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Research Article

Subclasses of Meromorphic Functions Associated with Convolution

Maisarah Haji Mohd,[1] Rosihan M. Ali,[1] Lee See Keong,[1] and V. Ravichandran[2]

[1] *School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia*
[2] *Department of Mathematics, University of Delhi, Delhi 110 007, India*

Correspondence should be addressed to Lee See Keong, sklee@cs.usm.my

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Several subclasses of meromorphic functions in the unit disk are introduced by means of convolution with a given fixed meromorphic function. Subjecting each convoluted-derived function in the class to be subordinated to a given normalized convex function with positive real part, these subclasses extend the classical subclasses of meromorphic starlikeness, convexity, close-to-convexity, and quasi-convexity. Class relations, as well as inclusion and convolution properties of these subclasses, are investigated.

1. Introduction

Let $\mathcal{H}$ be the set of all analytic functions defined in the unit disk $U = \{z : |z| < 1\}$. We denote by $\mathcal{A}$ the class of normalized analytic functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ defined in U. For two functions f and g analytic in U, the function f is subordinate to g, written as

$$f(z) \prec g(z), \tag{1.1}$$

if there exists a Schwarz function w, analytic in U with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$. In particular, if the function g is univalent in U, then $f(z) \prec g(z)$ is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$.

A function $f \in \mathcal{A}$ is starlike if $zf'(z)/f(z)$ is subordinate to $(1+z)/(1-z)$ and convex if $1+zf''(z)/f'(z)$ is subordinate to $(1+z)/(1-z)$. Ma and Minda [1] gave a unified presentation of these classes and introduced the classes

$$S^*(h) = \left\{ f \in \mathcal{A} \;\middle|\; \frac{zf'(z)}{f(z)} \prec h(z) \right\},$$
$$C(h) = \left\{ f \in \mathcal{A} \;\middle|\; 1 + \frac{zf''(z)}{f'(z)} \prec h(z) \right\}, \tag{1.2}$$

where h is an analytic function with positive real part, $h(0) = 1$, and h maps the unit disk U onto a region starlike with respect to 1.

The convolution or the Hadamard product of two analytic functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ is given by

$$(f * g)(z) = \sum_{n=1}^{\infty} a_n b_n z^n. \tag{1.3}$$

In term of convolution, a function f is starlike if $f * (z/(1-z))$ is starlike, and convex if $f * (z/(1-z)^2)$ is starlike. These ideas led to the study of the class of all functions f such that $f * g$ is starlike for some fixed function g in $\mathcal{A}$. In this direction, Shanmugam [2] introduced and investigated various subclasses of analytic functions by using the convex hull method [3–5] and the method of differential subordination. Ravichandran [6] introduced certain classes of analytic functions with respect to n-ply symmetric points, conjugate points, and symmetric conjugate points, and also discussed their convolution properties. Some other related studies were also made in [7–9], and more recently by Shamani et al. [10].

Let $\mathcal{M}$ denote the class of meromorphic functions f of the form

$$f(z) = \frac{1}{z} + \sum_{n=0}^{\infty} a_n z^n, \tag{1.4}$$

which are analytic and univalent in the punctured unit disk $U^* = \{z : 0 < |z| < 1\}$. For $0 \le \alpha < 1$, we recall that the classes of meromorphic starlike, meromorphic convex, meromorphic close-to-convex, meromorphic γ-convex (Mocanu sense), and meromorphic quasi-convex functions of order α, denoted by $\mathcal{M}^s$, $\mathcal{M}^k$, $\mathcal{M}^c$, $\mathcal{M}^k_\gamma$, and $\mathcal{M}^q$, respectively, are defined by

$$\mathcal{M}^s = \left\{ f \in \mathcal{M} \;\middle|\; -\Re \frac{zf'(z)}{f(z)} > \alpha \right\},$$
$$\mathcal{M}^k = \left\{ f \in \mathcal{M} \;\middle|\; -\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) > \alpha \right\},$$
$$\mathcal{M}^c = \left\{ f \in \mathcal{M} \;\middle|\; -\Re \frac{zf'(z)}{g(z)} > \alpha,\ g \in \mathcal{M}^s \right\}, \tag{1.5}$$
$$\mathcal{M}^k_\gamma = \left\{ f \in \mathcal{M} \;\middle|\; -\Re \left[(1-\gamma)\frac{zf'(z)}{f(z)} + \gamma \left(1 + \frac{zf''(z)}{f'(z)} \right) \right] > \alpha \right\},$$
$$\mathcal{M}^q = \left\{ f \in \mathcal{M} \;\middle|\; -\Re \frac{[zf'(z)]'}{g'(z)} > \alpha,\ g \in \mathcal{M}^k \right\}.$$

The convolution of two meromorphic functions f and g, where f is given by (1.4) and $g(z) = 1/z + \sum_{n=0}^{\infty} b_n z^n$, is given by

$$(f * g)(z) = \frac{1}{z} + \sum_{n=0}^{\infty} a_n b_n z^n. \tag{1.6}$$

Motivated by the investigation of Shanmugam [2], Ravichandran [6], and Ali et al. [7, 11], several subclasses of meromorphic functions defined by means of convolution with a given fixed meromorphic function are introduced in Section 2. These new subclasses extend the classical classes of meromorphic starlike, convex, close-to-convex, γ-convex, and quasi-convex functions given in (1.5). Section 3 is devoted to the investigation of the class relations as well as inclusion and convolution properties of these newly defined classes.

We will need the following definition and results to prove our main results.

Let $S^*(\alpha)$ denote the class of starlike functions of order α. The class R_α of prestarlike functions of order α is defined by

$$R_\alpha = \left\{ f \in \mathcal{A} \,\middle|\, f * \frac{z}{(1-z)^{2-2\alpha}} \in S^*(\alpha) \right\} \tag{1.7}$$

for $\alpha < 1$, and

$$R_1 = \left\{ f \in \mathcal{A} \,\middle|\, \Re \frac{f(z)}{z} > \frac{1}{2} \right\}. \tag{1.8}$$

Theorem 1.1 (see [12, Theorem 2.4]). *Let $\alpha \leq 1$, $f \in \mathcal{R}_\alpha$, and $g \in S^*(\alpha)$. Then, for any analytic function $H \in \mathcal{H}(U)$,*

$$\frac{f * Hg}{f * g}(U) \subset \overline{\mathrm{co}}(H(U)), \tag{1.9}$$

where $\overline{\mathrm{co}}(H(U))$ denotes the closed convex hull of $H(U)$.

Theorem 1.2 (see [13]). *Let h be convex in U and $\beta, \gamma \in \mathbb{C}$ with $\Re(\beta h(z) + \gamma) > 0$. If p is analytic in U with $p(0) = h(0)$, then*

$$p(z) + \frac{zp'(z)}{\beta p(z) + \gamma} \prec h(z) \quad \textit{implies } p(z) \prec h(z). \tag{1.10}$$

2. Definitions

In this section, various subclasses of $\mathcal{M}$ are defined by means of convolution and subordination. Let g be a fixed function in $\mathcal{M}$, and let h be a convex univalent function with positive real part in U and $h(0) = 1$.

Definition 2.1. The class $\mathcal{M}_g^s(h)$ consists of functions $f \in \mathcal{M}$ satisfying $(g*f)(z) \neq 0$ in U^* and the subordination

$$-\frac{z(g*f)'(z)}{(g*f)(z)} \prec h(z). \tag{2.1}$$

Remark 2.2. If $g(z) = 1/z + 1/(1-z)$, then $\mathcal{M}_g^s(h)$ coincides with $\mathcal{M}^s(h)$, where

$$\mathcal{M}^s(h) = \left\{ f \in \mathcal{M} \;\middle|\; -\frac{zf'(z)}{f(z)} \prec h(z) \right\}. \tag{2.2}$$

Definition 2.3. The class $\mathcal{M}_g^k(h)$ consists of functions $f \in \mathcal{M}$ satisfying $(g*f)'(z) \neq 0$ in U^* and the subordination

$$-\left\{1 + \frac{z(g*f)''(z)}{(g*f)'(z)}\right\} \prec h(z). \tag{2.3}$$

Definition 2.4. The class $\mathcal{M}_g^c(h)$ consists of functions $f \in \mathcal{M}$ such that $(g*\psi)(z) \neq 0$ in U^* for some $\psi \in \mathcal{M}_g^s(h)$ and satisfying the subordination

$$-\frac{z(g*f)'(z)}{(g*\psi)(z)} \prec h(z). \tag{2.4}$$

Definition 2.5. For γ real, the class $\mathcal{M}_{g,\gamma}^k(h)$ consists of functions $f \in \mathcal{M}$ satisfying $(g*f)(z) \neq 0$, $(g*f)'(z) \neq 0$ in U^* and the subordination

$$-\left\{\gamma\left(1 + \frac{z(g*f)''(z)}{(g*f)'(z)}\right) + (1-\gamma)\left(\frac{z(g*f)'(z)}{(g*f)(z)}\right)\right\} \prec h(z). \tag{2.5}$$

Definition 2.6. The class $\mathcal{M}_g^q(h)$ consists of functions $f \in \mathcal{M}$ such that $(g*\varphi)'(z) \neq 0$ in U^* for some $\varphi \in \mathcal{M}_g^k(h)$ and satisfying the subordination

$$\frac{[-z(g*f)'(z)]'}{(g*\varphi)'(z)} \prec h(z). \tag{2.6}$$

3. Main Results

This section is devoted to the investigation of class relations as well as inclusion and convolution properties of the new subclasses given in Section 2.

Theorem 3.1. *Let h be a convex univalent function satisfying $\Re h(z) < 2-\alpha$, $0 \leq \alpha < 1$, and $g \in \mathcal{M}$ with $z^2 g \in R_\alpha$. If $f \in \mathcal{M}^s(h)$, then $f \in \mathcal{M}_g^s(h)$. Equivalently, if $f \in \mathcal{M}^s(h)$, then $g*f \in \mathcal{M}^s(h)$.*

Proof. Define the function F by

$$F(z) = -\frac{zf'(z)}{f(z)}. \tag{3.1}$$

For $f \in \mathcal{M}^s(h)$, it follows that

$$-\Re\left\{\frac{zf'(z)}{f(z)}\right\} < 2-\alpha, \tag{3.2}$$

and therefore,

$$\Re\left\{\frac{z(z^2f)'(z)}{z^2f(z)}\right\} > \alpha. \tag{3.3}$$

Hence $z^2 f \in S^*(\alpha)$. A computation shows that

$$-\frac{z(g*f)'(z)}{(g*f)(z)} = \frac{(g*-zf')(z)}{(g*f)(z)} = \frac{(g*fF)(z)}{(g*f)(z)} = \frac{(z^2g*z^2fF)(z)}{(z^2g*z^2f)(z)}. \tag{3.4}$$

Theorem 1.1 yields

$$-\frac{z(g*f)'(z)}{(g*f)(z)} = \frac{(z^2g*z^2fF)(z)}{(z^2g*z^2f)(z)} \in \overline{\mathrm{co}}(F(U)), \tag{3.5}$$

and because $F(z) \prec h(z)$, it follows that

$$-\frac{z(g*f)'(z)}{(g*f)(z)} \prec h(z). \tag{3.6}$$

□

Theorem 3.2. *The function $f \in \mathcal{M}_g^k(h)$ if and only if $-zf' \in \mathcal{M}_g^s(h)$.*

Proof. The results follow from the equivalence relations

$$-\left(1+\frac{z(g*f)''(z)}{(g*f)'(z)}\right) \prec h(z) \Longleftrightarrow -\frac{(z(g*f)'(z))'}{(g*f)'(z)} \prec h(z) \Longleftrightarrow -\frac{z(g*-zf')'(z)}{(g*-zf')(z)} \prec h(z). \tag{3.7}$$

□

Theorem 3.3. *Let h be a convex univalent function satisfying $\Re h(z) < 2-\alpha$, $0 \le \alpha < 1$, and $\phi \in \mathcal{M}$ with $z^2\phi \in R_\alpha$. If $f \in \mathcal{M}_g^s(h)$, then $\phi * f \in \mathcal{M}_g^s(h)$.*

Proof. Since $f \in \mathcal{M}_g^s(h)$, it follows that

$$-\Re\left\{\frac{z(g*f)'(z)}{(g*f)(z)}\right\} < 2-\alpha, \tag{3.8}$$

and thus

$$\Re\left\{\frac{z\left(z^2(g*f)\right)'(z)}{z^2(g*f)(z)}\right\} > \alpha. \tag{3.9}$$

Let

$$P(z) = -\frac{z(g*f)'(z)}{(g*f)(z)}. \tag{3.10}$$

A similar computation as in the proof of Theorem 3.1 yields

$$-\frac{z(\phi*g*f)'(z)}{(\phi*g*f)(z)} = \frac{z^2\phi(z)*z^2(g*f)(z)P(z)}{z^2\phi(z)*z^2(g*f)(z)}. \tag{3.11}$$

Inequality (3.9) shows that $z^2(g*f) \in S^*(\alpha)$. Therefore Theorem 1.1 yields

$$-\frac{z(\phi*g*f)'(z)}{(\phi*g*f)(z)} \prec h(z), \tag{3.12}$$

hence $\phi * f \in \mathcal{M}_g^s(h)$. □

Corollary 3.4. *$\mathcal{M}_g^s(h) \subset \mathcal{M}_{\phi*g}^s(h)$ under the conditions of Theorem 3.3.*

Proof. The proof follows from (3.12). □

In particular, when $g(z) = 1/z + 1/(1-z)$, the following corollary is obtained.

Corollary 3.5. *Let h and ϕ satisfy the conditions of Theorem 3.3. If $f \in \mathcal{M}^s(h)$, then $f \in \mathcal{M}_\phi^s(h)$.*

Theorem 3.6. *Let h and ϕ satisfy the conditions of Theorem 3.3. If $f \in \mathcal{M}_g^k(h)$, then $\phi * f \in \mathcal{M}_g^k(h)$. Equivalently $\mathcal{M}_g^k(h) \subset \mathcal{M}_{\phi*g}^k(h)$.*

Proof. If $f \in \mathcal{M}_g^k(h)$, it follows from Theorem 3.2 that $-zf' \in \mathcal{M}_g^s(h)$. Theorem 3.3 shows that $\phi * (-zf') = -z(\phi * f)' \in \mathcal{M}_g^s(h)$. Hence $\phi * f \in \mathcal{M}_g^k(h)$. □

Theorem 3.7. *Under the conditions of Theorem 3.3, if $f \in \mathcal{M}_g^c(h)$ with respect to $\psi \in \mathcal{M}_g^s(h)$, then $\phi * f \in \mathcal{M}_g^c(h)$ with respect to $\phi * \psi \in \mathcal{M}_g^s(h)$.*

Proof. Theorem 3.3 shows that $\phi * \psi \in \mathcal{M}_g^s(h)$. Since $\psi \in \mathcal{M}_g^s(h)$, (3.9) yields $z^2(g*\psi) \in S^*(\alpha)$.

Let the function G be defined by

$$G(z) = -\frac{z(g * f)'(z)}{(g * \psi)(z)}. \tag{3.13}$$

A similar computation as in the proof of Theorem 3.1 yields

$$-\frac{z(\phi * g * f)'(z)}{(\phi * g * \psi)(z)} = \frac{z^2\phi(z) * z^2(g * \psi)(z)G(z)}{z^2\phi(z) * z^2(g * \psi)(z)}. \tag{3.14}$$

Since $z^2\phi \in R_\alpha$ and $z^2(g * \psi) \in S^*(\alpha)$, it follows from Theorem 1.1 that

$$-\frac{z(\phi * g * f)'(z)}{(\phi * g * \psi)(z)} \prec h(z). \tag{3.15}$$

Thus $\phi * f \in \mathcal{M}_g^c(h)$ with respect to $\phi * \psi$. □

Corollary 3.8. *$\mathcal{M}_g^c(h) \subset \mathcal{M}_{\phi * g}^c(h)$ under the assumptions of Theorem 3.3.*

Proof. The subordination (3.15) shows that $f \in \mathcal{M}_{\phi * g}^c(h)$. □

Theorem 3.9. *Let $\Re(\gamma h(z)) < 0$. Then*

(i) $\mathcal{M}_{g,\gamma}^k(h) \subset \mathcal{M}_g^s(h)$,

(ii) $\mathcal{M}_{g,\gamma}^k(h) \subset \mathcal{M}_{g,\beta}^k(h)$ *for* $\gamma < \beta \le 0$.

Proof. Define the function P by

$$P(z) = -\frac{z(g * f)'(z)}{(g * f)(z)} \tag{3.16}$$

and the function $J_g(\gamma; f)$ by

$$J_g(\gamma; f)(z) = -\left\{\gamma\left(1 + \frac{z(g * f)''(z)}{(g * f)'(z)}\right) + (1 - \gamma)\left(\frac{z(g * f)'(z)}{(g * f)(z)}\right)\right\}. \tag{3.17}$$

For $f \in \mathcal{M}_{g,\gamma}^k(h)$, it follows that $J_g(\gamma; f)(z) \prec h(z)$. Note also that

$$J_g(\gamma; f)(z) = P(z) - \frac{\gamma z P'(z)}{P(z)}. \tag{3.18}$$

(i) Since $\Re(\gamma h(z)) < 0$ and

$$P(z) - \frac{\gamma z P'(z)}{P(z)} \prec h(z). \tag{3.19}$$

Theorem 1.2 yields $P(z) \prec h(z)$. Hence $f \in \mathcal{M}_g^s(h)$.

(ii) Observe that

$$
\begin{aligned}
J_g(\beta; f)(z) &= -\left\{\beta\left(1 + \frac{z(g*f)''(z)}{(g*f)'(z)}\right) + (1-\beta)\left(\frac{z(g*f)'(z)}{(g*f)(z)}\right)\right\} \\
&= \left(1 - \frac{\beta}{\gamma}\right)P(z) + \frac{\beta}{\gamma}J_g(\gamma; f)(z).
\end{aligned}
\tag{3.20}
$$

Furthermore $J_g(\gamma; f)(z) \prec h(z)$ and $P(z) \prec h(z)$ from (i). Since $0 < \beta/\gamma < 1$ and $h(U)$ is convex, we deduce that $J_g(\beta; f)(z) \in h(U)$. Therefore, $J_g(\beta; f)(z) \prec h(z)$. □

Corollary 3.10. *The class $\mathcal{M}_g^k(h)$ is a subset of the class $\mathcal{M}_g^q(h)$.*

Proof. The proof follows from the definition of the classes by taking $f = \varphi$. □

Theorem 3.11. *The function $f \in \mathcal{M}_g^q(h)$ if and only if $-zf' \in \mathcal{M}_g^c(h)$.*

Proof. If $f \in \mathcal{M}_g^q(h)$, then there exists $\varphi \in \mathcal{M}_g^k(h)$ such that

$$
\frac{\left[-z(g*f)'(z)\right]'}{(g*\varphi)'(z)} \prec h(z). \tag{3.21}
$$

Also,

$$
\frac{-z(g*-zf')'(z)}{(g*-z\varphi')(z)} = \frac{\left[-z(g*f)'(z)\right]'}{(g*\varphi)'(z)} \prec h(z). \tag{3.22}
$$

Since $\varphi \in \mathcal{M}_g^k(h)$, by Theorem 3.2 , $-z\varphi' \in \mathcal{M}_g^s(h)$. Hence $-zf' \in \mathcal{M}_g^c(h)$.

Conversely, if $-zf' \in \mathcal{M}_g^c(h)$, then

$$
-\frac{z(g*-zf')'(z)}{(g*\varphi_1)(z)} \prec h(z) \tag{3.23}
$$

for some $\varphi_1 \in \mathcal{M}_g^s(h)$. Let $\varphi \in \mathcal{M}_g^k(h)$ be such that $-z\varphi' = \varphi_1 \in \mathcal{M}_g^s(h)$. The proof is completed by observing that

$$
\frac{\left[-z(g*f)'(z)\right]'}{(g*\varphi)'(z)} = -\frac{z(g*-zf')'(z)}{(g*-z\varphi')(z)} \prec h(z). \tag{3.24}
$$

□

Corollary 3.12. *Let h and ϕ satisfy the conditions of Theorem 3.3. If $f \in \mathcal{M}_g^q(h)$, then $\phi * f \in \mathcal{M}_g^q(h)$.*

Proof. If $f \in \mathcal{M}_g^q(h)$, Theorem 3.11 gives $-zf' \in \mathcal{M}_g^c(h)$. Theorem 3.7 next gives $\phi * (-zf') = -z(\phi * f)' \in \mathcal{M}_g^c(h)$. Thus, Theorem 3.11 yields $\phi * f \in \mathcal{M}_g^q(h)$. □

Corollary 3.13. *$\mathcal{M}_g^q(h) \subset \mathcal{M}_{\phi * g}^q(h)$ under the conditions of Theorem 3.3.*

Proof. If $f \in \mathcal{M}_g^q(h)$, it follows from Corollary 3.12 that $\phi * f \in \mathcal{M}_g^q(h)$. The subordination

$$\frac{\left[-z(\phi * g * f)'(z)\right]'}{(\phi * g * \varphi)'(z)} \prec h(z) \tag{3.25}$$

gives $f \in \mathcal{M}_{\phi * g}^q(h)$. Therefore $\mathcal{M}_g^q(h) \subset \mathcal{M}_{\phi * g}^q(h)$. □

Open Problem

An analytic convex function in the unit disk is necessarily starlike. For the meromorphic case, is it true that $\mathcal{M}_g^k(h) \subset \mathcal{M}_g^s(h)$?

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