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Correction to: On statistical convergence and strong Cesàro convergence by moduli

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Abstract

We correct a logic mistake in our paper "On statistical convergence and strong Cesàro convergence by moduli" (León-Saavedra et al. in J. Inequal. Appl. 23:298, 2019).

Keywords: Strong Cesàro convergence; f-density; f-statistical convergence; f-strong Cesàro convergence

1 Introduction

It has come to our attention that there is a logic mistake with the converse of some results in our paper [1]. These converse of these results are not central in the papers, but they could be interested in its own right. The next result correct Proposition 2.9 in [1].

Proposition 1.1

(a) *If all statistical convergent sequences are f-statistical convergent then f is a compatible modulus function.*

(b) *If all strong Cesàro convergent sequences are f-strong Cesàro convergent then f is a compatible modulus function.*

Proof Let ε_n be a decreasing sequence converging to 0. Since f is not compatible, there exists $c > 0$ such that, for each k, there exists m_k such that $f(m_k\varepsilon_k) > cf(m_k)$. Moreover, we can select m_k inductively satisfying

$$1 - \varepsilon_{k+1} - \frac{1}{m_{k+1}} > \frac{(1-\varepsilon_k)m_k}{m_{k+1}}. \tag{1.1}$$

Now we use an standard argument used to construct subsets with prescribed densities. Let us denote $\lfloor x \rfloor$ the integer part of $x \in \mathbb{R}$. Set $n_k = \lfloor m_k\varepsilon_k \rfloor + 1$. And extracting a subsequence if it is necessary, we can assume that $n_1 < n_2 < \cdots$, $m_1 < m_2 < \cdots$. Thus, set $A_k = [m_{k+1} - (n_{k+1} - n_k)] \cap \mathbb{N}$. Condition (1.1) guarantee that $A_k \subset [m_k, m_{k+1}]$.

Let us denote $A = \bigcup_k A_k$, and $x_n = \chi_A(n)$. Let us prove that x_n is statistical convergent to 0, but not f-statistical convergent, a contradiction. Indeed, for any m, there exists k such that $m_k < m \le m_{k+1}$. Moreover, we can suppose without loss that $m \in A$, that is,

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$m_{k+1} - n_{k+1} + n_k \leq m$. Thus for any $\varepsilon > 0$:

$$\frac{\#\{l \leq m : |x_l| > \varepsilon\}}{m} \leq \frac{\#\{l \leq m_k : |x_k| > \varepsilon\}}{m_k} + \frac{n_{k+1} - n_k}{m_{k+1} - n_{k+1} + n_k}$$
$$\leq \frac{n_k}{m_k} + \frac{1}{\frac{m_{k+1}}{n_{k+1}-n_k} - 1} \to 0$$

as $k \to \infty$. On the other hand, since $\varepsilon_{k+1} < \frac{n_{k+1}}{m_{k+1}}$

$$\frac{f(\#\{n < m_{k+1} : |x_n| > 1/2\})}{f(m_{k+1})} = \frac{f(n_{k+1})}{f(m_{k+1})} \geq \frac{f(m_{k+1}\varepsilon_{k+1})}{f(m_{k+1})} > c,$$

which yields (a) as promised. The part (b) is same proof. Indeed, for the sequence (x_n) defined in part (a), we have that $\frac{f(\sum_{k=1}^{n}|x_n|)}{f(n)} = \frac{f(\{k \leq n | x_k| > \varepsilon\})}{f(n)}$. □

The following result corrects the converse of Theorem 3.4 in [1].

Proposition 1.2 *If all f-strong Cesàro convergent sequences are f-statistically and uniformly bounded then f must be compatible.*

Proof Assume that f is not compatible. Thus, as in the proof in Proposition 1.1 we can construct sequences (ε_k), (m_k) such that $f(m_k\varepsilon_k) \geq cf(m_k)$ for some $c > 0$. Moreover, we can construct (m_k) inductively, such that the sequence

$$r_k = \frac{m_{k+1}\varepsilon_{k+1} - m_k\varepsilon_k}{m_{k+1} - m_k}$$

is decreasing and converging to 0. Let us consider $x_n = \sum_{k=0}^{\infty} r_{k+1}\chi_{(m_k,m_{k+1}]}(n)$. Since (x_n) is decreasing, (x_n) if f-statistically convergent to 0. On the other hand $f(\sum_{l=1}^{m_k}|x_l|) = f(m_k\varepsilon_k) \geq cf(m_k)$, which gives that (x_n) is not f-strong Cesàro convergent, as we desired. □

The corrections have been indicated in this article and the original article [1] has been corrected.

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