Dehghanian et al. *Journal of Inequalities and Applications* (2024) 2024:27
https://doi.org/10.1186/s13660-024-03104-6

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

# A Pexider system of additive functional equations in Banach algebras

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**Abstract**

In this paper, we solve the system of functional equations

$$\begin{cases} f(x+y)+g(y-x)=2f(x), \\ g(x+y)-f(y-x)=2g(y) \end{cases}$$

and we investigate the stability of $g$-derivations in Banach algebras.

**Mathematics Subject Classification:** Primary 47B47; 17B40; 39B72; 47H10; secondary 39B52

**Keywords:** Hyers–Ulam stability; $g$-derivation; Additive mapping; Fixed point method; System of functional equations

## 1 Introduction

Let $\mathcal{B}$ be a complex Banach algebra and let $g:\mathcal{B}\to\mathcal{B}$ be a $\mathbf{C}$-linear mapping. Mirzavaziri and Moslehian [1] introduced the concept of $g$-derivation $f:\mathcal{B}\to\mathcal{B}$ as follows:

$$f(xy)=f(x)g(y)+g(x)f(y) \tag{1.1}$$

for all $x,y\in\mathcal{B}$. Park *et al.* [2] introduced the concept of hom-derivation on $\mathcal{B}$, i.e., $g:\mathcal{B}\to\mathcal{B}$ is a homomorphism and $f$ satisfies (1.1) for all $x,y\in\mathcal{B}$.

The stability problem of functional equations originated from a question of Ulam [3] concerning the stability of group homomorphisms. Hyers [4] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Aoki [5] for additive mappings and by Th.M. Rassias [6] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Th.M. Rassias theorem was obtained by Găvruta [7] by replacing the unbounded Cauchy difference by a general control function in the spirit of Th.M. Rassias' approach. Recently, Lee *et al.* [8, 9] extended more general functional equations, which were mixed types of additive, quadratic and cubic functional equations in Banach spaces, and Park and Rassias [10] applied the functional equation theory to study partial multipliers in $C^*$-algebras. Many mathematicians developed the Hyers results in various directions [11–18].

The method provided by Hyers [4] which produces the additive function will be called a direct method. This method is the most significant and strongest tool to study the stability of different functional equations. That is, the exact solution of the functional equation is explicitly constructed as a limit of a sequence, starting from the given approximate solution [19, 20]. The other significant method is the fixed point theorem, that is, the exact solution of the functional equation is explicitly created as a fixed point of some certain map (see [21, 22]).

We consider a fixed point alternative theorem.

**Theorem 1.1** [23] *Assume that $(\mathcal{B}, d)$ is a complete generalized metric space and $\mathcal{I} : \mathcal{B} \to \mathcal{B}$ is a strictly contractive mapping, that is,*

$$d(\mathcal{I}u, \mathcal{I}v) \leq Ld(u, v)$$

*for all $u, v \in \mathcal{B}$ and a Lipschitz constant $L < 1$. Then for each given element $u \in \mathcal{B}$, either*

$$d\left(\mathcal{I}^n u, \mathcal{I}^{n+1} u\right) = +\infty, \quad \forall n \geq 0,$$

*or*

$$d\left(\mathcal{I}^n u, \mathcal{I}^{n+1} u\right) < +\infty, \quad \forall n \geq n_0,$$

*for some positive integer $n_0$. Furthermore, if the second alternative holds, then:*

(i) *the sequence $(\mathcal{I}^n u)$ is convergent to a fixed point $p$ of $\mathcal{I}$;*

(ii) *$p$ is the unique fixed point of $\mathcal{I}$ in the set $V := \{v \in \mathcal{B}, d(\mathcal{I}^{n_0} u, v) < +\infty\}$;*

(iii) *$d(v, p) \leq \frac{1}{1-L} d(v, \mathcal{I}v)$ for all $u, v \in V$.*

In this paper, we consider the following system of functional equations:

$$\begin{cases} f(x+y) + g(y-x) = 2f(x), \\ g(x+y) - f(y-x) = 2g(y) \end{cases} \tag{1.2}$$

for all $x, y \in \mathcal{B}$.

The aim of the present paper is to solve the system of functional equations (1.2) and prove the Hyers–Ulam stability of $g$-derivations in complex Banach algebras by using the fixed point method.

Throughout this paper, we assume that $\mathcal{B}$ is a complex Banach algebra.

## 2 Stability of the system of functional equations (1.2)

We solve and investigate the system of additive functional equations (1.2) in complex Banach algebras.

**Lemma 2.1** [24, *Theorem* 2.1] *Let $\mathcal{B}$ be a complex Banach algebra and let $\mathcal{F} : \mathcal{B} \to \mathcal{B}$ be an additive mapping such that $\mathcal{F}(\alpha x) = \alpha \mathcal{F}(x)$ for all $\alpha \in \mathbf{T}^1 := \{\zeta \in \mathbf{C} : |\zeta| = 1\}$ and all $x \in \mathcal{B}$. Then $\mathcal{F}$ is $\mathbf{C}$-linear.*

**Lemma 2.2** *Let $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying (1.2) for all $x, y \in \mathcal{B}$. Then the mappings $f, g : \mathcal{B} \to \mathcal{B}$ are additive.*

*Proof* Letting $x = y = 0$ in (1.2), we get

$$f(0) = g(0) = 0.$$

Putting $y = x$ in (1.2), we have

$$f(2x) = 2f(x)$$

for all $x \in \mathcal{B}$. Setting $y = 0$ in (1.2), we obtain

$$g(x) = f(-x) \tag{2.1}$$

for all $x \in \mathcal{B}$.

Replacing $g(y - x)$ by $f(x - y)$ in (1.2), we have

$$f(x + y) + f(x - y) = f(2x)$$

for all $x, y \in \mathcal{B}$. Hence the mapping $f : \mathcal{B} \to \mathcal{B}$ is additive and thus by (2.1) the mapping $g : \mathcal{B} \to \mathcal{B}$ is additive. □

Using the fixed point technique, we prove the Hyers–Ulam stability of the system of the additive functional equations (1.2) in complex Banach algebras.

**Theorem 2.3** *Suppose that $\Delta : \mathcal{B}^2 \to [0, \infty)$ is a function such that there exists an $L < 1$ with*

$$\Delta\left(\frac{x}{2}, \frac{y}{2}\right) \le \frac{L}{2}\Delta(x, y) \tag{2.2}$$

*for all $x, y \in \mathcal{B}$. Let $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying*

$$\begin{cases} \|f(x + y) + g(y - x) - 2f(x)\| \le \Delta(x, y), \\ \|g(x + y) - f(y - x) - 2g(y)\| \le \Delta(x, y) \end{cases} \tag{2.3}$$

*for all $x, y \in \mathcal{B}$. Then there exist unique additive mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that*

$$\|F(x) - f(x)\| \le \frac{L}{2(1 - L)}\Delta(x, x),$$

$$\|G(x) - g(x)\| \le \frac{L}{2(1 - L)}\Delta(x, x)$$

*for all $x \in \mathcal{B}$.*

*Proof* Putting $x = y = 0$ in (2.3), we get

$$\begin{cases} \|g(0) - f(0)\| \leq \Delta(0,0) = 0, \\ \|f(0) + g(0)\| \leq \Delta(0,0) = 0, \end{cases}$$

so $f(0) = g(0) = 0$.

Letting $y = x$ in (2.3), we obtain

$$\begin{cases} \|f(2x) - 2f(x)\| \leq \Delta(x,x), \\ \|g(2x) - 2g(x)\| \leq \Delta(x,x). \end{cases} \tag{2.4}$$

Let $\Gamma = \{\gamma : \mathcal{B} \to \mathcal{B} : \gamma(0) = 0\}$. We define a generalized metric $d : \Gamma \times \Gamma \to [0,\infty]$ by

$$d(\delta,\gamma) = \inf\{\mu \in \mathbf{R}_+ : \|\delta(x) - \gamma(x)\| \leq \mu\Delta(x,x), \forall x \in \mathcal{B}\}$$

and we consider $\inf\emptyset = +\infty$. Then $d$ is a complete generalized metric on $\Gamma$ (see [25]).

Now, we define the mapping $\mathcal{J} : (\Gamma,d) \to (\Gamma,d)$ such that

$$\mathcal{J}\delta(x) := 2\delta\left(\frac{x}{2}\right)$$

for all $x \in \mathcal{B}$.

Actually, let $\delta,\gamma \in (\Gamma,d)$ be given such that $d(\delta,\gamma) = \mu$. Then

$$\|\delta(x) - \gamma(x)\| \leq \mu\Delta(x,x)$$

for all $x \in \mathcal{B}$. Hence

$$\|\mathcal{J}\delta(x) - \mathcal{J}\gamma(x)\| = \left\|2\delta\left(\frac{x}{2}\right) - 2\gamma\left(\frac{x}{2}\right)\right\| \leq 2\mu\Delta\left(\frac{x}{2},\frac{x}{2}\right) \leq L\mu\Delta(x,x)$$

for all $x \in \mathcal{B}$. It follows that $d(\mathcal{J}\delta,\mathcal{J}\gamma) \leq L\mu$. So

$$d(\mathcal{J}\delta,\mathcal{J}\gamma) \leq Ld(\delta,\gamma)$$

for all $x \in \mathcal{B}$ and all $\delta,\gamma \in \Gamma$.

Using (2.4), we obtain

$$\begin{cases} \|f(x) - 2f(\frac{x}{2})\| \leq \Delta(\frac{x}{2},\frac{x}{2}) \leq \frac{L}{2}\Delta(x,x), \\ \|g(x) - 2g(\frac{x}{2})\| \leq \Delta(\frac{x}{2},\frac{x}{2}) \leq \frac{L}{2}\Delta(x,x) \end{cases}$$

for all $x \in \mathcal{B}$, which imply that $d(f,\mathcal{J}f) \leq \frac{L}{2}$ and $d(g,\mathcal{J}g) \leq \frac{L}{2}$.

Using the fixed point alternative, we deduce the existence of unique fixed points of $\mathcal{J}$, that is, the existence of mappings $F, G : \mathcal{B} \to \mathcal{B}$, respectively, such that

$$F(x) = 2F\left(\frac{x}{2}\right), \qquad G(x) = 2G\left(\frac{x}{2}\right)$$

with the following property: there exist $\mu, \eta \in (0, \infty)$ satisfying

$$\left\|f(x) - F(x)\right\| \le \mu\Delta(x,x), \qquad \left\|g(x) - G(x)\right\| \le \eta\Delta(x,x)$$

for all $x \in \mathcal{B}$.

Since $\lim_{n\to\infty} d(\mathcal{J}^n f, F) = 0$ and $\lim_{n\to\infty} d(\mathcal{J}^n g, G) = 0$,

$$\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right) = F(x), \qquad \lim_{n\to\infty} 2^n g\left(\frac{x}{2^n}\right) = G(x)$$

for all $x \in \mathcal{B}$.

Next, $d(f,F) \le \frac{1}{1-L} d(f, \mathcal{J}f)$ and $d(g,G) \le \frac{1}{1-L} d(g, \mathcal{J}g)$, which imply

$$\left\|f(x) - F(x)\right\| \le \frac{L}{2(1-L)}\Delta(x,x), \qquad \left\|g(x) - G(x)\right\| \le \frac{L}{2(1-L)}\Delta(x,x) \tag{2.5}$$

for all $x \in \mathcal{B}$.

Using (2.2) and (2.3), we conclude that

$$\begin{aligned}\left\|F(x+y) + G(y-x) - 2F(x)\right\| &= \lim_{n\to\infty} 2^n \left\|f\left(\frac{x+y}{2^n}\right) + g\left(\frac{y-x}{2^n}\right) - 2f\left(\frac{x}{2^n}\right)\right\| \\ &\le \lim_{n\to\infty} 2^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \le \lim_{n\to\infty} L^n \Delta(x,y) = 0\end{aligned}$$

and

$$\begin{aligned}\left\|G(x+y) - F(y-x) - 2G(x)\right\| &= \lim_{n\to\infty} 2^n \left\|g\left(\frac{x+y}{2^n}\right) - f\left(\frac{y-x}{2^n}\right) - 2g\left(\frac{x}{2^n}\right)\right\| \\ &\le \lim_{n\to\infty} 2^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \le \lim_{n\to\infty} L^n \Delta(x,y) = 0\end{aligned}$$

for all $x, y \in \mathcal{B}$, since $L < 1$. Hence

$$\begin{cases} F(x+y) + G(y-x) = 2F(x), \\ G(x+y) - F(y-x) = 2G(x) \end{cases}$$

for all $x, y \in \mathcal{B}$, since $L < 1$. Therefore by Lemma 2.2, the mappings $F, G : \mathcal{B} \to \mathcal{B}$ are additive. □

**Corollary 2.4** *Let $\eta$, $p$ be nonnegative real numbers with $p \ge 1$ and let $f, g : \mathcal{B} \to \mathcal{B}$ be mappings satisfying*

$$\begin{cases} \|f(x+y) + g(y-x) - 2f(x)\| \le \eta(\|x\|^p + \|y\|^p), \\ \|g(x+y) - f(y-x) - 2g(y)\| \le \eta(\|x\|^p + \|y\|^p) \end{cases}$$

*for all $x, y \in \mathcal{B}$. Then there exist unique additive mappings $F, G : \mathcal{B} \to \mathcal{B}$ such that*

$$\left\|F(x) - f(x)\right\| \le \frac{\eta}{2^{p-1} - 1}\|x\|^p,$$

$$\left\| G(x) - g(x) \right\| \leq \frac{\eta}{2^{p-1} - 1} \|x\|^p$$

*for all* $x \in \mathcal{B}$.

*Proof* The proof follows from Theorem 2.3 by taking $L = 2^{1-p}$ and $\Delta(x, y) = \eta(\|x\|^p + \|y\|^p)$ for all $x, y \in \mathcal{B}$. □

## 3 Stability of *G*-derivations in Banach algebras

In this section, by using the fixed point technique, we prove the Hyers–Ulam stability of $g$-derivations in complex Banach algebras.

**Lemma 3.1** *Let* $f, g : \mathcal{B} \to \mathcal{B}$ *be mappings satisfying*

$$\begin{cases} f(\lambda(x+y)) + g(\lambda(y-x)) = 2\lambda f(x), \\ g(\lambda(x+y)) - f(\lambda(y-x)) = 2\lambda g(y) \end{cases} \tag{3.1}$$

*for all* $x, y \in \mathcal{B}$ *and all* $\lambda \in \mathbf{T}^1$. *Then the mappings* $f, g : \mathcal{B} \to \mathcal{B}$ *are* $\mathbf{C}$-*linear.*

*Proof* If we put $\lambda = 1$ in (3.1), then $f$ and $g$ are additive by Lemma 2.2.

Letting $y = x$ in (3.1), we have

$$\begin{cases} f(2\lambda x) = 2\lambda f(x), \\ g(2\lambda x) = 2\lambda g(x) \end{cases}$$

for all $x \in \mathcal{B}$ and all $\lambda \in \mathbf{T}^1$. Since the mappings $f$ and $g$ are additive,

$$\begin{cases} f(\lambda x) = \lambda f(x), \\ g(\lambda x) = \lambda g(x) \end{cases}$$

for all $x \in \mathcal{B}$ and all $\lambda \in \mathbf{T}^1$. So by Lemma 2.1 the mappings $f$ and $g$ are $\mathbf{C}$-linear. □

**Theorem 3.2** *Suppose that* $\Delta : \mathcal{B}^2 \to [0, \infty)$ *is a function such that there exists an* $L < 1$ *with*

$$\Delta(x, y) \leq \frac{L}{4} \Delta(2x, 2y) \tag{3.2}$$

*for all* $x, y \in \mathcal{B}$. *Let* $f, g : \mathcal{B} \to \mathcal{B}$ *be mappings satisfying*

$$\begin{cases} \|f(\lambda(x+y)) + g(\lambda(y-x)) - 2\lambda f(x)\| \leq \Delta(x, y), \\ \|g(\lambda(x+y)) - f(\lambda(y-x)) - 2\lambda g(y)\| \leq \Delta(x, y), \end{cases} \tag{3.3}$$

$$\left\| f(xy) - f(x)g(y) - g(x)f(y) \right\| \leq \Delta(x, y) \tag{3.4}$$

*for all* $x, y \in \mathcal{B}$ *and all* $\lambda \in \mathbf{T}^1$. *Then there exist unique* $\mathbf{C}$-*linear mappings* $F, G : \mathcal{B} \to \mathcal{B}$ *such that* $F$ *is a* $G$-*derivation and*

$$\left\| F(x) - f(x) \right\| \leq \frac{L}{2(2-L)} \Delta(x, x), \tag{3.5}$$

$$\left\| G(x) - g(x) \right\| \leq \frac{L}{2(2-L)} \Delta(x,x) \tag{3.6}$$

*for all* $x \in \mathcal{B}$.

*Proof* Let $\lambda = 1$ in (3.3). By Theorem 2.3, there are unique additive mappings $F, G : \mathcal{B} \to \mathcal{B}$ satisfying (3.5) and (3.6) given by

$$\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right) = F(x), \qquad \lim_{n\to\infty} 2^n g\left(\frac{x}{2^n}\right) = G(x)$$

for all $x \in \mathcal{B}$.

Using (3.2) and (3.3), we conclude that

$$\begin{aligned}
&\left\| F\big(\lambda(x+y)\big) + G\big(\lambda(y-x)\big) - 2\lambda F(x) \right\| \\
&\quad = \lim_{n\to\infty} 2^n \left\| f\left(\frac{\lambda(x+y)}{2^n}\right) + g\left(\frac{\lambda(y-x)}{2^n}\right) - 2\lambda f\left(\frac{x}{2^n}\right) \right\| \\
&\quad \leq \lim_{n\to\infty} 2^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \leq \lim_{n\to\infty} L^n \Delta(x,y) = 0
\end{aligned}$$

and

$$\begin{aligned}
&\left\| G\big(\lambda(x+y)\big) - F\big(\lambda(y-x)\big) - 2\lambda G(x) \right\| \\
&\quad = \lim_{n\to\infty} 2^n \left\| g\left(\frac{\lambda(x+y)}{2^n}\right) - f\left(\frac{\lambda(y-x)}{2^n}\right) - 2\lambda g\left(\frac{x}{2^n}\right) \right\| \\
&\quad \leq \lim_{n\to\infty} 2^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \leq \lim_{n\to\infty} L^n \Delta(x,y) = 0
\end{aligned}$$

for all $x, y \in \mathcal{B}$, since $L < 1$. Hence

$$\begin{cases} F(\lambda(x+y)) + G(\lambda(y-x)) = 2\lambda F(x), \\ G(\lambda(x+y)) - F(\lambda(y-x)) = 2\lambda G(x) \end{cases}$$

for all $x, y \in \mathcal{B}$ and all $\lambda \in \mathbf{T}^1$, since $L < 1$. Therefore by Lemma 3.1, the mappings $F, G : \mathcal{B} \to \mathcal{B}$ are $\mathbf{C}$-linear.

It follows from (3.4) that

$$\begin{aligned}
&\left\| F(xy) - F(x)G(y) - G(x)F(y) \right\| \\
&\quad = \lim_{n\to\infty} 4^n \left\| f\left(\frac{xy}{4^n}\right) - f\left(\frac{x}{2^n}\right) g\left(\frac{y}{2^n}\right) - g\left(\frac{x}{2^n}\right) f\left(\frac{y}{2^n}\right) \right\| \\
&\quad \leq \lim_{n\to\infty} 4^n \Delta\left(\frac{x}{2^n}, \frac{y}{2^n}\right) \\
&\quad \leq \lim_{n\to\infty} L^n \Delta(x,y) = 0
\end{aligned}$$

for all $x, y \in \mathcal{B}$. So

$$F(xy) = F(x)G(y) + G(x)F(y)$$

for all $x, y \in \mathcal{B}$. Thus the $\mathbf{C}$-linear mapping $F$ is a $G$-derivation. □

**Corollary 3.3** *Let $p$, $q$, $\eta$ be nonnegative real numbers with $p+q>2$ and let $f,g:\mathcal{B}\to\mathcal{B}$ be mappings satisfying*

$$\begin{cases} \|f(\lambda(x+y))+g(\lambda(y-x))-2\lambda f(x)\| \le \eta\|x\|^p\|y\|^q, \\ \|g(\lambda(x+y))-f(\lambda(y-x))-2\lambda g(y)\| \le \eta\|x\|^p\|y\|^q \end{cases}$$

*and*

$$\big\|f(xy)-f(x)g(y)-g(x)f(y)\big\| \le \|x\|^p\|y\|^q$$

*for all $x,y\in\mathcal{B}$ and all $\lambda\in\mathbf{T}^1$. Then there exist unique $\mathbf{C}$-linear mappings $F,G:\mathcal{B}\to\mathcal{B}$ such that $F$ is a $G$-derivation and*

$$\big\|F(x)-f(x)\big\| \le \frac{\eta}{2^{p+q}-2}\|x\|^{p+q},$$
$$\big\|G(x)-g(x)\big\| \le \frac{\eta}{2^{p+q}-2}\|x\|^{p+q}$$

*for all $x\in\mathcal{B}$.*

*Proof* The proof follows from Theorem 3.2 by taking $\Delta(x,y)=\eta\|x\|^p\|y\|^q$ for all $x,y\in\mathcal{B}$. Choosing $L=2^{2-p-q}$, we obtain the desired result. □

## 4 Conclusion

We solved the system of functional equations (1.2) and we proved the Hyers–Ulam stability of $g$-derivations in Banach algebras.

**Acknowledgements**
We would like to express our sincere gratitude to the anonymous referee for his/her helpful comments, which helped to improve the quality of the manuscript.

**Abbreviations**
Not applicable.

**Data availability**
Not applicable.

## Declarations

**Ethics approval and consent to participate**
Not applicable.

**Consent for publication**
The authors declare that they have no competing interests.

**Competing interests**
The authors declare no competing interests.

**Author contributions**
M.D. and Y.S. wrote the main manuscript and S.P. and C.P. revised the manuscript. All authors reviewed the manuscript.

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Received: 21 June 2023 Accepted: 5 February 2024 Published online: 23 February 2024

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