = \left[a_{ij}^m\right]_{n\times n}, \quad A^M = \left[a_{ij}^M\right]_{n\times n} \text{ satisfying } a_{ij}^m < a_{ij}^M, \\ B^m = \left[b_{ij}^m\right]_{n\times p}, \quad B^M = \left[b_{ij}^M\right]_{n\times p} \text{ satisfying } b_{ij}^m < b_{ij}^M. \end{gathered} \tag{2.4}$$

Let

$$\begin{gathered} A_0 := \frac{A^M + A^m}{2}, \qquad \overline{A}_{ij} := \frac{A^M - A^m}{2} : \left[\overline{a}_{ij}\right], \\ B_0 := \frac{B^M + B^m}{2}, \qquad \overline{B}_{ij} := \frac{B^M - B^m}{2} : \left[\overline{b}_{ij}\right], \end{gathered} \tag{2.5}$$

where A^m, A^M, B^m, and B^M are known real matrices; $\overline{A}_{ij}$ denotes that the i, jth component is $\overline{a}_{ij}$ with all other entries being zeros and can be factorized into $\overline{a}_{ij} e_i \times {e_j}^T$. A and B can be described as an equivalent form:

$$\begin{gathered} A = A_0 + \Delta A = A_0 + \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{ij}\overline{A}_{ij} = A_0 + \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{ij}\overline{a}_{ij} e_i \times {e_j}^T = A_0 + D_1 F_1 E_1, \\ A_d = A_{d0} + \Delta A_d = A_{d0} + \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij}\overline{A}_{dij} = A_{d0} + \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij}\overline{a}_{dij} e_i \times {e_j}^T = A_{d0} + D_d F_d E_d, \\ B = B_0 + \Delta B = B_0 + \sum_{i=1}^{n}\sum_{j=1}^{p} \beta_{ij}\overline{B}_{ij} = B_0 + \sum_{i=1}^{n}\sum_{j=1}^{p} \beta_{ij}\overline{b}_{ij} e_i \times {e_j}^T = B_0 + D_2 F_2 E_2, \end{gathered} \tag{2.6}$$

where $|\lambda_{ij}| \leq 1$, $i, j = 1 \cdots n$, $|\alpha_{ij}| \leq 1$, $i, j = 1 \cdots n$, $|\beta_{ij}| \leq 1$, $i = 1 \cdots n, j = 1 \cdots p$, and uncertainty matrices satisfy

$$F_1^T F_1 \leq I_{n^2}, \quad F_d^T(t) F_d(t) \leq I_{n^2}, \quad F_2^T F_2 \leq I_{(n\times p)\times(n\times p)}. \tag{2.7}$$

The actual control input implemented is assumed to be $u = (K_{p\times n} + \Delta K)x = \widehat{K}x$, where D_k, E_k are known constant dimension matrices and $F_k(t)$ satisfies

$$F_k^{\mathrm{T}} F_k \leq \mu I_{(p\times n)\times(p\times n)}, \tag{2.8}$$

where $\mu > 0$ is controller gain perturbation uncertainty bound.

The cost function associated with this system is

$$J = \int_0^{\infty} \left[x^T(t) R_1 x(t) + u^T(t) R_2 u(t)\right] dt, \tag{2.9}$$

where $R_1 > 0, R_2 > 0$ are given weighting matrices.

Substituting (2.1) into (2.10) the resulting closed-loop system is

$$\begin{aligned} \dot{x}(t) = {} & [(A_0 + D_1 F_1(t) E_1) + (B_0 + D_2 F_2(t) E_2)(K + D_k F_k(t) E_k)] x(t) \\ & + (A_{d0} + D_d F_d(t) E_d) x(t - h(t)) + B_1 w(t). \end{aligned} \tag{2.10}$$

Our controllers design objective is described as follows.

The closed loop system (2.10) is asymptotically stable with disturbance attenuation γ, nonfragility μ, if the following is fulfilled for all time-varying delay and admissible uncertainties satisfying (2.7) and (2.8).

(1) The closed close system (2.10) is asymptotically stable.

(2) The closed loop system (2.10) guarantees, under aeroinitial conditions,

$$\|z(t)\|_2 < \gamma \|w(t)\|_2 \tag{2.11}$$

for all nonzero $w(t) \in L_2[0, \infty]$.

(3) The closed-loop cost function satisfies $J \leq J^*$.

Therefore, the objective of this paper is to design a nonfragile guaranteed cost H_∞ controller in the presence of time-varying delay, time-varying parameter uncertainty of an interval system, and uncertainty of the controller. Also the controller guarantees disturbance attenuation of the closed loop system from $w(t)$ to $z(t)$.

Lemma 2.1 (see [26]). *Let Y_1, M, N, and ψ be matrices of appropriate dimensions and assume ψ is symmetric, satisfying $\psi^T \psi \leq I$, then*

$$Y_1 + M\psi N + N^T \psi^T M^T < 0 \tag{2.12}$$

if and only if there exists a scalar $\varepsilon > 0$ satisfying

$$Y_1 + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0. \tag{2.13}$$

3. Main Results

Defining $h_a = (h_M + h_m)/2$, $\delta = (h_M - h_m)/2$, where h_a and δ can be taken as the mean value and range of variation of the time-varying delay. First consider a delay-dependent stability for the following nominal system of (2.1) $(w(t) \equiv 0)$:

$$\dot{x}(t) = A_0 x(t) + A_{d0} x(t - h(t)) + B_0 u(t). \tag{3.1}$$

Using the Leibniz-Newton formula, we can write

$$x(t - h_a) - x(t - h(t)) = \int_{t-h(x)}^{t-h_a} \dot{x}(s) ds \tag{3.2}$$

and system (3.1) can be rewritten as

$$\dot{x}(t) = \left(A_0 + B_0 \widehat{K}\right) x(t) + A_{d0} x(t - h_a) - A_{d0} \int_{t-h(x)}^{t-h_a} \dot{x}(s) ds. \tag{3.3}$$

The following theorem presents a sufficient condition for the existence of the nonfragile guaranteed cost controller.

Theorem 3.1. *A control law $u = \widehat{K}x$ is said to be a non-fragile guaranteed cost control associated with cost matrix $P, Q, R, S > 0$ and N_i $(i = 1, 2)$ of appropriate dimensions for the system* (3.1) *and cost function* (2.9) *and given scalars h_m and h_M. Suppose that the disturbance input is zero for all times $(w(t) \equiv 0)$, if the following matrix inequality*

$$\Gamma = \begin{bmatrix} \Delta_1 & PA_{d0} - N_1^T + N_2 & -h_a N_1^T & \delta P A_{d0} & h_a \left(A_0 + B_0 \widehat{K}\right)^T R & \delta \left(A_0 + B_0 \widehat{K}\right)^T S \\ * & -Q - N_2^T - N_2 & -h_a N_2^T & 0 & h_a A_{d0}^T R & \delta A_{d0}^T S \\ * & * & -h_a R & 0 & 0 & 0 \\ * & * & * & -\delta S & \delta h_a A_{d0}^T R & \delta^2 A_{d0}^T S \\ * & * & * & * & -h_a R & 0 \\ * & * & * & * & * & -\delta S \end{bmatrix} < 0, \tag{3.4}$$

where

$$\begin{aligned} \Delta_1 = {} & Q + P\left(A_0 + B_0 \widehat{K}\right) + \left(A_0 + B_0 \widehat{K}\right)^T P + \left(A_0 + B_0 \widehat{K}\right)^T (h_a R + \delta S) \left(A_0 + B_0 \widehat{K}\right) \\ & + \left(R_1 + \widehat{K}^T R_2 \widehat{K}\right) + N_1^T + N_1 \end{aligned} \tag{3.5}$$

holds for all admissible uncertainty (2.7), (2.8), *and any* $\tau(t)$ *satisfying* (2.2). *The closed-loop cost function satisfies*

$$J \leq J^* = x_0^T P x_0 + \int_{-h_a}^{0} \varphi^T(t) Q \varphi(t) dt + \int_{-h_a}^{0} d\beta \int_{\beta}^{0} \dot{\varphi}^T(t) R \dot{\varphi}(t) dt + \int_{-h_a}^{-h_m} d\beta \int_{\beta}^{0} \dot{\varphi}^T(t) S \dot{\varphi}(t) dt. \tag{3.6}$$

Proof. Choose a Lyapunov function as

$$\begin{aligned} V(x(t)) &= V_1(x(t)) + V_2(x(t)), \\ V_1(x(t)) &= x^{\mathrm{T}}(t) P x(t) + \int_{t-h_a}^{t} x^{\mathrm{T}}(\alpha) Q x(\alpha) \mathrm{d}\alpha + \int_{-h_a}^{0} \mathrm{d}\beta \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\theta) R \dot{x}(\theta) d\theta, \\ V_2(x(t)) &= \begin{cases} \displaystyle\int_{-h_a}^{-h_m} \mathrm{d}\beta \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta, & h_m \leq h(t) < h_a, \\ 0 & h(t) < h_a, \\ \displaystyle\int_{-h_M}^{-h_a} \mathrm{d}\beta \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta, & h_m < h(t) \leq h_M, \end{cases} \end{aligned} \tag{3.7}$$

where $P > 0, Q > 0, R > 0$, and $S > 0$.

Case 1 ($h_m \leq h(t) \leq h_a$). Taking the time derivative of $V(\cdot)$ along any trajectory of the closed-loop system (3.3) is given by

$$\begin{aligned} \dot{V}(x(t)) = {} & x^T(t) \left[P\left(A + B\widehat{K}\right) + \left(A + B\widehat{K}\right)^T P \right] x(t) + x^T(t) Q x(t) + 2x^T(t) P A_{d0}(t) x(t - h_a) \\ & - 2x^T(t) P A_{d0} \int_{t-h(x)}^{t-h_a} \dot{x}(s) ds - x^T(t - h_a) Q x(t - h_a) + \dot{x}^T(t)(h_a R + \delta S)\dot{x}(t) \\ & - \int_{t-h_a}^{t} \dot{x}^T(s) R \dot{x}(s) ds - \int_{t-h_a}^{t-h_m} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta. \end{aligned} \tag{3.8}$$

It is easy to see that

$$-\int_{t-h_a}^{t-h_m} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta \leq -\int_{t-h_a}^{t-h(t)} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta. \tag{3.9}$$

According to (3.2), for any matrices $N_i,\ i = 1, 2$, with appropriate dimensions, the following equations hold:

$$2\left[x^T(t) N_1^T + x^T(t - h_a) N_2^T\right] \times \left[x(t) - x(t - h_d) - \int_{t-h_a}^{t} \dot{x}(s) ds\right] = 0. \tag{3.10}$$

Then we have

$$\begin{aligned}\dot{V}(x(t)) \le{}& x^T(t)\left[P\left(A+B\widehat{K}\right)+\left(A+B\widehat{K}\right)^T P+Q\right]x(t)+2x^T(t)PA_{d0}(t)x(t-h_a)\\&-2x^T(t)PA_{d0}\int_{t-h(t)}^{t-h_a}\dot{x}(s)ds-x^T(t-h_a)Qx(t-h_a)+\dot{x}^T(t)(h_aR+\delta S)\dot{x}(t)\\&-\int_{t-h_a}^{t}\dot{x}^T(s)R\dot{x}(s)ds-\int_{t-h_a}^{t-h(t)}\dot{x}^{\mathrm{T}}(\theta)S\dot{x}(\theta)d\theta+2\left[x^T(t)N_1^T+x^T(t-h_a)N_2^T\right]\\&\times\left[x(t)-x(t-h_d)-\int_{t-h_a}^{t}\dot{x}(s)ds\right]\le\frac{1}{h_a}\int_{t-h_a}^{t}\eta^T(t,s)\Phi_1\eta(t,s)ds\\&+\frac{1}{h_a-h(t)}\int_{t-h_a}^{t-h(t)}\eta^T(t,s)\Phi_2\eta(t,s)ds,\end{aligned}\tag{3.11}$$

where

$$\eta^T(t,s)=\left[x^T(t)x^T(t-h_a)\dot{x}^T(s)\right],$$

$$\Phi_1=\begin{bmatrix}\mathfrak{A} & PA_{d0}+\left(A_0+B_0\widehat{K}\right)^T(h_aR+\delta S)A_{d0}-N_1^T+N_2+Z_{12} & -h_aN_1^T\\ * & -Q+A_{d0}^T(h_aR+\delta S)A_{d0}-N_2^T-N_2+Z_{22} & -h_aN_2^T\\ * & * & -h_aR\end{bmatrix},$$
$$\Phi_2=\begin{bmatrix}-Z_{11} & -Z_{12} & \delta\left[PA_{d0}+\left(A_0+B_0\widehat{K}\right)^T(h_aR+\delta S)A_{d0}\right]\\ * & -Z_{22} & \delta A_{d0}^T(h_aR+\delta S)A_{d0}\\ * & * & \delta^2A_{d0}^T(h_aR+\delta S)A_{d0}-\delta S\end{bmatrix},\tag{3.12}$$

where$\mathfrak{A}$ denotes $Q+P(A_0+B_0\widehat{K})+(A_0+B_0\widehat{K})^TP+(A_0+B_0\widehat{K})^T(h_aR+\delta S)(A_0+B_0\widehat{K})+N_1^T+N_1+Z_{11}$. With $Z_{11}>0, Z_{22}>0, Z_{12}$ are some parameter matrices of appropriate dimensions. The asymptotic stability is achieved if $\Phi_1<0$ and $\Phi_2<0$ which is equivalent. From Schur complement, (3.4) holds if and only if $\Phi_1<0$ and $\Phi_2<0$ simultaneously.

From (3.11), we have

$$\dot{V}(x(t))+x^T(t)\left(R_1+\widehat{K}^TR_2\widehat{K}\right)x(t)\le\eta^T(t,s)\Gamma\eta(t,s).\tag{3.13}$$

According to inequality (3.4), $\Gamma<0$ implies $\Phi_1<0$ and $\Phi_2<0$, then

$$\dot{V}(x(t))<-x^{\mathrm{T}}(t)R_1+\left((K+\Delta K)^{\mathrm{T}}R_2(K+\Delta K)\right)x(t)<0\tag{3.14}$$

can be obtained so that

$$\dot{V}(x(t)) < -\left[x(t)^T R_1 x(t) + u(t)^T R_2 u(t)\right] < 0. \tag{3.15}$$

Therefore, the closed-loop system (3.1) is asymptotically stable. Furthermore, by integrating both sides of the above inequality from 0 to T and using the initial condition,

$$\begin{aligned} J &= \int_0^\infty \left[x^T(t) R_1 x(t) + u^T(t) R_2 u(t)\right] dt \\ &\leq -\int_0^\infty \dot{V}(x(t)) dt = V(x(0)) \leq x_0^T P x_0 + \int_{-h_a}^0 \varphi^T(t) Q \varphi(t) dt \\ &\quad + \int_{-h_a}^0 \mathrm{d}\beta \int_\beta^0 \dot{\varphi}^{\mathrm{T}}(t) R \dot{\varphi}(t) dt + \int_{-h_a}^{-h_m} \mathrm{d}\beta \int_\beta^0 \dot{\varphi}^{\mathrm{T}}(t) S \dot{\varphi}(t) dt \end{aligned} \tag{3.16}$$

is obtained. Thus, Theorem 3.1 is true.

Case 2 ($h(t) = h_a$). Similar to the above analysis, it is easy to see that $\Phi_1, \Phi_2$are the leading minor of obtained (3.11) and (3.13), respectively. Therefore, system (3.1) is asymptotically stable if and onlyif (3.4) holds. The closed-loop cost function satisfies (3.6).

Case 3 ($h_m \leq h(t) \leq h_M$). Similar to Case 1, one can prove that system (3.1) is asymptotically stable. □

The objective of this paper is to develop a procedure for determining a state feedback gain matrix $\widehat{K}$which contains controller gain perturbation such that the control law $u = \widehat{K}x$ is a non-fragile guaranteed cost H_∞control of the system (2.1), cost function (2.9), and disturbance attenuation γ.

Theorem 3.2. *A control law $u = \widehat{K}x$ is said to be a non-fragile guaranteed cost H_∞ control associated with cost matrix $P, Q, R, S > 0$, and N_i $(i = 1, 2)$ of appropriate dimensions for the system* (2.1) *and cost function* (2.9)*, disturbance attenuation $\gamma > 0$, and given scalars h_m and h_M, if the following matrix inequality*

$$\Sigma = \begin{bmatrix} \Delta_1 & PA_{d0} - N_1^T + N_2 & -h_a N_1^T & \delta P A_{d0} & h_a \mathfrak{B} R & \delta \mathfrak{B} S & \left(C + D\widehat{K}\right)^T & PB_1 \\ * & -Q - N_2^T - N_2 & -h_a N_2^T & 0 & h_a A_{d0}^T R & \delta A_{d0}^T S & 0 & 0 \\ * & * & -h_a R & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta S & \delta h_a A_{d0}^T R & \delta^2 A_{d0}^T S & 0 & 0 \\ * & * & * & * & -h_a R & 0 & 0 & 0 \\ * & * & * & * & * & -\delta S & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{3.17}$$

where $\mathfrak{B}$ *denotes* $(A_0 + B_0\widehat{K})^T$, *and*

$$\Delta_1 = Q + P\left(A_0 + B_0\widehat{K}\right) + \left(A_0 + B_0\widehat{K}\right)^T P + \left(R_1 + \widehat{K}^T R_2 \widehat{K}\right) + N_1^T + N_1 \tag{3.18}$$

holds for all admissible uncertainty (2.7) *and* (2.8). *The closed-loop cost function satisfies* (3.6).

Proof. It has been noticed that (3.17) implies (3.4). Therefore (3.17) ensures the asymptotic stability of the closed loop system (2.1). Under zero initial condition $x(t) = 0, t \in [-h_M, 0]$,

$$J = \int_0^\infty \left[z(t)^T z(t) - \gamma^2 w(t)^T w(t)\right] dt \tag{3.19}$$

can be introduced. Then for any nonzero $w(t) \in L_2[0,\infty]$,

$$J \le \int_0^\infty \left[z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \dot{V}x(t)\right] dt = \int_0^\infty \phi(t)^T \psi \phi(t) dt, \tag{3.20}$$

where $\phi(t) = \left[\, \eta(t)^T \quad w(t)^T \,\right]$ and $\psi = \Sigma$. The condition of $\psi < 0$ implies $w(t) \in L_2[0,\infty]$. Therefore when $\psi < 0$, the closed loop system (2.1) is asymptotically stable with disturbance attenuation $\gamma > 0$ and nonfragility $\mu > 0$. □

Theorem 3.3. *There exist non-fragile guaranteed cost* H_∞ *controllers for the system* (2.1) *and the cost function* (2.9), *disturbance attenuation* $\gamma > 0$, h_m, h_M, *and* $\mu > 0$, $\alpha > 0$, *if there exists scalars* $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\rho_1 > 0$, $\rho_2 > 0$, *symmetric positive matrices* $U, \widetilde{S}, \widetilde{Q}$ *and,* $\widetilde{N}_i$ $(i = 1, 2)$ *of appropriate dimensions and a matrix* W*such that*

$$\begin{bmatrix}
\Delta_{11} & \Delta_{12} & -h_a\widetilde{N}_1^T & \delta A_{d0}\widetilde{S} & h_a\mathfrak{C} & \delta\mathfrak{C} & W^T & \mathfrak{D} & B_0D_k & D_d & UE_k^T & 0 & D_1 & D_2 & UE_1^T & W^TE_2^T & U & B_1 \\
\ast & \Delta_{22} & -h_a\widetilde{N}_2 & 0 & h_aUA_{d0}^T & \delta UA_{d0}^T & 0 & 0 & 0 & 0 & 0 & UE_d^T & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & -\alpha h_aU & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & -\delta\widetilde{S} & \delta h_a\widetilde{S}A_{d0}^T & \delta^2\widetilde{S}A_{d0}^T & 0 & 0 & 0 & 0 & 0 & \delta^{-1}\widetilde{S}E_d^T & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & -\alpha^{-1}h_aU & 0 & 0 & 0 & h_aB_0D_k & h_aD_d & 0 & 0 & h_aD_1 & h_aD_2 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & -\delta\widetilde{S} & 0 & 0 & \delta B_0D_k & \delta D_d & 0 & 0 & \delta D_1 & \delta D_2 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & -R_2^{-1} & 0 & D_k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & -I & DD_k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\rho_1^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & D_k^TE_2^T & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\rho_2^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\mu^{-1}\rho_1I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\rho_2I & 0 & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_1^{-1}I & 0 & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_2^{-1}I & 0 & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_1I & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\varepsilon_2I & 0 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -R_1^{-1} & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & \ast & -\gamma^2I
\end{bmatrix} < 0, \tag{3.21}$$

where $\mathfrak{C}$ *denotes* $(A_0U + B_0W)^T$, $\mathfrak{D}$ *denotes* $(CU + DW)^T$, *and*

$$\begin{aligned}\Delta_{11} &= \widetilde{Q} + (A_0U + B_0W) + (A_0U + B_0W)^T + \widetilde{N}_1^T + \widetilde{N}_1,\\ \Delta_{12} &= A_{d0}U - \widetilde{N}_1^T + \widetilde{N}_2,\\ \Delta_{22} &= -\widetilde{Q} - \widetilde{N}_2^T - \widetilde{N}_2.\end{aligned} \tag{3.22}$$

Furthermore, if $(\varepsilon_i, \rho_i, U, \widetilde{S}, \widetilde{N}_i, W)$, $i = 1,2$ *is a feasible solution to the inequality* (3.21)*, then* $u = \widehat{K}x$ *is a non-fragile guaranteed cost* H_∞ *controller of the system* (2.1)*, where the feedback gain matrix* $\widehat{K}$ *is given by* $\widehat{K} = WU^{-1}$*and the corresponding closed-loop cost function satisfies* (3.6)*.*

Proof. Let $A_{d0} = A_{d0} + D_dF_dE_d, \widehat{K} = K + D_kF_kE_k$.

By manipulating the left-hand side in inequality (3.17), it follows that the inequality (3.17) is equivalent to

$$\Upsilon_1 + \Sigma_1 + \Sigma_1^{\mathrm{T}} + \Sigma_2 + \Sigma_2^{\mathrm{T}} < 0, \tag{3.23}$$

where

$$\Upsilon_1 = \begin{bmatrix} \widetilde{\Delta}_1 & PA_{d0} - N_1^T + N_2 & -h_aN_1^T & \delta PA_{d0} & h_a\mathfrak{C}R & \delta\mathfrak{C}S & K^T & (C+DK)^T & PB_1 \\ * & -Q - N_2^T - N_2 & -h_aN_2^T & 0 & h_aA_{d0}^TR & \delta A_{d0}^TS & 0 & 0 & 0 \\ * & * & -h_aR & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta S & \delta h_aA_{d0}^TR & \delta^2A_{d0}^TS & 0 & 0 & 0 \\ * & * & * & * & -h_aR & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\delta S & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_2^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\gamma^2I \end{bmatrix},$$

$$\widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0K) + \mathfrak{C}P + N_1^T + N_1,$$

$$\Sigma_1 = \begin{bmatrix} PB_0D_k \\ 0 \\ 0 \\ 0 \\ h_aR^TB_0D_k \\ \delta S^TB_0D_k \\ D_k \\ DD_k \\ 0 \end{bmatrix} F_k\left[E_k \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right], \qquad \Sigma_2 = \begin{bmatrix} PD_d \\ 0 \\ 0 \\ 0 \\ h_aR^TD_d \\ \delta S^TD_d \\ 0 \\ 0 \\ 0 \end{bmatrix} F_d\left[0\ E_d\ 0\ \delta E_d\ 0\ 0\ 0\ 0\ 0\right], \tag{3.24}$$

where $\mathfrak{C}$ denotes $(A_0 + B_0K)^T$.

By applying Lemma 2.1, the above inequality (3.23) holds for all F_k, F_d, satisfying $F_k(t)F_k^{\mathrm{T}}(t) < \mu I$, $F_d {F_d}^{\mathrm{T}} \le I_{n^2}$ if and only if there exists a constant $\rho_1 > 0$, $\rho_2 > 0$ such that

$$\Upsilon_1 + \rho_1 \begin{bmatrix} PB_0D_k \\ 0 \\ 0 \\ 0 \\ h_aR^TB_0D_k \\ \delta S^TB_0D_k \\ D_k \\ DD_k \\ 0 \end{bmatrix} \begin{bmatrix} PB_0D_k \\ 0 \\ 0 \\ 0 \\ h_aR^TB_0D_k \\ \delta S^TB_0D_k \\ D_k \\ DD_k \\ 0 \end{bmatrix}^T + \mu\rho_1^{-1}[E_k\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T[E_k\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

$$+\rho_2 \begin{bmatrix} PD_d \\ 0 \\ 0 \\ 0 \\ h_aR^TD_d \\ \delta S^TD_d \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} PD_d \\ 0 \\ 0 \\ 0 \\ h_aR^TD_d \\ \delta S^TD_d \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \rho_2^{-1}[0\ E_d\ 0\ \delta E_d\ 0\ 0\ 0\ 0\ 0]^T[0\ E_d\ 0\ \delta E_d\ 0\ 0\ 0\ 0\ 0] < \mathbf{0} \tag{3.25}$$

It follows from the Schur complement that the above inequality is further equivalent to the following inequality:

$$\begin{bmatrix} \tilde{\Delta}_1 & PA_{d0} - N_1^T + N_2 & -h_aN_1^T & \delta PA_{d0} & h_a\mathfrak{E}R & \delta\mathfrak{E}S & K^T & \mathfrak{F} & PB_0D_k & PD_d & E_k^T & 0 & PB_1 \\ * & -Q - N_2^T - N_2 & -h_aN_2^T & 0 & h_aA_{d0}^TR & \delta A_{d0}^TS & 0 & 0 & 0 & 0 & 0 & E_d^T & 0 \\ * & * & -h_aR & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta S & \delta h_aA_{d0}^TR & \delta^2A_{d0}^TS & 0 & 0 & 0 & 0 & 0 & \delta^{-1}E_d^T & 0 \\ * & * & * & * & -h_aR & 0 & 0 & 0 & h_aR^TB_0D_k & h_aR^TD_d & 0 & 0 & 0 \\ * & * & * & * & * & -\delta S & 0 & 0 & \delta S^TB_0D_k & \delta S^TD_d & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_2^{-1} & 0 & D_k & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & DD_k & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\rho_1^{-1}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\rho_2^{-1}I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\mu^{-1}\rho_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\rho_2 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$< 0, \tag{3.26}$$

where $\mathfrak{F}$ denotes $(C + DK)^T$, and

$$\widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0K) + (A_0 + B_0K)^T P + N_1^T + N_1. \tag{3.27}$$

Let $A_0 = A_0 + D_1F_1E_1$, $B_0 = B_0 + D_2F_2E_2$. By manipulating the left-hand side in inequality (3.17) again, it follows that the above inequality is equivalent to

$$\Upsilon_2 + \Xi_1 + \Xi_1^T + \Xi_2 + \Xi_2^T < 0, \tag{3.28}$$

where

$$\Upsilon_2 = \begin{bmatrix} \widetilde{\Delta}_1 & PA_{d0} - N_1^T + N_2 & -h_a N_1^T & \delta PA_{d0} & h_a \mathfrak{E} R & \delta \mathfrak{E} S & K^T & \mathfrak{F} & PB_0D_k & PD_d & E_k^T & 0 & PB_1 \\ * & -Q - N_2^T - N_2 & -h_a N_2^T & 0 & h_a A_{d0}^T R & \delta A_{d0}^T S & 0 & 0 & 0 & 0 & 0 & E_d^T & 0 \\ * & * & -h_a R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta S & \delta h_a A_{d0}^T R & \delta^2 A_{d0}^T S & 0 & 0 & 0 & 0 & 0 & \delta^{-1} E_d^T & 0 \\ * & * & * & * & -h_a R & 0 & 0 & 0 & h_a R^T B_0 D_k & h_a R^T D_d & 0 & 0 & 0 \\ * & * & * & * & * & -\delta S & 0 & 0 & \delta S^T B_0 D_k & \delta S^T D_d & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_2^{-1} & 0 & D_k & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & DD_k & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\rho_1^{-1} I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\rho_2^{-1} I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\mu^{-1}\rho_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\rho_2 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}, \tag{3.29}$$

and

$$\begin{gathered} \widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0K) + (A_0 + B_0K)^T P + N_1^T + N_1, \\ \Xi_1 = \begin{bmatrix} PD_1 \\ 0 \\ 0 \\ 0 \\ h_a R^T D_1 \\ \delta S^T D_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_1 [E_1 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0], \qquad \Xi_2 = \begin{bmatrix} PD_2 \\ 0 \\ 0 \\ 0 \\ h_a R^T D_2 \\ \delta S^T D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_2 [E_2K \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, 0 \;\, E_2D_K \;\, 0 \;\, 0]. \end{gathered} \tag{3.30}$$

Applying Lemma 2.1 again, the inequality (3.28) holds for all F_1, F_2 satisfying $F_1^T(t)F_1(t) \leq I_{n^2}$, $F_2^T(t)F_2(t) \leq I_{n\times p}$, if and only if there exists a constant $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that

$$\begin{aligned}
&\Upsilon_2 + \varepsilon_1 \begin{bmatrix} PD_1 \\ 0 \\ 0 \\ 0 \\ h_a R^T D_1 \\ \delta S^T D_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} PD_1 \\ 0 \\ 0 \\ 0 \\ h_a R^T D_1 \\ \delta S^T D_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\
&\quad + \varepsilon_1^{-1} \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\quad + \varepsilon_2 \begin{bmatrix} PD_2 \\ 0 \\ 0 \\ 0 \\ h_a R^T D_2 \\ \delta S^T D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} PD_2 \\ 0 \\ 0 \\ 0 \\ h_a R^T D_2 \\ \delta S^T D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\
&\quad + \varepsilon_2^{-1} \begin{bmatrix} E_2 K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_2 D_K & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_2 K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_2 D_K & 0 & 0 \end{bmatrix} \\
&\quad < 0.
\end{aligned} \tag{3.31}$$

It follows from the Schur complement again that the above inequality is further equivalent to the following inequality:

$$\begin{bmatrix}
\tilde{\Delta}_1 & PA_{d0}-N_1^T+N_2 & -h_aN_1^T & \delta PA_{d0} & h_a\mathfrak{E}R & \delta\mathfrak{E}S & K^T & \mathfrak{F} & PB_0D_k & PD_d & E_k^T & 0 & PD_1 & PD_2 & E_1^T & K^TE_2^T & PB_1 \\
* & -Q-N_2^T-N_2 & -h_aN_2^T & 0 & h_aA_{d0}^TR & \delta A_{d0}^TS & 0 & 0 & 0 & 0 & 0 & E_d^T & 0 & 0 & 0 & 0 & 0 \\
* & * & -h_aR & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\delta S & \delta h_aA_{d0}^TR & \delta^2A_{d0}^TS & 0 & 0 & 0 & 0 & 0 & \delta^{-1}E_d^T & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -h_aR & 0 & 0 & 0 & h_aRB_0D_k & h_aRD_d & 0 & 0 & h_aRD_1 & h_aRD_2 & 0 & 0 & 0 \\
* & * & * & * & * & -\delta S & 0 & 0 & \delta SB_0D_k & \delta SD_d & 0 & 0 & \delta SD_1 & \delta SD_2 & 0 & 0 & 0 \\
* & * & * & * & * & * & -R_2^{-1} & 0 & D_k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -I & DD_k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\rho_1^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & D_k^TE_2^T & 0 \\
* & * & * & * & * & * & * & * & * & -\rho_2^{-1}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\mu^{-1}\rho_1I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\rho_2I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_1^{-1}I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2^{-1}I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_1I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2I
\end{bmatrix} < 0, \tag{3.32}$$

and

$$\tilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0K) + (A_0 + B_0K)^{\mathrm{T}}P + N_1^T + N_1. \tag{3.33}$$

By pre- and postmultiplying the left-hand side matrix in the above inequality by the matrix $\operatorname{diag}\{P^{-1}\ \ P^{-1}\ \ P^{-1}\ \ S^{-1}\ \ R^{-1}\ \ I\ \ \cdots\ \ P^{-1}\ \ I\}$, respectively, and defining the matrix $U = P^{-1}$, $W = KU$, $\widetilde{N}_1 = UN_1U$, $\widetilde{N}_2 = UN_2U$, $\tilde{Q} = UQU$ and, $\tilde{S} = S^{-1}$. If we set $R = \alpha U^{-1}$, then $URU = \alpha U$, and $R^{-1} = \alpha^{-1}U$, and it can be concluded that the above matrix inequality is equivalent to (3.21). The proof is complete. □

4. A Numerical Example

Consider system (2.1) with [17]

$$A = \begin{bmatrix} 4 & 0.1 & -0.3 \\ -0.2 & 3 & -0.2 \\ 0.2 & -0.3 & 2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.4 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 0.2 & 0 \\ 0 & 3 & 1 \\ 0.1 & 0 & 3 \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \tag{4.1}$$

$$E = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad E_b = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

and $1.2 \leq h(t) \leq 1.8$. By applying Theorem 3.3 with $\alpha = 0.16$ and $\gamma = 1$, the controller gain is obtained:

$$K = \begin{bmatrix} 2.5536 & -0.1039 & -0.2143 \\ 0.1895 & -2.5017 & 1.1060 \\ -0.1513 & 0.4258 & -2.4936 \end{bmatrix}, \tag{4.2}$$

where it is also a result $\varepsilon_1 = 3, \varepsilon_2 = 4, \rho_1 = 5, \rho_2 = 6$:

$$U = \begin{bmatrix} 0.2911 & 0.0213 & 0.0045 \\ 0.0213 & 0.3233 & 0.0392 \\ 0.0045 & 0.0392 & 0.351 \end{bmatrix}, \quad W = \begin{bmatrix} 0.6318 & 0.0036 & -0.0402 \\ 0.0036 & -0.7133 & -0.0251 \\ -0.0402 & -0.0251 & -0.5452 \end{bmatrix}, \quad \widetilde{S} = 8. \tag{4.3}$$

For $h_m = 1.2$, the maximum allowable upper bound of the delay is $h_M = 3.0679$ which is larger than $h_M = 1.8467$ derived in Jiang and Han in [17]. This means that any $h(t)$ satisfyies $1.2 \leq h(t) \leq 3.0679$.

The associated upper bound over the closed-loop cost function is $J^* = 24.5368$.

The obtained robust and non-fragile optimal guaranteed cost H_∞controller guarantees the asymptotic stability, disturbance attenuation γ, $\|z(t)\|_2 < 0.385\|w(t)\|_2$, and the upper bound of cost function J^*.

5. Conclusion

This paper has considered the problem of delay-dependent stability and guaranteed cost H_∞ control with interval time-varying delay for an interval system based on Lyapunov–Krasovskii functional approach. The delay-dependent stabilization criterion for guaranteed cost H_∞ control has been formulated in terms of LMIs. The derivative of the interval time-varying delay is not a restriction, which allows a fast time-varying delay and also is more close to practices control object. A numerical example has shown the effectiveness of the method.

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